Financial Planning and Risk-return profiles

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Abstract

The importance of funded private or occupational old age provision will increase due to demographic changes and the resulting problems for government-run pay-as-you-go systems. Clients and advisors therefore need reliable methodologies to match offered products and clients' needs and risk appetite. In this paper, we analyze existing approaches such as sample illustrations and historical backtesting that are often used for comparing and explaining products. We find that the information provided is often insufficient or even misleading. We introduce an alternative methodology based on risk-return profiles, i.e. the (forward-looking) probability distribution of benefits. In a model with stochastic interest rates and equity returns including stochastic equity volatility, we derive risk-return profiles for various types of existing unit-linked/equity-linked products with and without embedded guarantees. We highlight the differences between actual product characteristics and the impression generated by existing approaches and explain the resulting misleading incentives for product developers and financial advisors.

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1 Introduction

The demographic transition resulting from a continuing increase in life expectancy combined with rather low fertility rates constitutes a severe challenge for government-run pay-as-you-go pension systems in many countries. Therefore, the importance of funded private and/or occupational old age provision has been increasing and will – according to industry surveys – continue to increase. Competing for clients' money, providers of old age provision products (e.g. life insurers and asset managers) have come up with a variety of products that claim to be suitable 'packaged products' for old age provision. Often, equity investments equipped with certain guarantees are marketed as 'optimal' solutions combining the upside potential of stock markets with the security of certain guarantees, e.g. money back guarantees or guaranteed returns.

However, clients as well as financial advisors face a severe problem: The information that is (or legally has to be) provided by product providers is usually not sufficient to assess and to compare the risk-return profile of different products since important product characteristics are ignored. Thus, product characteristics cannot be matched with the client's risk aversion.

Information provided to clients about a product's performance potential and investment risk often falls in one of the following two categories: Deterministic sample illustrations or historical backtesting. A deterministic sample illustration gives the hypothetical development and maturity benefit of a product, assuming that the underlying investment performs steadily with some constant return every year. Different products are then ranked by comparing their performance assuming the underlying investment vehicles perform at this rate of return. Often, equity funds are projected with the same constant annual rate as balanced funds, money market funds, or guaranteed funds that are managed according to some strategy, e.g. CPPI. In contrast, a historical backtesting shows how the product would have performed, had it been offered in the past, or at several times in the past. Both, sample illustrations and backtestings are obviously of very limited explanatory value for future return potential and risk of a product. Furthermore, backtestings lead to a certain cyclical behavior of financial advisors and clients because products that would have performed well in the immediate past are typically preferred.

There are two main streams of literature in the field of individual financial planning. On the one hand, there exists a large body of literature on the question whether and what portion of wealth people should annuitize when reaching retirement age, e.g. a seminal paper by Yaari (1965) or more recent studies by Milevsky (1998), Milevsky et al. (2005) or Gerrard et al. (2010).[1] On the other hand, some authors deal with finding optimal investment strategies for accumulating money for retirement. This research is usually concerned with deriving the optimal (often dynamic) allocation among given asset classes by maximizing expected utility.

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[1] For a more detailed literature overview on this topic, we refer to the references therein.

Accordingly, e.g. Cairns et al. (2006) find optimal asset allocations (called stochastic "lifestyling") regarding a defined contribution plan, whereas e.g. Boyle and Tian (2009) derive "optimal" parameters of an equity indexed annuity (a product consisting of some minimum guarantee, a performance cap and an index participation rate) such that the investor's expected utility is maximized given certain constraints. However, these important theoretical results are often too complex for clients and/or advisors for a practical use by clients and/or advisors and highly depend on the particular choice of a utility function. Further, a practical implementation of such approaches would often require an ongoing management of clients' accounts which is often too complex, not feasible for rather small contract volumes or might result in tax disadvantages upon each transaction. Therefore, in practice often so-called 'packaged products' where certain strategies are implemented that do not require any action on the client's side during the term of the product, are offered and many financial advisors try to find the most suitable product out of a variety of such products for each client. We are not aware of any literature dealing with the practical issue of how to compare existing packaged products and how the relevant information necessary for a client-individual product selection can be derived from information provided.

In the present paper, we therefore deal with the question of how clients should select their best choice from (a limited number of) different old-age provision products offered in the market. We explain the weaknesses of product illustrations and comparisons that are predominant in many countries and propose a new methodology based on a comparison of forward-looking risk-return profiles. By this, we mean the probability distribution of the (maturity) benefit calculated within a stochastic model for the financial market. Using risk-return profiles, products that match a client's risk aversion can be identified. Besides presenting this methodology, we derive numerical results comparing basic product categories that can be found in many countries. In particular, we focus on comparing different ways of generating investment guarantees such as dynamic strategies like CPPI or static option-based strategies. We explain why typical product information used hitherto is not only insufficient but often even misleading.

The remainder of this paper is organized as follows: In Section 2, we present the products considered in our analysis. Section 3 then shows how these products are typically explained using sample illustrations and historical backtestings. Section 4 introduces the financial model used for pricing the derivatives embedded in some products in Section 5 and for the derivation of forward-looking risk-return profiles in Section 6. Finally, Section 7 concludes.

2 Considered products

In our analyses, we consider several product types that are (sometimes in different variants) common in retirement planning and offered by various financial institutions such as banks, insurers or asset managers in many countries. We distinguish products with and without embedded investment guarantees and consider products with single premium and with regular monthly premium payment P and a term of T years. Further, for the sake of simplicity and to focus on the main effects, the same (rather simple) fee structure is applied to all different packaged products: Premium proportional charges $\beta \cdot P$ reduce the amount invested to $(1-\beta)\cdot P$ (this amount is also referred to as "savings premium"). Account proportional charges γ – quoted as an annual fee – are deducted on a monthly basis from the client's account. Additionally, fund management charges *c* (also quoted as an annual charge but deducted daily) are applied within mutual funds if such funds are used in the packaged product. Additional fees for the guarantee may apply for the products with guarantee (see below).

The considered products are displayed in Table 1:

<table>
<tr><th>Products without embedded guarantee</th><th colspan="2">Products with embedded guarantee</th></tr>
<tr><td>▪ Investment in mutual funds
o equity fund
o balanced fund
o lifecycle managed funds</td><td>Money back guarantee</td><td>▪ Static option based guarantee ("underlying plus put" or "zero plus call")
▪ Semi-static guaranteed product of type "zero plus underlying"
▪ Dynamic CPPI strategy used on a client by client individual basis</td></tr>
<tr><td></td><td>'Savings premium guarantee'</td><td>▪ Dynamic CPPI strategy implemented in a mutual fund and thus managed on a collective basis (so-called CPPI guaranteed fund with high watermark guarantee)</td></tr>
</table>

Table 1: Considered products

In products without embedded guarantee, the savings premium is contributed to a mutual fund investing in equities or bonds depending on the fund's profile. In our analysis we consider equity, balanced and lifecycle funds. In our numerical analyses in Sections 5 and 6, we assume that an equity fund is completely invested in the modelled equity-process, whereas a balanced fund has a constant portion $x_S \in [0,1]$ (in our numerical analyses: 50%) of its capital invested in equity shares and the remaining part in bonds. We assume that the bond investment has some target duration d and therefore model the bond portfolio by a simple zero-bond with time to maturity $d = 5$. In addition, a lifecycle managed fund applies a time-dependent (however not path-dependent) asset allocation, where $(x_{S,t})_t \in [0,1]$ models the equity portion at time *t*. We assume that the lifecycle fund starts with 100% equity investment and then linearly decreases the equity portion each year to 0% in the last year. Finally, the current asset allocation of all considered funds is rebalanced daily to match required portions and the zero-bond's time to maturity respectively.

