Risk Neutral Valuation of With-Profits Life Insurance Contracts

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Abstract

The valuation of life insurance contracts using concepts from financial mathematics has recently attracted considerable interest in academia as well as among practitioners. In this paper, we will investigate the valuation of participating contracts, which are characterized by embedded interest rate guarantees and some bonus distribution rules. We will model these under the specific regulatory framework in Germany; however, our analysis can be applied to any insurance market with cliquet-style guarantees.

We will present a framework, in which different kinds of guarantees or options can be analyzed separately. Also, the practical implementation of such models is discussed. We use two different numerical approaches to derive fair parameter settings of such contracts and price the embedded options.

The sensitivity of the contract value with respect to multiple parameters is studied. In particular, we find that life insurers offer interest rate guarantees below their risk-neutral value. Furthermore, the financial strength of an insurance company considerably affects the value of a contract.

Keywords: participating life insurance contracts, risk-neutral valuation, interest rate guarantees, embedded options

1 Introduction

Participating life insurance policies often contain an interest rate guarantee. In many products, this guarantee is given on a point-to-point basis, i.e. the guarantee is only relevant at maturity of the contract and not increased by bonus distribution during the term of the contract. In other products (which are predominant e.g. in the German market), there is a so-called cliquet-style guarantee. This means that the policy holders have an account to which each year a certain rate of return has to be credited. Usually, the life insurance companies provide the guaranteed rate of interest plus some surplus on the policy holders' account every year. Considering the big market share of such products in many countries, the analysis of life insurance contracts with a cliquet-style guarantee is very important. However, a comparatively large portion of the academic literature focuses on guarantees in unit-linked, equity-linked or variable life insurance contracts (e.g. Brennan and Schwartz (1976) or Aase and Persson (1994)).

The analysis of participating policies with a clique style guarantee requires a realistic model of bonus payments. For instance the approach presented in Grosen and Jørgensen (2000) explicitly models a bonus account, which permits smoothing the reserve-dependent bonus payments. Smoothing the returns is often referred to as the *"average interest principle"*. Aside from the guarantee and a distribution mechanism for excessive returns, Grosen and Jørgensen's model includes the option for the policy holder to surrender and *"walk away"*. In this case, the policy holder obtains his account value whereas the reserves remain with the company. Since the account value is path dependent, they are not able to present closed form solutions for the risk-neutral value of the liabilities. Monte Carlo methods are used for the valuation and the analysis.

Similarly, in Miltersen and Persson (2003) a cliquet-style guarantee, a bonus account and a distribution mechanism are considered. Here the return exceeding the guaranteed level is distributed between the policy holders' account, the company's account and an account for terminal bonus. If in some year the return on assets is below the guaranteed rate, the bonus account can be used to fulfil the guarantee. In particular, the bonus account can become negative, but the insurer has to consolidate a negative balance at the end of the insurance period. However, a positive balance is completely credited to the policy holders.

In Hansen and Miltersen (2002), a hybrid of the models by Miltersen and Persson (2003) and Grosen and Jørgensen (2000) is presented. They use the same model for the distribution mechanism as in Grosen and Jørgensen (2000), but the account structure from Miltersen and Persson (2003). Besides a variety of numerical results, they focus on the analysis of the *"pooling effect"*, i.e. they analyze the consequence of pooling the undistributed surplus over two inhomogeneous customers.

In Bacinello (2001) and Bacinello (2003), also cliquet-style guarantees are considered. In Bacinello (2001), she prices participating insurance contracts with a guaranteed interest rate in a Black-Scholes market model. Here, as in Miltersen and Persson (2003), the bonus is modelled as a fixed fraction of the excessive return. She finds closed form solutions for the prices of various policies. In Bacinello (2003), she additionally allows for the surrender of the policy and presents numerical results in a Cox-Ross-Rubinstein framework.

Grosen et al. (2001) introduce a different numerical approach to their valuation problem from Grosen and Jørgensen (2000) using the Black-Scholes Partial Differential Equation and arbitrage arguments. They show that the value function follows a known differential equation which can be solved by a finite difference method. This approach is extended and generalized in Tanskanen and Lukkarinen (2004). They use a discretization method in order to solve the partial differential equation. Their model permits multiple distribution mechanisms, including those considered in Miltersen and Persson (2003) and Grosen and Jørgensen (2000).

However, these models can not be used to analyze some important features of contracts in insurance markets, where accounting rules allow for building and dissolving valuation reserves which can be used to stabilize the return on book values and, thus, the surplus distribution. In this case, insurers should consider the reserve quota when deciding how much surplus is distributed. Thus, the reserve situation is of great influence on the value of an insurance contract.

The present paper fills this gap: Surplus at time t can be determined and credited depending on the development of the assets (book or market value) and any management decision rule based on information available at time t. Furthermore, minimum surplus distribution laws that exist in many countries may be considered. In particular, our model can represent all relevant features of the German market, including legal and supervisory issues as well as predominant management decision rules. On the other hand, our framework is general enough to include most of the above models and, therefore, products of other insurance markets as special cases.

We use a distribution mechanism that is typical for the German market which has been introduced in Kling et al. (2004). As opposed to their work, where the authors investigate, how the different parameters, such as the initial reserve quota, legal requirements, etc., affect the shortfall probability of a contract and how these factors interact, we are interested in the risk-neutral value of the corresponding contracts.

The rest of this paper is organized as follows. In section 2 we introduce our model and the distribution mechanisms, i.e. the rules according to which earnings are distributed among policy holders, shareholders, and the insurance company. The goal of this paper is to find a fair price for an insurance policy using methods from financial mathematics. However, certain conditions must be fulfilled in order to obtain a "meaningful" price. In section 3, we will discuss under which circumstances a risk-neutral valuation is appropriate. Since the considered insurance contracts are complex and path-dependent derivatives, it is not possible to find closed form solutions for their price. In section 4, we present a Monte Carlo Algorithm, which allows for the separate valuation of the embedded options, and an extension of discretization approach presented in Tanskanen and Lukkarinen (2004), that allows us to consider a *surrender* or *walk-away option*. Our results are presented in section 5. Besides the values of the contracts and the embedded options, we examine the influence of several parameters and give economic interpretations. Section 6 closes with a summary of the main results and an outlook for future research.