Products equipped with "money back guarantee" guarantee that at least the client's contributions are paid back at maturity. However, the way of generating this guarantee varies throughout the considered products:

We let A_t denote the client's account value at time t and G_t the guaranteed amount at time t to be paid at the contract's maturity T, i.e. the sum of paid premiums up to time t in our analysis.

The static option based strategy invests the savings premiums into the introduced equity fund. Each month – additionally to the fees introduced above – an account proportional guarantee fee[2] g – quoted as an annual charge – is deducted from the client's account and invested in some derivative security (or hedge portfolio) on the introduced equity fund delivering $\max\{G_T - A_T, 0\}$ at contract's maturity. Therefore – from a client's point of view – the product corresponds to an equity investment plus a put option on the equity fund. Note that the focus of our analyses is not on the hedging and its performance but rather on the option's payoff from client's perspective at maturity.

The zero plus underlying type product consists of a riskless asset (e.g. a zero-bond[3]) and a risky financial instrument (e.g. an equity fund). The portion invested to the risk-free asset is determined such that its payoff at the contract's maturity T coincides with the guarantee G_T. If a single premium is paid at contract's outset, the guarantee's present value – derived by discounting with the zero-bond's risk-free rate of return less account proportional fees – is invested in the riskless asset which then generates the guarantee at the contract's maturity. The remaining part is invested in the underlying equity fund. Regarding regular contributions, the analogous (semi-static) asset allocation is performed as follows: Each month[4] – say at time t – the client's account value A_t is split up into both assets. First, the guarantee's present value – the so-called floor F_t (calculated analogously as in the single premium case) – is invested the riskless asset which then exactly ensures the issued guarantee at maturity. Second, the remaining cushion $C_t = A_t - F_t$ is invested in the risky asset which provides

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[2] Note g is fixed throughout the contract's term at outset and not adjusted later on.

[3] We ignore potential default risk in our approach.

[4] In general, a readjustment of the underlying portfolio is only necessary at time of new contributions. Since, we consider single premium and regular payment we however perform monthly rebalancing permanently.

further upside potential. Even in case of total loss in equities, the underlying concept ensures the guarantee by respective zero-bond investment.

Constant proportion portfolio insurance (CPPI[5]) essentially further develops the idea of creating guarantees by investing in riskless and risky assets. However, here the asset allocation dynamically changes over time. At each rebalancing time t (usually and also in our numerical analyses daily) the provider determines the “optimal” asset allocation for the client’s account. However, in contrast to the zero plus underlying strategy, not only the cushion $C_t = A_t - F_t$, but a certain multiple thereof, i.e. $\max(m \cdot C_t, A_t)$, is invested in the risky asset. This is built on the assumption that the risky asset will not loose more than a ”worst-case-performance” of $1/m$ until the next rebalancing occurs. Therefore, the client’s account will not loose more than $1/m \cdot \max(m \cdot C_t, A_t) \leq C_t$ and thus the account value will not drop below the floor. Obviously, the product provider faces two sources of risk within a CPPI structure: First, the risky asset might loose more than $1/m$ during one period (this risk is often referred to as gap risk or overnight risk) and second, the floor might have changed within one period due to interest rate fluctuations. Most of the providers hedge the first risk by purchasing so-called crash-protection puts (essentially $1/m$-out-of-the-money-puts) but do not hedge the second risk. Tankov (2010) provides useful insights in pricing and hedging related crash-protection-puts in a Lévy-type framework. In our approach we will however skip valuation of the crash-puts and rather assume a flat additional charge of 0.2% p.a. on the risky asset within CPPI products.

We distinguish a CPPI strategy managed individually for each client (so-called individual CPPI or iCPPI) where a certain minimum value of each client’s account is guaranteed and CPPI managed within a mutual fund where a certain guaranteed maturity value per unit of the fund is guaranteed. Such fixed-term funds are very popular in several European countries, e.g. in Germany. They essentially promise a money back guarantee at maturity for all contributions ever made to the fund (which is typically less than the contributions made to the product that invests in the fund). In order to guarantee all contributions made to the fund by different clients (which may start their contracts at different times, have different premium patterns like single premium vs. monthly premium, or constant premiums vs. increasing premiums), the guaranteed maturity value of the fund has to be increased whenever the current net asset value (and thus the price at which new money is invested in the fund) exceeds the previous guaranteed value (high watermark guarantee). Further note, only contributions kept within the fund *until* maturity are guaranteed. Therefore if any charges are

[5] Cf. Black and Perold (1992).

deducted throughout the product's lifetime by selling parts of the purchased fund units even less than the savings premiums might be guaranteed at contract's maturity.

3 Product comparisons – Existing approaches

We now summarize approaches typically used for comparing the characteristics of the considered products. We start with a methodology very common e.g. in Germany or Austria based on deterministic sample illustrations and continue with an approach using historical data which is often used e.g. in the United Kingdom and recently gains popularity also in other parts of Europe. To the best of our knowledge, while there are differences in details, most existing approaches worldwide to compare the payoff characteristics of old age provision products are either based on the assumption of some hypothetic capital market scenario[6] or on historical data. There are only very few forward-looking stochastic approaches for analysing packaged products, e.g. in Italy where the regulator demands calculating certain "shortfall probabilities" and percentiles of return distributions[7]. However, these probabilities have to be computed using risk-neutral probabilities which to our opinion significantly reduces the explanatory value.

Sections 3.1 and 3.2 show how the considered products are characterized using existing methodologies. Throughout the following sections we apply charges as displayed in Table 2.

Charge	Value
Premium proportional charge	$\beta = 5\%$
Account proportional charge	$\gamma = 0.5\%\,p.a.$
Management fee within the funds	$c = 1.3\%\,p.a.$
Guarantee fee for option based product	As priced in Section 5
Crash-put protection fee	0.2% p.a. per unit of exposure

Table 2: General charges

The multiplier within the CPPI constructs is set to 4 corresponding to an assumed worst case of 25% within one rebalancing period.