2 Model

This section introduces our model. First, we model the insurance company's balance sheet. Then, we introduce the considered insurance contract and the corresponding liabilities. Here, we refer to some specific aspects of German regulation and present two different kinds of surplus distribution schemes.

2.1 Insurance Company

We use a simplified illustration of the insurer's financial situation given in table 1.

Assets	Liabilities
A_t	L_t
	R_t
A_t	A_t

Table 1: Model of insurer's financial situation

By A_t we denote the market value of the insurer's assets at time t. The liability side comprises two entries: L_t is the time t book value of the policy holders' account; the second account is the reserve account R_t, which is given by $R_t = A_t - L_t$, and consists of asset valuation reserves as well as other components, e.g. equity.

To simplify notation, we assume that dividend payments to equity holders d_t occur once a year, at times $t = 1, 2, ..., T$. Thus, $A_t^+ = A_t^- - d_t$ denotes the value of the assets after these payments.

2.2 Insurance Contract

For the sake of simplicity, we look at a very simple life insurance contract, a single-premium term-fix insurance, and ignore any charges. The premium P is paid at $t = 0$, and a benefit is paid at time T, regardless if the policy holder is still alive or not. Thus, no mortality effects are considered. The benefit paid at time T depends on the development of the insurer's liabilities only and is given by $P\frac{L_T}{L_0}$.

Besides allowing for a focused analysis of financial guarantees, such a contract permits the nice interpretation of an insurer in a steady state: Given that the cost of insurance and the death benefit payments neutralize each other in every period, this term-fix insurance is an image of the company's general financial situation and thus the evolution of the liabilities can be thought of as the development of the insurer's liabilities as a whole.

2.3 Development of the Liabilities

The decision which surplus (i.e. interest exceeding the guaranteed rate) is given to the policy holders has to be made by the insurance company's management each year. Our general model allows for any management decision rule based on the information available at that time. In the numerical analysis, however, we will focus on two different bonus schemes. The first one only considers obligatory payments to the policy holders as required in the German market (MUST-case). Additionally, we present a surplus distribution mechanism, which closely models the behaviour of typical insurance companies in the German market over recent years (IS-case). Distinguishing these two methodologies is motivated by the different points of view of people interested in values of an insurance contract: Of course obligatory payments should be considered in any meaningful valuation. In addition, corporate political issues might be of interest for companies' actuaries, who are interested in the value of their product.

2.3.1 The MUST-Case

In what follows, we include important features of the current German regulatory and legal framework. Nevertheless, specific aspects of other countries may be considered analogously.

Under German legislation, there must be a year-by-year cliquet-style guarantee on the liabilities. Currently, German life insurance companies guarantee the policy holders a minimum rate of interest of $g = 2.75\%$[1] p.a., i.e.

$$L_t \overset{!}{\geq} L_{t-1} (1+g), \; t = 1, 2, 3, ..., T.$$

This guarantee has to be given for the whole term of the policy, even if the guaranteed rate will be changed by the regulators for new business. Thus, all policies that were sold when guaranteed rates were higher are still entitled to the guaranteed rate that prevailed when the contracts were sold (e.g. 3.25% or even 4% p.a.). Therefore, the average guaranteed interest rate over the policy-portfolio of a typical German insurer is currently about 3.5% p.a..

Furthermore, the law requires that at least $\delta = 90\%$ of the earnings on book values have to be credited to the policy holders' accounts. Hence, δ is called the minimal participation rate. Since earnings on book values are subject to accounting rules, they are not necessarily equal to earnings on the market value $A_t^- - A_{t-1}^+$. Following Kling et al. (2004), we assume that the insurer can always dissolve hidden reserves without restrictions by selling the corresponding assets. Building up reserves is, however, subject to restrictions. We assume that at least a portion y of any increase in market value has to be identified as earnings on book values in the balance sheet. Thus, we get

$$L_t = (1+g) L_{t-1} + \left[\delta y \left(A_t^- - A_{t-1}^+\right) - g L_{t-1}\right]^+, \; t = 1, 2, 3, ..., T. \quad (1)$$

We assume that in the MUST-case the remaining portion of the earnings on book values is being paid out as dividends, i.e.

$$\begin{aligned} d_t &= (1-\delta)\, y \left(A_t^- - A_{t-1}^+\right) 1_{\left\{\delta y \left(A_t^- - A_{t-1}^+\right) > g L_{t-1}\right\}} \\ &\quad + \left[y \left(A_t^- - A_{t-1}^+\right) - g L_{t-1}\right] 1_{\left\{\delta y \left(A_t^- - A_{t-1}^+\right) \leq g L_{t-1} \leq y \left(A_t^- - A_{t-1}^+\right)\right\}} \end{aligned}$$

Finally, the policy holder has the possibility to cancel the contract at any policy anniversary date t_0, receiving his account value L_{t_0} (walk-away option).

[1] More precisely, there is a maximum technical interest rate for discounting future benefits and premiums in the prospective calculation of policy reserves as well as surrender values, which is used by almost all insurance companies. Therefore, at least this technical rate has to be credited to the policyholders' accounts each year, implying a year-by-year interest rate guarantee.

2.3.2 The IS-Case

In the IS-case, we model actual behaviour of typical German insurers. Obviously, bonus payments may not be lower than in the MUST-case. Thus, we get

$$L_t \geq (1+g) L_{t-1} + \left[\delta y \left(A_t^- - A_{t-1}^+\right) - g L_{t-1}\right]^+ . \tag{2}$$

We will now focus on a specific bonus distribution scheme that appears to prevail in Germany. In the past, German insurance companies have credited a stable rate of interest to the policy reserves each year. In adverse market conditions, they used hidden reserves that had been accumulated in earlier years to keep the surplus stable. Only when the reserves reached a rather low level, they started reducing the surplus. Therefore, we apply the following decision rule that has been introduced in Kling et al. (2004):

A target rate of interest $z > g$ is credited to the policy reserves, as long as the so called reserve quota $x_t = \frac{R_t}{L_t}$ stays within a given range $[a, b]$. Only when the reserve quota becomes too low (too high), will the surplus be reduced (increased). We assume, that the dividends amount to a portion α of any surplus credited to the policy reserves. Thus,

$$L_t = (1+z) L_{t-1} \text{ and } d_t = \alpha (z-g) L_{t-1},\ t = 1, 2, 3, ..., T$$

as long as this leads to $a \leq x_t \leq b$.

If crediting the target rate z would result in a reserve quota below a and crediting the guaranteed rate g would lead to a reserve quota above a, then the company credits exactly the rate of interest that leads to $x_t = a$ (after crediting interest and dividends). Hence, we have

$$\begin{aligned} L_t &= (1+g) L_{t-1} + \frac{1}{1+a+\alpha}\left[A_t^- - (1+g)(1+a) L_{t-1}\right], \text{ and} \\ d_t &= \frac{\alpha}{1+a+\alpha}\left[A_t^- - (1+g)(1+a) L_{t-1}\right]. \end{aligned}$$

If even crediting only the guaranteed rate of interest leads to a reserve quota level below a, then the guaranteed rate of interest is granted and no dividends are paid, i.e.

$$L_t = (1+g) L_{t-1} \text{ and } d_t = 0.$$

If crediting the target rate of interest would result in a reserve quota above the upper limit b, the company credits exactly the rate of interest to the policy holders that meets the upper reserve quota boundary $x_t = b$ (after crediting interest and dividends), i.e.,

$$\begin{aligned} L_t &= (1+g)\, L_{t-1} + \frac{1}{1+b+\alpha}\left[A_t^- - (1+g)(1+b)\, L_{t-1}\right] \text{ and} \\ d_t &= \frac{\alpha}{1+b+\alpha}\left[A_t^- - (1+g)(1+b)\, L_{t-1}\right]. \end{aligned}$$

Finally, it still needs to be checked, whether these rules comply with the minimum participation rate, i.e. if condition (2) is fulfilled. Whenever necessary, the company increases the surplus defining L_t as in (1) and by letting

$$d_t = \alpha\left[\delta y\left(A_t^- - A_{t-1}^+\right) - gL_{t-1}\right]^+ .$$

3 Risk-Neutral Valuation

As usual in this context, we assume that there exists a probability space $(\Omega, \mathcal{F}, Q)$ equipped with a filtration $\mathbf{F} = (\mathcal{F}_t)_{t\in[0,T]}$, where Q is a risk-neutral measure under which payment streams can be valued as expected discounted values. Existence of this measure also implies that the financial market is arbitrage free. We use a bank account $(B_t)_{t\in[0,T]}$ as the numéraire process. In this setting, the general pricing formula for our contract (without walk-away option) is

$$P^* = E_Q\left[B_T^{-1} P \frac{L_T}{L_0}\right] \overset{L_0=P}{=} E_Q\left[B_T^{-1} L_T\right]. \tag{3}$$

In contrast to unit linked products, the underlying security in our case is not traded on the financial markets. It is a portfolio of assets which is subject to company's investment and, therefore, management decisions. However, it is possible for a financial intermediator, e.g. an investment bank, to approximate the insurers' reference portfolio by a traded benchmark portfolio, which permits the risk-neutral valuation approach. In what follows, we call the relevant portfolio the reference portfolio.

The above formula (3) is only useful to price the contract in total, but not appropriate to analyze the embedded features, e.g. the interest rate guarantee. Hence, we choose a different approach. We assume that the insurer invests his capital in the reference portfolio A and leaves it there. Using this reference portfolio we can model the following two cash flows:

- Dividends are paid to the shareholders. These payments leave the company and thereby reduce the value of the reference portfolio. However, the composition of the reference portfolio is not changed.

- If the return of the reference portfolio is so poor, that granting the minimum interest guarantee at time t would result in negative reserves, capital is needed in order to fulfil the obligations. This capital shot c_t enters the company's balance sheet and increases the value of the reference portfolio. Again, the composition stays the same.

Within the risk-neutral valuation approach, we can now price these cash-flows. In case of the capital shots[2], the risk-neutral value at time 0 is

$$C_0 = E_Q\left[\sum_{t=1}^{T} B_t^{-1} c_t\right].$$

Thus, C can be interpreted as the value of the minimum interest rate guarantee.

Similarly, one can determine the fair value at time 0 of the dividend payments

$$D_0 = E_Q\left[\sum_{t=1}^{T} B_t^{-1} d_t\right]$$

and the value at time 0 of the change of the reserve situation

$$\Delta R_0 = E_Q\left[B_T^{-1} R_T\right] - R_0.$$

For a "fair" contract, the value of the guarantee should coincide with the values of the dividend payments and the change of the reserve account, i.e.

$$C_0 \stackrel{!}{=} D_0 + \Delta R_0, \tag{4}$$

since then the policy holder's benefit from the guarantee is levelled out by the financial disadvantage due to dividend payments and undistributed final reserves. Thus, (4) represents an equilibrium condition for a fair contract. We also obtain an equivalent representation by taking into account, that for the value of the contract we have

$$P^* = E_Q\left[B_T^{-1} P \frac{L_T}{L_0}\right] = P + C_0 - D_0 - \Delta R_0. \tag{5}$$

Therefore, the equilibrium condition (4) is equivalent to

$$P^* \stackrel{!}{=} P. \tag{6}$$

As mentioned above, the policy holder has the possibility to surrender his contract at time t_0 and obtain his account value L_{t_0}. The reserves and the

[2]Note that in case no capital shot is needed at time t, we let $c_t = 0$.

unneeded capital for granting the minimum interest guarantee in $[t_0, T]$ remain with the insurer. For the sake of simplicity, we only allow for surrenders at the policy anniversary dates, i.e. $t_0 \in \{0, 1, 2, 3, ..., T\}$. If D_{t_0} denotes the value at t_0 of dividend payments in $[t_0, T]$, and, analogously, C_{t_0} denotes the value at t_0 of future capital shots, the policy holders gain from surrendering at time t_0 is

$$w_{t_0} = \max\left\{D_{t_0} + B_t\, E_Q\left[B_T^{-1} R_T \mid \mathcal{F}_t\right] - R_t - C_{t_0}, 0\right\}.$$

Thus, the value of the walk-away option (surrender option) at $t = 0$ is given by

$$W_0 = \sup_{\tau \in \Upsilon_{[0,T]}} E_Q\left[B_\tau^{-1} w_\tau\right],$$

where $\Upsilon_{[0,T]}$ denotes all stopping times with values in $\{0, 1, 2, 3, ..., T\}$[3].