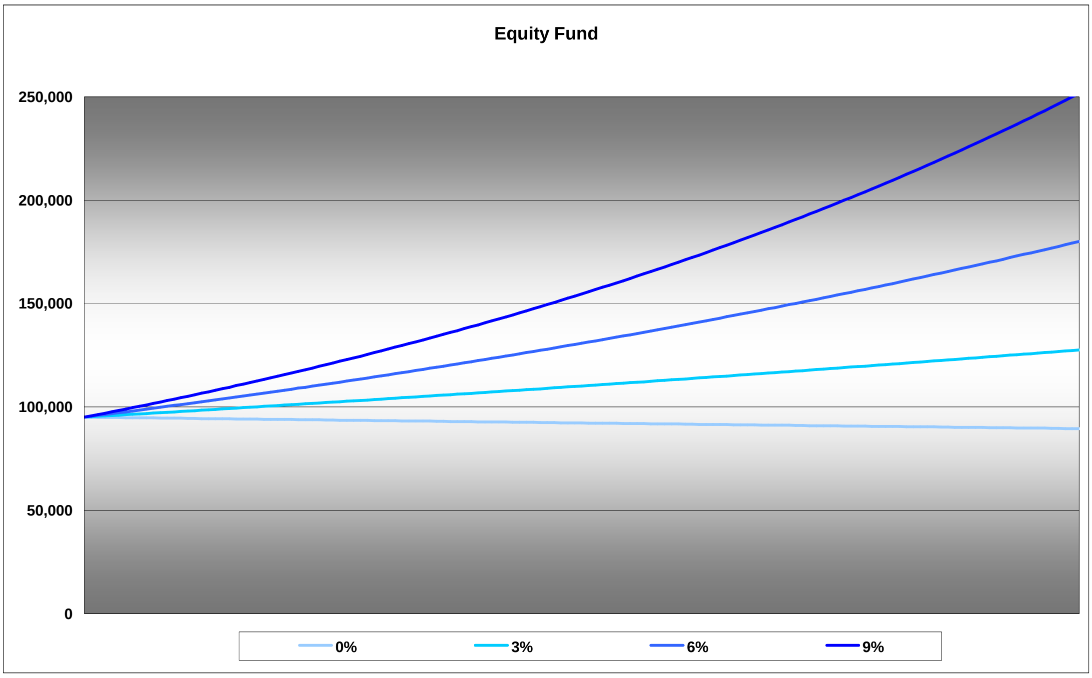
3.1 Sample illustrations

In this section, we investigate deterministic sample calculations. Under this approach, products are typically compared based on their projected maturity benefit derived by

[6] Bernard et al. (2009) shows impressive examples of quite optimistic scenarios used by providers of equity indexed annuities in the US.

[7] Minenna et al. (2009) provides a detailed description of the underlying methodology.

assuming that the product's underlying investment steadily performs according to some deterministic illustration rate. This approach does typically not distinguish funds according to their characteristics such as the invested asset classes or any special investment mechanisms. In consequence, e.g. equity funds are projected at the same rate of return as balanced funds. Further, path-dependent effects such as an ongoing rebalancing of riskless and risky assets (that happens e.g. in CPPI strategies) are not revealed by this approach since the considered sample paths exhibit no volatility. To make things even worse, typical illustrations in Germany or Austria assume that the considered fund (and not the fund's underlying) performs at the considered rate. Therefore all charges that occur inside funds are neglected. For instance, different funds investing in the same underlying but having different charges (such as management fees or crash-protection puts) will result in the same maturity benefit. Figure 1 shows the development of an equity fund investment (upper) and the account value of an iCPPI type product respectively (lower) regarding a product with a single premium of $P = 100{,}000$ and 12 year term. We assumed constant equity fund performances of 0%, 3%, 6% and 9% and a flat interest rate curve of 4.5% p.a.[8].



[8] In Germany rates of 0%, 3%, 6% and 9% are commonly used.

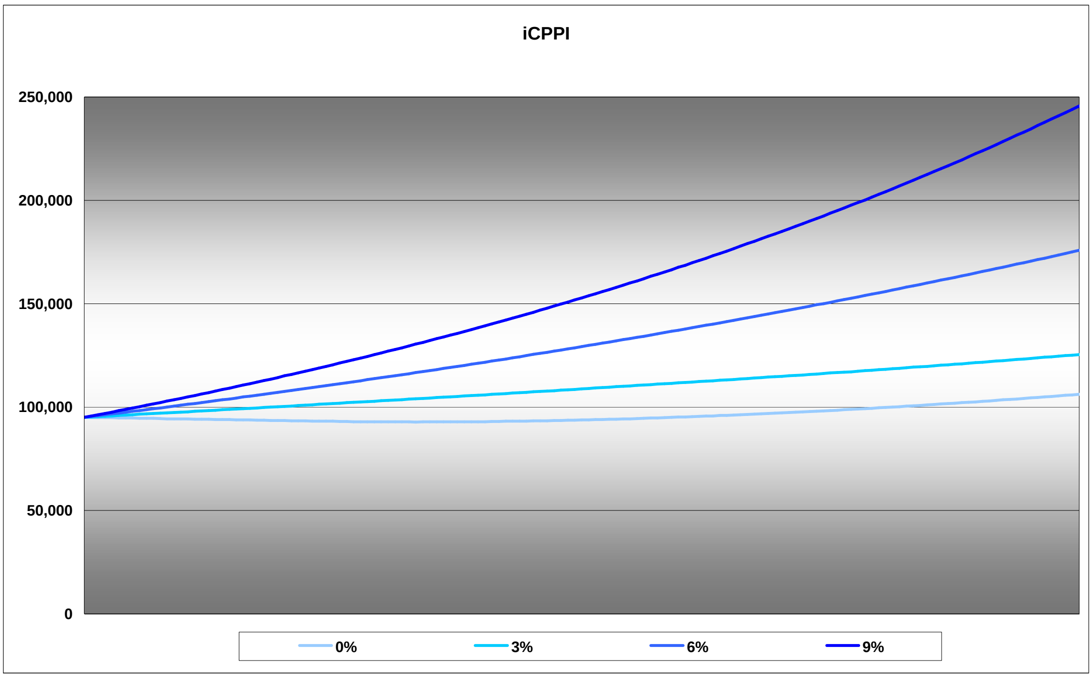


Figure 1: Sample Illustration of equity fund investment and iCPPI

Due to the absence of volatility in the considered scenarios, the iCPPI product looks almost identical to the pure equity fund investment with a slightly lower projected benefit considering the strictly positive illustration rates resulting from the additionally charged crash-protection fees[9] and a slightly higher projected benefit considering 0% equity fund performance due to the guarantee mechanism. Note that e.g. an investment into a high watermark CPPI fund would show exactly the same projected benefit as the pure equity fund although our analyses in Section 6 will show that the high watermark guarantee fund constitutes a very different investment with significantly lower expected return and of course also with lower risk. Path-dependent effects are completely neglected and therefore the impression is generated that the iCPPI's (or high watermark CPPI fund's) guarantee has very little (completely no) effect on the return potential. Further, obviously by using only non-negative illustration rates the downside-risk appears smaller than it is, in particular since clients with a rather low financial education might perceive the lowest illustration as some kind of worst case. Section 6 reveals the tremendous differences between a deterministic sample illustration and forward-looking stochastic analyses of considered products and shows that the probability of getting the benefit projected in a sample illustration may be very small for some concepts.

Even these simple examples are sufficient to conclude that deterministic sample calculations are not suitable to asses and compare the risk-return profile of different product types and that more meaningful information is required for practical use in financial planning.

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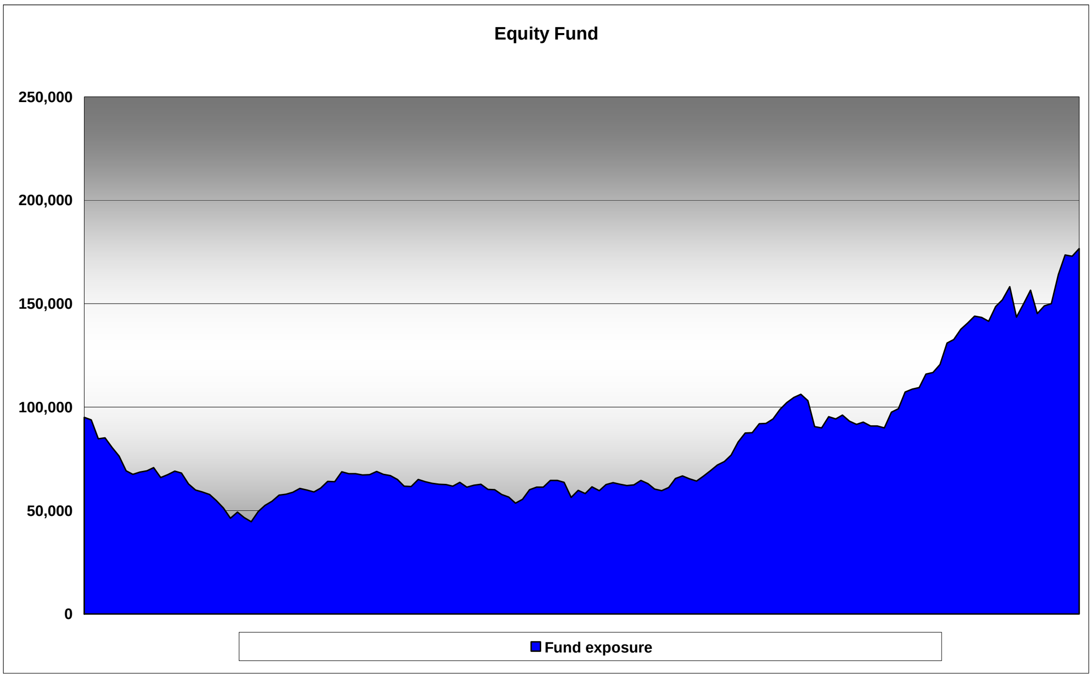
[9] Note that we assume that these fees are charged at the level of the packaged product. If they were charged within the equity fund, they would be neglected in this comparison.

3.2 Backtesting

We have seen that a major disadvantage of sample illustrations is their lack of volatility in the underlying assumptions. In consequence, another wide-spread methodology uses realized historical scenarios of equity and interest rate markets – that is one realized “sample” path – and then derives the product’s payoff assuming the product had been offered in the past.

The following examples use historical data from 01 January 1973 to 31 December 2009 of the MSCI World equity index and of German government bonds[10]. Figure 2 shows the ”backtested” account value of the same products that have already been analyzed in Section 3.1 assuming they have been issued at 01 January 1973.



[10] We use monthly data of government bonds with time-to-maturities 0.5, 1, 2… 15 years obtained from German Federal Bank and calculate bond prices needed in the considered products from these interest rates.

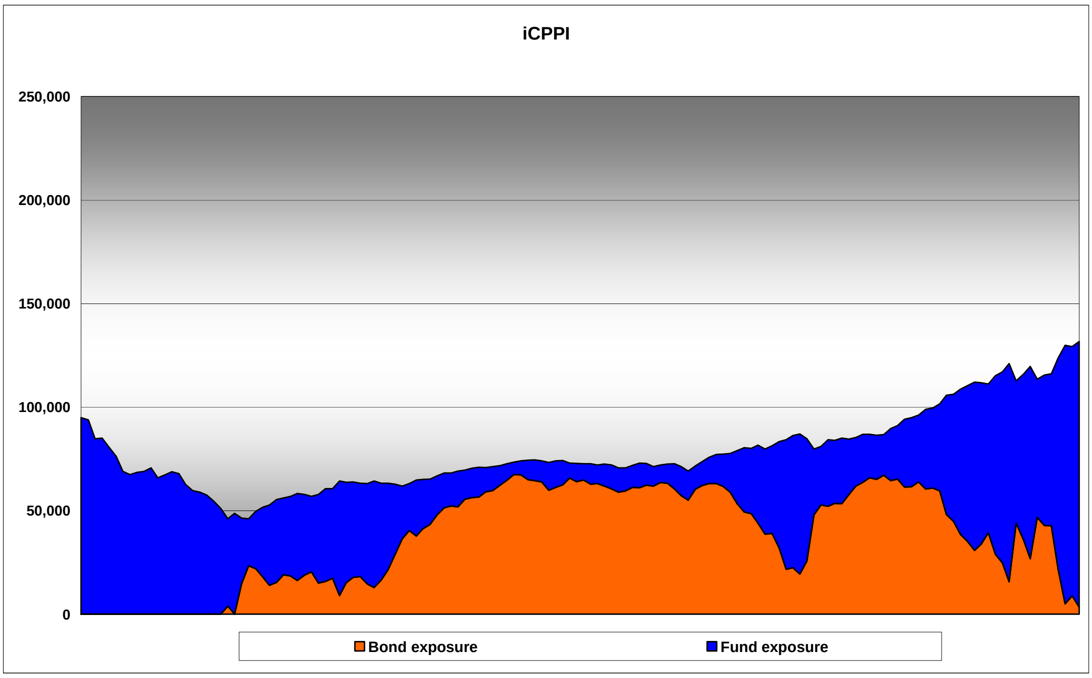


Figure 2: Backtesting of equity fund investment and iCPPI incepted on 01 January 1973

In contrast to a sample illustration (Figure 1), a historical backtesting uncovers the different mechanics between the pure equity linked and the path-dependent iCPPI product. Whereas a deterministic sample illustration completely neglects volatility and hence the downside risk of the risky asset, the backtesting shows the essential feature of CPPI type products, namely an ongoing reallocation between risky and riskless asset to assure the given guarantee. When equity values declined in the considered time-frame, a significant part of the client's account value had to be moved to the riskless asset to secure the guarantee. In consequence, the client could not participate in the following recovery of equity prices which resulted in a lower maturity benefit. We also see that, although the iCPPI product was at some stage almost completely invested in the risk-free asset, at maturity, the product was completely invested in the risky asset, due to leveraging the risky exposure with a multiplier of 4. Summarizing, compared to a sample illustration, the key characteristics and behaviour of different products can be uncovered and shown transparently using historical data.