Considering the surrender option, the equilibrium condition (4) changes to

$$C_0 + W_0 \stackrel{!}{=} D_0 + \Delta R_0.$$

However, (6) remains the same, since

$$P^* = E_Q\left[B_T^{-1} P \frac{L_T}{L_0}\right] + W_0 = P + C_0 - D_0 - \Delta R_0 + W_0. \tag{7}$$

Implicitly, by (5) and (7) we also answered the initial question for the risk-neutral value of an insurance contract in our model. Even though, at least in the case without a walk-away option, this representation is equivalent to the one initially given by (3), it is more meaningful in our approach, since it takes into account the special circumstances when pricing insurance contracts. In particular, it allows for a separate valuation and thus hedging of the components of the contract.

4 Numerical Analysis

We assume a frictionless, arbitrage-free and continuous market. Ignoring payments to equity holders for a moment, we let A_t evolve according to a geometric Brownian motion with constant coefficients

$$dA_t = A_t\left(r\, dt + \sigma\, dW_t\right),$$

where W_t denotes a Wiener process on the stochastic basis $(\Omega, \mathcal{F}, Q, \mathbf{F})$, r denotes the risk free rate of interest, and σ denotes the volatility of the

[3]Note that surrendering at time $t = 0$ can be interpreted as not concluding the contract.

reference portfolio. Taking into account dividend payments and capital shots as defined in sections 2 and 3, we obtain

$$\begin{aligned} A_t^- &= A_{t-1}^+ \exp\left\{\left(r - \frac{\sigma}{2}\right) + \sigma\left(W_t - W_{t-1}\right)\right\} \text{ and} \\ A_t^+ &= \max\left\{A_t^- - d_t, L_t\right\}, \; t = 1, 2, 3, ..., T. \end{aligned}$$

This can be conveniently used in Monte Carlo algorithms. Furthermore, we let $B_0 = 1$ and obtain $B_t = e^{rt}$.

We implemented a standard Monte Carlo algorithm, which provides the possibility to separately determine the risk-neutral price of a contract without surrender option, and the corresponding C, D and ΔR as defined in section 3. For pricing contracts with surrender option, we also implemented a discretization approach extending the ideas of Tanskanen and Lukkarinen (2004). They present a method based on discretization via a finite mesh. In what follows, we provide the basic ideas of this approach and our extensions.[4]

A Discretization Approach

Between two policy anniversaries, differences in the value process $V(t)$ of the contract are only caused by movements in the asset process. Thus, $V(t)$ fulfils the Black-Scholes PDE. By a change of variables, this equation can be transformed into a one dimensional heat equation, from which an integral representation of the solution can be derived given the final value. The value process at $t = T$ is known ($V(T) = L_T$), and the value process is a function of the prior account values as state variables. Thus, we can obtain the value process at $t = T - 1$ given the state variables at $T - 1$.

Arbitrage arguments show that the value process has to be continuous at any policy anniversary, since the distribution of the surplus is carried out according to deterministic, known functions. This enables us to obtain $V(t^-)$ for given state variables by a transformation of the state variables according to the distribution mechanism. Again by solving the Black-Scholes PDE with given state variables at $t - 1$, one can obtain $V(t - 1)$ given these state variables. This leads to an iterative algorithm to obtain $V(0)$ for given state variables.

Since there are an infinite number of possible state variables, discretization methods are used. Account values are obtained by interpolation, and the integral representation of the solution for the Black-Scholes PDE is also approximated by interpolation methods.

[4]For a more detailed description of their model and our extensions, respectively, we refer to Tanskanen and Lukkarinen (2004) and Bauer (2005).

As opposed to Tanskanen and Lukkarinen (2004), in our model x_t describing the insurer's current reserve situation is needed as an additional state variable besides L_t and A_t. Since including this state variable would increase the dimensionality of the problem and thus slow down the computations considerably, we examined the arbitrage arguments for a discontinuous asset process, and found that the argumentation remains valid. Thus, we can lower the dimensionality by letting A be discontinuous. However, additional and different interpolation schemes are needed, since now not only the transformation of the policy holders' account value L is necessary, but also A_t^- has to be transformed to A_t^+.

This method enables us to consider contracts with a surrender option (non-European contracts). We can price the value of the surrender option as the difference between the values of a European and a non-European contract with identical parameters. We used `C++` for the practical implementation. Other distribution mechanisms can easily be incorporated in our program.

5 Discussion of Results

In what follows, we will provide risk-neutral values of the contract and the considered embedded options. Furthermore, we will perform sensitivity analyses with respect to the most important parameters. Some dependencies are obvious and rather trivial; therefore, we will focus on the more complex and interesting relationships, in particular the interaction of several parameters. Unless stated otherwise, for all our calculations, we let the guaranteed rate of interest $g = 3.5\%$, the minimum participation rate $\delta = 90\%$ as motivated earlier, the insurer's initial reserve quota $x_0 = 10\%$, the target distribution $z = 5\%$, the reserve corridor $[a, b] = [5\%, 30\%]$, the portion of earnings that is provided to equity holders $\alpha = 5\%$, the asset volatility $\sigma = 7.5\%$, and the risk free rate of interest $r = 4\%$, since these values represent the situation and behaviour of a typical German insurance company. Furthermore, we let the time horizon $T = 10$ and the initial investment $P = 10,000$. Besides, we assume the portion of market value earnings that has to be identified as book value earnings to be $y = 50\%$. An estimation of y seems to be rather hard to perform, since the accounting rules are rather complex. However, the sensitivity of our results with respect to changes in y is rather low.