However, just one (no matter how "realistic") scenario cannot explain the (stochastic) up- and downside potential of such a product properly. Furthermore, results are highly dependent on the chosen historical time interval. To illustrate this effect, Figure 3 gives similar computations assuming the products commenced at 01 January 1998.

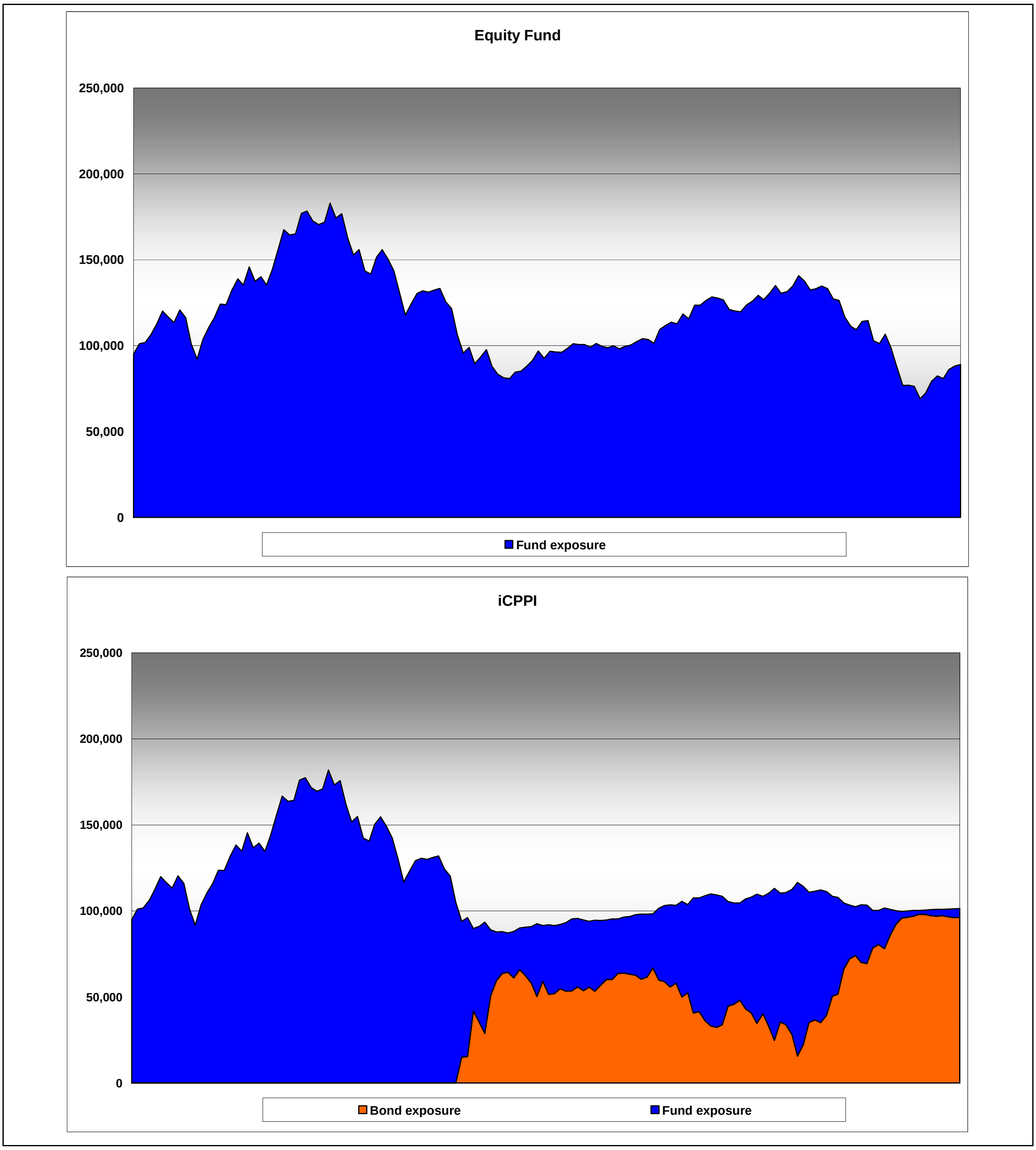


Figure 3: Backtesting of equity fund investment and iCPPI incepted on 01 January 1998

Whereas Figure 2 showed the equity fund investment resulting in a higher maturity benefit than the iCPPI product, Figure 3 shows a backtesting where the iCPPI-product yields a higher return than an equity fund investment. So while backtestings are somewhat superior to sample illustrations introduced in Section 3.1 in that they help explaining the product's characteristics, a major disadvantage is that the results – in particular the relation between products – highly depends on the chosen time interval. In particular, when using backtestings, there is a strong incentive to design products that would have performed particularly well in the immediate past thus resulting in good backtestings.

In addition, we can conclude that financial advisors relying on backtesting as a decision tool might act cyclically. This is illustrated by Figure 4.