Our Monte Carlo algorithm allows for the calculation of all values except the value of the surrender option. Although the discretization approach allows for the calculation of all values, it is less accurate[5]. Therefore, we use

[5]Due to discretization and interpolation errors within the discretization approach, there are slight differences between the results of the different methods. However, the differences

the discretization method for the calculation of the surrender option only. All other results are calculated using Monte Carlo techniques.

Base Case Results

Table 2 shows the risk-neutral value of the contract and its components for the base case parameters shown above.

	MUST-Case	**IS-Case**
initial investment	10,000.00	10,000.00
+ Value of Interest Guarantee	+ 865.92	+ 1,004.19
- Value of Dividends	- 238.08	- 75.05
- Change of Reserve	- 267.47	- 10.05
Fair Value European Contract	10,360.37	10,919.09
+ Value of Surrender Option	0	0
Fair Value American Contract	10,360.37	10,919.09

Table 2: Contract Values

One can see from table 2, that the value of the contract $(10,360.37)$ exceeds the initial investment $(10,000)$ even in the MUST-case. This shows that for our base case parameters, the contracts are underpriced according to our equilibrium condition from section 3. Furthermore, table 2 shows that the interest rate guarantee is an essential part of a life insurance contract. Its value equals 865.92 in the MUST-case and 1004.19 in the IS-case.

In the MUST-case, any earnings on book values that are not required to be credited to the policy holders' accounts are paid out as dividends. This is the reason why the value of the dividend payments is rather high in this case. The value of the final reserve and therefore the change of reserve tends to be higher in the MUST-case. This is obvious since higher ongoing surplus in the IS-case leads to a lower reserve quota.

For our base case assumptions, the surrender option is worthless in the MUST-case as well as in the IS-case. In what follows, we therefore only will consider European type contracts unless stated otherwise.

The Influence of the Guaranteed Rate of Interest

Figure 1 shows the risk-neutral value of the liabilities in the MUST-case and in the IS-case as a function of the guaranteed rate of interest g.

are negligibly small.

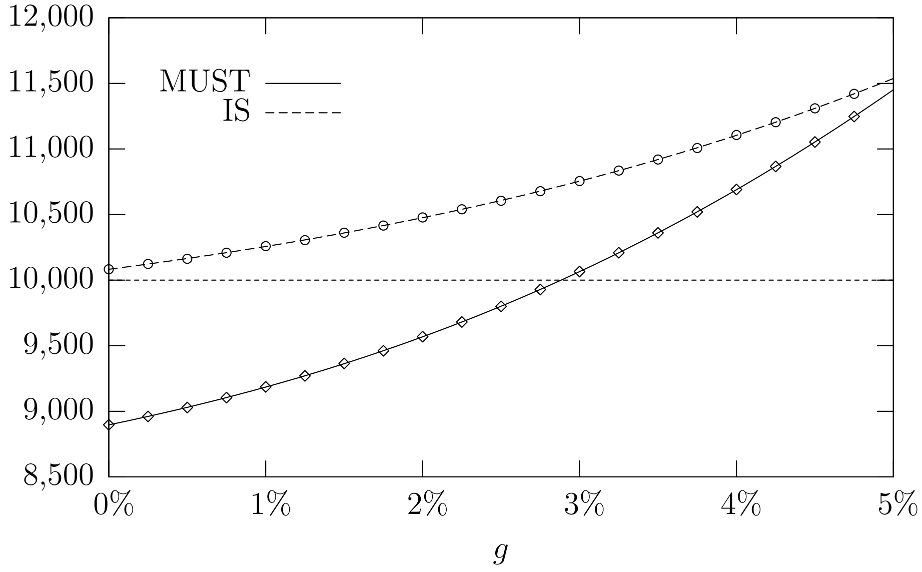

Figure 1: Influence of the guaranteed rate g on the contract value

The value of a contract in the IS-case, i.e. under a realistic distribution scheme, exceeds the initial investment even for a guaranteed rate of 0%. This is because the rather high target rate of interest is credited as long as the reserve quota doesn't fall below 5%. This shows that in times of rather low market interest rates, life insurance companies give away valuable interest rate guarantees below value.

In the MUST-case, for approximately $g = 2.75\%$ the contract value coincides with the initial premium. Thus, at current guaranteed rates a contract would be fairly priced if insurers only credited to the policy holders' accounts what they are required to. However, under any realistic assumptions about surplus distribution mechanisms, contracts currently offered by German insurance companies appear to be underpriced.

Furthermore, the influence of g is more pronounced in the MUST-case. In the IS-case, the guaranteed rate is of less importance, since often the target rate is credited, which is assumed to be the same for all values of g. However, for growing g, the influence becomes more dominant in the IS-case, too. In particular, the two curves become close for large values of g, since then the guarantee is the dominant factor under both schemes.

The Influence of the Risk Free Rate of Interest

In table 3 contract values and the value of the surrender option are shown for different levels of the risk free rate r.

	$r = 3.5\%$	$r = 4\%$	$r = 5\%$
Fair Value European MUST-Case	10,775.70	10,360.37	9,612.50
Surrender Option	0	0	387.50
Fair Value European Is-Case	11,296.90	10,919.09	10,255.10
Surrender Option	0	0	92.90

Table 3: Contract values at different levels of risk free rate r

Obviously, the European contract value decreases when market interest rates increase since the guarantees become less valuable. In particular, the values decrease by about 6% when r is increased from 4% to 5%. This shows that the fair value of life insurance liabilities is rather sensitive to changes in r. Thus, extending our model to include stochastic interest rates in a next step seems worthwhile.

However, in the IS-case even for $r = 5\%$ the risk-neutral value of the European contract exceeds the initial investment. Also, it is worth noting that the surrender option becomes more valuable with increasing r since alternative assets become more and more attractive, in particular when the insurer's reserves are low.

The Influence of the Insurer's Initial Financial Situation

Figure 2 shows the risk-neutral value of the European contract as a function of the initial reserve quota x_0.

The insurer's initial financial situation has a big impact on the contract value. In the IS-case, the contract value increases in x_0 since companies with higher reserves are more likely to be able to credit the target rate over a long period. This implies that customers should consider the financial situation of a company when buying participating life insurance not only for credit risk reasons. Clearly, higher policy interest rates can be expected from insurers with higher reserves.

Also in the MUST-case, the insurer's financial situation is of importance. Since return on the reserves is part of the total return of a period, on average higher surplus has to be credited for higher values of x_0. This influence is, however, rather small.

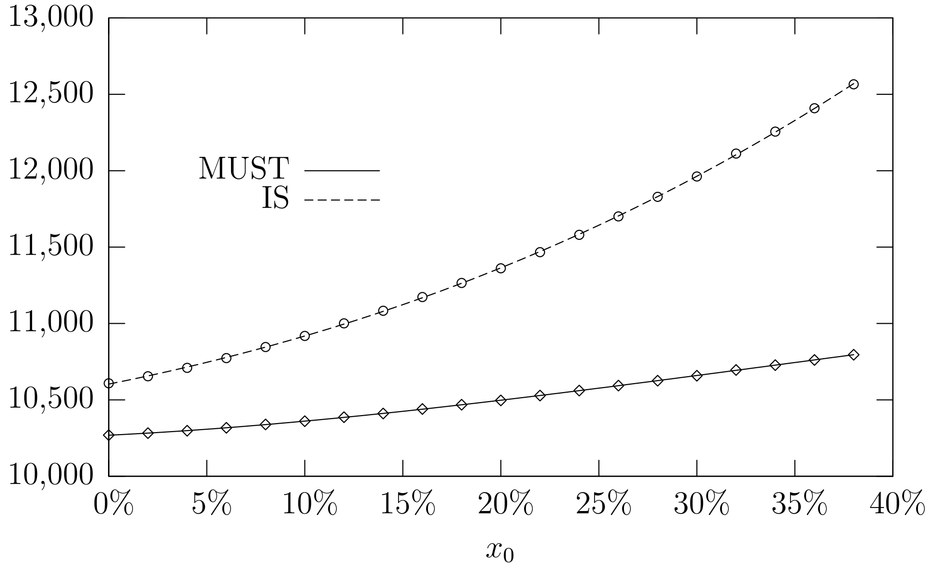

Figure 2: Influence of the initial reserve quota x_0 on the contract value in the MUST-case

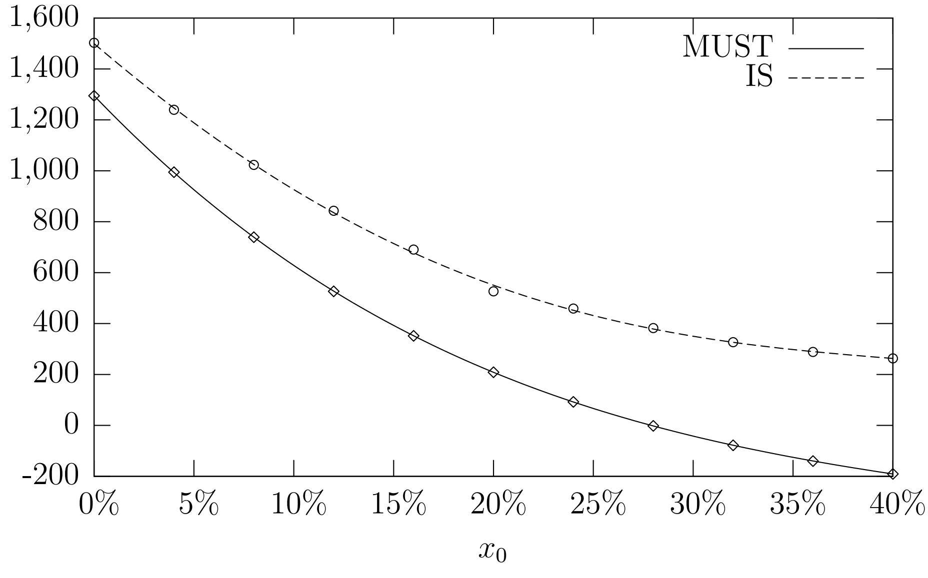

Figure 3: Influence of the initial reserve quota x_0 on the subsidy in the MUST-case

Figure 3 shows the risk-neutral value at $t = 0$ of the subsidy that has to be provided to fulfil the interest rate guarantee as a function of the initial reserve quota x_0. By subsidy, we mean the value of the capital shots less the value of the dividend payments.

As opposed to the contract value, the value of the subsidy is decreasing in the initial reserve quota x_0. Companies with higher reserves need significantly lower capital shots, since money only has to be provided from outside if reserves fall below 0. Thus, a strong financial situation is an advantage for equity holders as well as for the policy holders. Since this advantage is financed by the reserves, future policy generations may possibly not have a similar advantage.

The Insurer's Flexibility in the Distribution of Asset Returns

The parameters y and δ describe the flexibility insurers have in distributing the returns. While y determines which portion of the market returns has to be shown as book return, δ determines which portion of the book return has to be given to the policy holders. Current German accounting rules give a rather high degree of freedom (i.e. low values of y), but minimum participation rates imposed by regulators are rather high ($\delta = 90\%$). Since it is obvious that the contract value is increasing in δ and y, the interaction of these parameters is worth analyzing. If asset returns can no longer be used to increase hidden reserves (via y), insurers need the possibility to keep part of the book returns, i.e. to build reserves via δ.