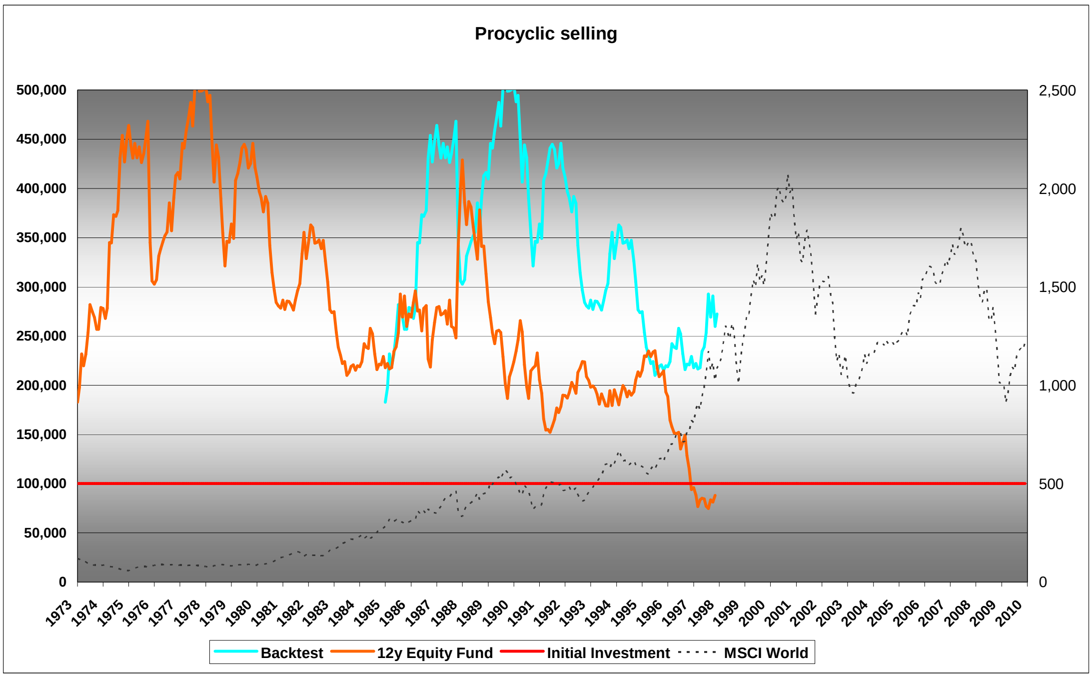


Figure 4: Backtesting – fooled by randomness[11]

The dotted black line gives the MSCI World's development from 01 January 1973 to 01 January 2010. The orange line accordingly shows the final value of the considered 12-year equity fund investment when taken out at some time *t*. For example if the contract started at the beginning of 1973, the maturity benefit would have amounted to roughly 182,000. The light blue line depicts the result of a backtesting performed at the same time, e.g. at the beginning of 1985 the corresponding maturity benefit obtained by backtesting equals 182,000.

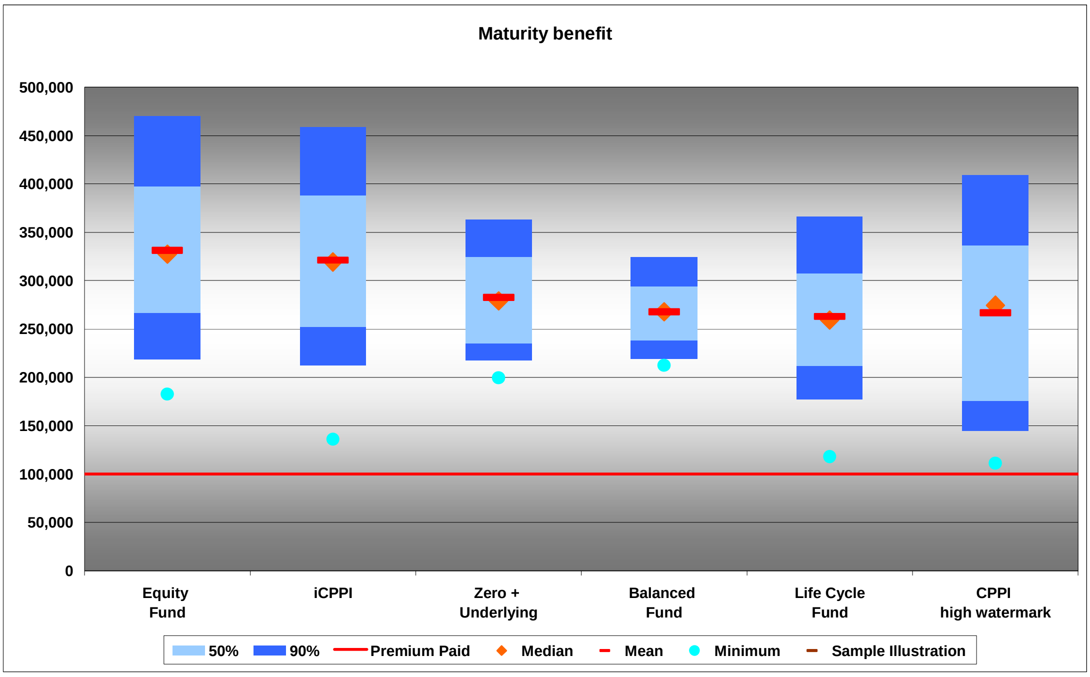
In 1998, both, immediate backtesting and so-called repeated backtesting (cf. Section 3.3) showed very attractive results. However, an investor who invested in 1998 actually lost money, an event that was labelled as only "theoretically" possible based on backtesting.

3.3 Repeated backtesting

Figure 2 and Figure 3 showed that for assessing the product's risk-return profile just investigating one or several sample paths is not sufficient. Therefore, a common approach is to perform historical backtestings using many different historical scenarios. The resulting frequency distribution is then often used as a proxy for the probability distribution of the product's payoff and for assessing a product's likelihood to achieve a certain return. Since

[11] Taleb (2005) compares designing products that show good backtestings to "throwing monkeys on typewriters, without specifying what book you want the monkey to write and hence hitting on hypothetical gold somewhere".

old-age-provision contracts generally have a rather long term to maturity, a large number of historical scenarios can usually only be generated by using overlapping time intervals stemming from the same historical time series at different starting points. We have calculated the resulting distributions of various products'[12] maturity benefits and their corresponding internal rates of return using all possible 12-year time-intervals starting on the first of each month within the underlying time series from 01 January 1973 to 31 December 1999. Hence, the first observed maturity benefit is derived from a contract starting at 01 January 1973, the second observation is deduced from a contract starting at 01 February 1973, and so on. Certain percentiles and the expected value of the distribution of the maturity benefits (upper) and the corresponding internal rates of return (lower) are displayed in Figure 5. Along with the 5-, 25-, 75- and 95-percentile the observed minimum (turquoise dot), the median (orange diamond) and the mean (red bar) are displayed. Further, the dark blue (light blue) area contains 90% (50%) of the observations. The corresponding numbers for the products' internal rates of return are given in Table 3 where the "expected return" is determined as internal rate of return of the expected maturity benefit.



[12] We omit the option based product in these calculations.