Figure 4 shows combinations of δ and y, which lead to identical contract values of $10,000$ units (fair contracts according to the equilibrium condition) and combinations of δ and y that lead to identical contract values of $10,360$ units (value of the base contract in the MUST-case).

It is obvious that more restrictive asset valuation rules (high y) have to be compensated by reducing the minimal participation rate δ, to keep the value of the liabilities on the same level. Regulatory authorities have to pay attention to the interdependence of the various parameters.

The Interaction of Guarantees and Surplus Distribution

Historically, German life insurers used to credit the same total rate of interest (i.e. guaranteed rate plus surplus) to different "generations" of contracts. Recently, some insurers credited a higher total rate to contracts with a lower guaranteed rate, since these contracts obviously bear a higher downside risk. Regulators intervened and claimed that this is an unfair discrimination against certain generations of policy holders.

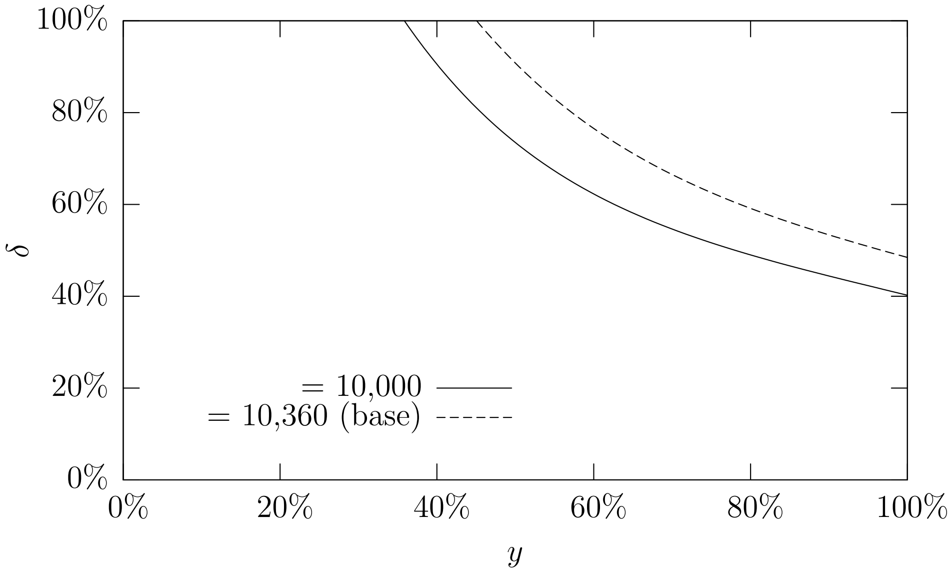

Figure 4: Parameter combinations of the minimum participation rate δ and y in the MUST-case

However, our model indicates the contrary. As can be seen in figure 5, a decrease in g should be compensated by an increase in the target rate, in order to keep the value of the liabilities on the same level. Thus, splitting the target rate does not constitute a discrimination of customers with higher guarantees but rather not splitting seems to be to the disadvantage of customers with lower guarantees.

However, this relationship is more complex, since other parameters also have an impact on the interaction of the guaranteed rate of interest g and the target rate of interest z. In figure 6, the same relationship is shown, but this time for a company with higher reserves. We can see that the difference in target rates for different guarantees should be the lower, the higher reserves are. Thus, the insurer's individual situation has to be considered when making such a decision.

The Interaction of Asset Allocation and Surplus Distribution

There are many other parameters that influence the contract value, for example, the asset allocation determining the volatility of the reference portfolio.

Figure 7 shows combinations of the asset volatility σ and the target rate of interest z, which lead to identical contract values of $10,000$ units, and $10,919$ units (base contract, IS-case), respectively.

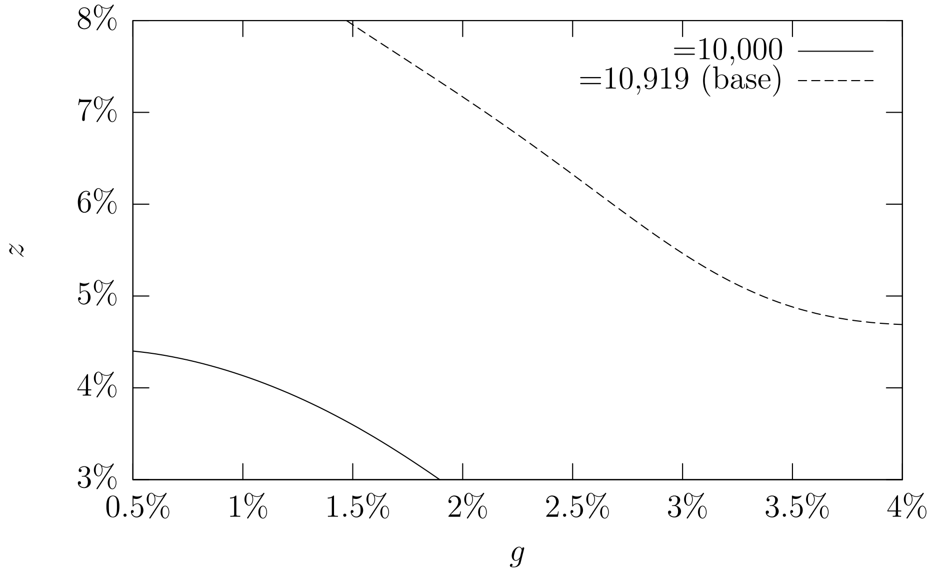

Figure 5: Parameter combinations of the guaranteed rate g and the target rate z in the IS-case with initial reserve quota of x_0=10%

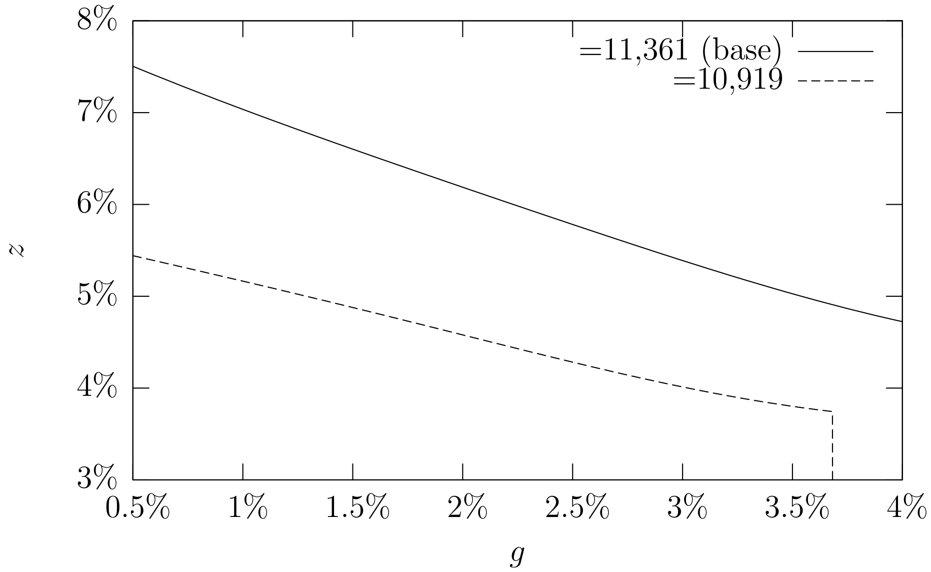

Figure 6: Parameter combinations of the guaranteed rate g and the target rate z in the IS-Case with initial reserve quota of x_0=20%

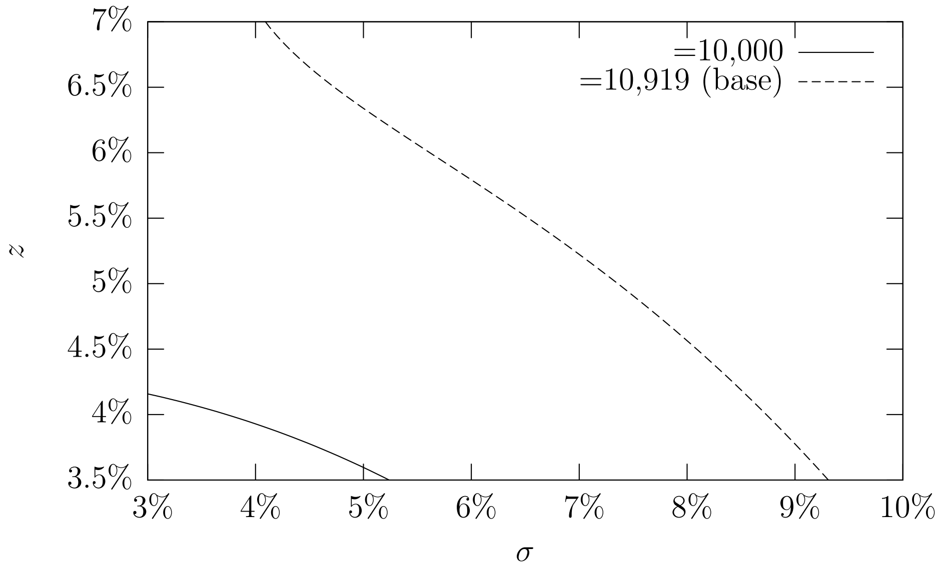

Figure 7: Parameter combinations of the volatility σ and the target rate z in the IS-case

With higher volatility, the policy holders' possibility to participate in high earnings is increased, while the downside risk is not increased because of the minimum interest rate guarantee. Thus, a conservative asset allocation strategy is advisable for the insurer, whereas the policy holder would benefit from high volatilities.[6] For the base contract, changing the volatility by 1% can be compensated by changing the target rate by about 1.5% in the opposite direction.

If the insurance company aims to offer fair contracts, only rather conservative combinations of σ and z are acceptable, e.g. an asset volatility of $\sigma = 5\%$ combined with a target rate of interest $z = 3.5\%$ or an asset volatility of $\sigma = 3\%$ combined with a target rate of interest $z = 4.2\%$.

6 Conclusion

We presented a model for evaluating and analyzing participating insurance contracts and adapted it to the German regulatory framework. Besides considering obligatory payments (MUST-Case), we also included a distribution mechanism which is typical for German insurers (IS-Case). We applied the model to valuate and analyze contracts. Furthermore, we discussed under

[6] If default risk is ignored.

which conditions and prerequisites a risk-neutral approach is meaningful. We presented a cash flow model, which takes into account the special circumstances of the valuation of German insurance contracts and also provides the possibility to separately valuate and analyze embedded options and other components of the contract.

Since these types of contracts are complex and path-dependent contingent claims, we relied on numerical methods for the evaluations. Besides an efficient Monte Carlo algorithm, which enables us to consider the contract components separately, we presented a discretization algorithm based on the Black Scholes PDE, which allows us to include a surrender option.

We examined the impact of various parameters on the value of a contract. We found that this value is significantly influenced by the insurer's financial situation and the provided minimum interest and participation guarantees, whereas the surrender option is of negligible value in most realistic cases. In particular, a contract with a financially strong company in general is more valuable. Under current market conditions, the value of the contract exceeds the initial investment – and therefore the price – of a contract. This is alarming and partially explains current problems of the German life insurance industry. Furthermore, we found the ability to build up and dissolve hidden reserves is crucial for offering these types of contracts.

Our model supports the venture of several insurance companies in the German market to provide different target rates of interest for different contract generations with different guarantee levels. However, other factors than just the interaction of target and guarantee rates have to be taken into account in order to not disadvantage either "old" or "new" customers.

We particularly found that the interactions of the parameters describing the regulatory framework, the financial market, the insurance company's situation, and the insurance contract are rather complex. An isolated analysis of the impact of one (set of) parameters does not seem appropriate. Since our results are rather sensitive to changes in the risk free rate of interest r and since the considered time horizon is rather long, including stochastic interest rates in the model should be a next step. Also, an analysis of the corresponding hedging strategies would be appropriate and of practical relevance.

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