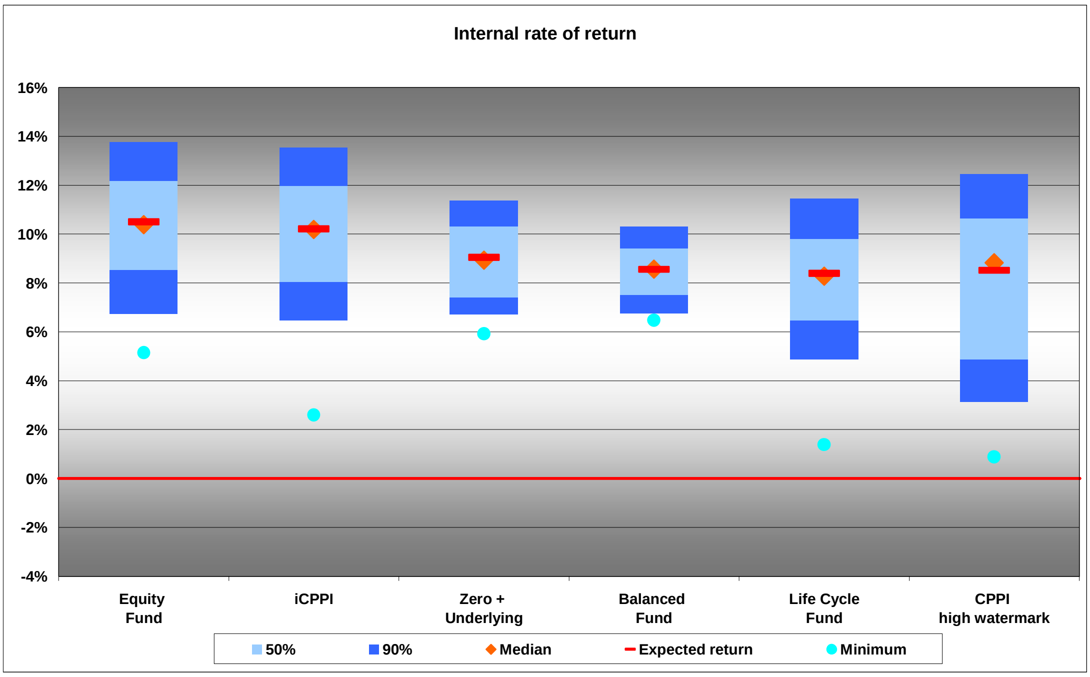


Figure 5: Distributions resulting from "repeated backtesting" – 12 year single premium

	Equity Fund	iCPPI	Zero + Underlying	Balanced Fund	Life Cycle Fund	CPPI high watermark
Minimum	5.15%	2.60%	5.92%	6.48%	1.39%	0.88%
5%	6.73%	6.47%	6.71%	6.76%	4.87%	3.13%
25%	8.53%	8.05%	7.41%	7.52%	6.48%	4.87%
Median	10.38%	10.19%	8.93%	8.56%	8.28%	8.83%
75%	12.18%	11.98%	10.32%	9.42%	9.80%	10.65%
95%	13.76%	13.55%	11.37%	10.31%	11.45%	12.46%
Maximum	14.66%	14.43%	12.51%	11.01%	12.16%	13.32%
Expected return	10.49%	10.21%	9.04%	8.55%	8.39%	8.52%

Table 3: Distributions resulting from "repeated backtesting" – 12 year single premium

The equity fund investment shows the highest upside potential. Further, regarding the equity fund's downside risk, the minimum return is found as 5.15% p.a. within the considered time-period. This may be perceived by less educated clients as some kind of "worst case" resulting in a wrong assessment of the product's actual risk. In addition, considering the displayed percentiles, the equity fund appears to dominate the lifecycle strategy, a results that is clearly contradicted by the products' risk-return profiles (cf. Section 6).

Further, all products without embedded guarantees (equity fund, balanced fund and lifecycle fund) display a very low downside risk. Negative returns did not occur in the considered time-period. Hence, the derived distributions suggest that there is no need for guarantees, since shortfall is only theoretically possible. Even very risk averse clients might conclude from such results that products with guarantees are not desirable since they reduce the upside potential in turn for protection against what seems to be a very unlikely event.

Considering the products with embedded guarantees, the iCPPI structure displays almost the same upside potential as the pure equity investment and additionally provides capital protection (that is however not triggered). The zero plus underlying type product induces less skewness as compared to iCPPI still offering fairly high upside potential and rather stable returns. The high-watermark-CPPI strategy displays the smallest minimum return of all considered products with guarantees and is dominated by the individual CPPI.

It is intuitively clear, that major weakness of “repeated backtestings” stems from the fact that the time intervals used for the different backtestings show a significant overlap. For instance the first and the second backtesting from the results above (starting at 01 January 1973 and 01 February 1973 respectively) are two 12-year intervals that have 143 out of 144 months in common. To quantify this effect, Figure 6 shows the autocorrelation function of the underlying time-series of projected maturity benefits regarding the equity fund investment.

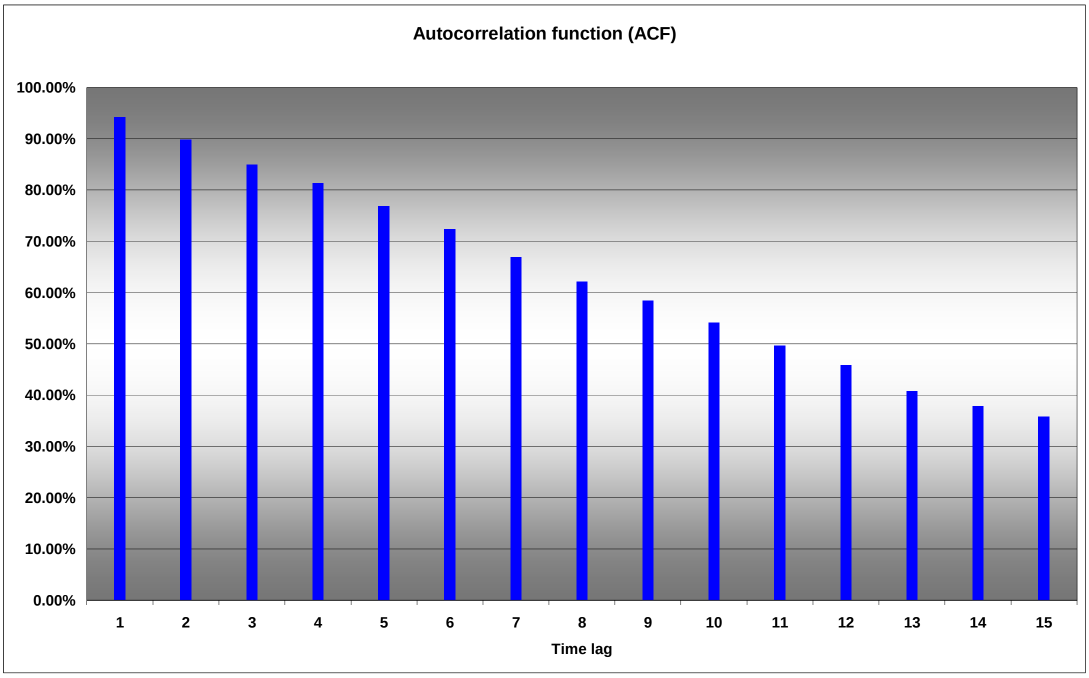


Figure 6: Autocorrelation (ACF) of equity fund’s time series

As expected, massive autocorrelation is detected in the time-series. Therefore, after having observed a good performance of a product in one time interval, it is very likely to observe a good performance in the next interval, as well. Hence, the “different” scenarios are not independent and therefore might provide questionable incentives to financial planners or product developers relying on such methodologies.

Summarizing, the historical backtesting approach is in many ways superior to a deterministic sample illustration e.g. because it can help revealing characteristics (in particular of path-dependent products) more transparently. However, by solely relying on past scenarios inappropriate incentives may be set. Distributions generated from repeated backtesting show too little variability because the underlying scenarios have a very high auto-correlation (compared to Figure 7).

Therefore, a forward-looking stochastic methodology projecting the product's benefits and deriving their risk-return profile under independent scenarios using appropriate capital market models and parameters appears desirable.

Whereas the approaches in Sections 3.1, 3.2 and 3.3 either relied on deterministic or (massively correlated) historical scenarios, appropriate risk-return profiles have to be derived using stochastic modelling of future capital market scenarios. In Section 4 we first present the financial model used in our analyses whereas Section 6 quantitatively investigates resulting risk-return profiles. We further show how the probability distribution can be reduced to certain key figures (suitable for practical use) describing up- and especially downside potential. In practical applications, in particular for analyzing individual products (as opposed to product types which is the purpose of this paper) all product specific information e.g. about charges has to be considered, especially underlying guarantee fees in option based products. In this paper however, Section 5 determines fair prices of the guarantee rider that are consistent with the model used to compare the products.

4 Financial model

We start with an introduction of the real-world asset model used for our analysis and then perform a change of measure to a risk-neutral pricing measure necessary for valuation of the option based product's guarantee fee g.

The considered financial products (cf. Section 2) invest in stocks and zero-bonds. Hence, we need a model for equity and interest rates. Due to the long-term nature of the considered products modelling stochastic volatility also seems appropriate. Thus, we use a slightly modified version of the Heston model (cf. Heston (1993)) for stock markets and the Cox-Ingersoll-Ross model (cf. Cox et al. (1985)) for interest rate markets denoted as Heston-CIR hybrid model by Grzelak and Oosterlee (2010) and first studied by Bakshi et al. (2000) in a more general setup.

Therefore, let $(\Omega, F, \mathbf{F}, \mathbf{P})$ be a filtered probability space equipped with the natural filtration $\mathbf{F} = (F_t)_t = (\sigma((W_1(s), W_2(s), W_3(s)), s \leq t))_t$ generated by $\mathbf{P}$ – Brownian Motions $W_1(t)$, $W_2(t)$ and $W_3(t)$. Further let $r(t)$ denote the short-rate and $S(t)$ denote the equity's spot price at time t, respectively. The (real-world) asset model is then summarized with the $\mathbf{P}$ – dynamics

$$
\begin{aligned}
dr(t) &= \kappa_r(\theta_r - r(t))dt + \sigma_r\sqrt{r(t)}dW_1(t) \\
dS(t) &= S(t)\big((r(t) + \lambda_S)dt + \sqrt{V(t)}dW_2(t)\big) \\
dV(t) &= \kappa_V(\theta_V - V(t))dt + \sigma_V\sqrt{V(t)}dW_3(t)
\end{aligned}
$$

where λ_S denotes the equity risk premium. We further assume $dW_1(t)dW_2(t) = dW_1(t)dW_3(t) = 0$ and $dW_2(t)dW_3(t) = \rho dt$ with $\rho \in [-1,1]$ denoting the coefficient of correlation between the equity value and its instantaneous variance $V(t)$.

For pricing derivatives such as zero-bonds or equity derivatives, we further need to specify some pricing measure $\mathbf{Q}$. The underlying asset model lacks completeness due to the non-tradable short rate and the non-tradable stochastic variance. Hence, in contrast to a conventional Black-Scholes model, the risk-neutral measure is not uniquely defined. There exists a whole set of probability measures that ensure discounted (traded) assets being martingales and therefore provide arbitrage free derivative prices instead. Hence, let λ_r and λ_V denote the market price of interest rate and volatility risk, respectively. Risk-neutral dynamics can then be introduced as[13]

$$
\begin{aligned}
dr(t) &= \tilde{\kappa}_r\left(\tilde{\theta}_r - r(t)\right)dt + \sigma_r\sqrt{r(t)}\,d\tilde{W}_1(t) \\
dS(t) &= S(t)\left((r(t))dt + \sqrt{V(t)}\,d\tilde{W}_2(t)\right) \\
dV(t) &= \tilde{\kappa}_V\left(\tilde{\theta}_V - V(t)\right)dt + \sigma_V\sqrt{V(t)}\,d\tilde{W}_3(t)
\end{aligned}
$$

with $\tilde{\kappa}_r = \kappa_r + \lambda_r\sigma_r, \tilde{\kappa}_V = \kappa_V + \lambda_V\sigma_V$, $\tilde{\theta}_r = \frac{\kappa_r\theta_r}{\kappa_r + \lambda_r\sigma_r}, \tilde{\theta}_V = \frac{\kappa_V\theta_V}{\kappa_V + \lambda_V\sigma_V}$ and $\mathbf{Q}-$Brownian Motions $\tilde{W}_1(t), \tilde{W}_2(t)$ and $\tilde{W}_3(t)$.

Within this setting, zero-bond prices with time to maturity d at time t are given by[14]

$$P(t,d) = A(d)\exp\left(-B(d)r(t)\right) \quad \text{with} \quad A(d) = \left[\frac{2\cdot h\cdot\exp\left((\tilde{\kappa}_r + h)\cdot d/2\right)}{(\tilde{\kappa}_r + h)\cdot(\exp(h\cdot d)-1) + 2\cdot h}\right]^{\frac{2\tilde{\kappa}_r\tilde{\theta}_r}{\sigma_r^2}},$$

$$B(d) = \frac{2\cdot(\exp(h\cdot d)-1)}{(\tilde{\kappa}_r + h)(\exp(h\cdot d)-1) + 2\cdot h} \quad \text{where} \quad h = \sqrt{\tilde{\kappa}_r^{\,2} + 2\cdot\sigma_r^{\,2}}.$$

In our following quantitative analyses we assume an equity risk premium of $\lambda_S = 3\%$ according to quantitative research by the European Central Bank (cf. Capiello et al. (2008)).

Estimating the interest rate parameters – especially in a "$\mathbf{P}-\mathbf{Q}-$setting" – is rather crucial and heavily depending on data and methodology chosen. The related literature often proposes a two-step calibration[15]: First, "real-world parameters" are estimated using overnight rates as proxy for the short rate. Second, after having fixed the real-world parameters one then calibrates the market price of risk to fit observed yield curves.

However, regarding the first step, overnight rates often lack robustness and possible "outliers" due to liquidity issues may distort the estimation results. Therefore, estimates based on this approximation are likely to be numerically instable. Furthermore, Bernashi et

[13] For technical details on respective measure change, refer to Cox et al. (1985), Wong and Heide (1996) and Paulsen et al. (2009).

[14] Cf. e.g. Bingham and Kiesel (2004).

[15] Cf. e.g. Bernashi et al. (2007).

al. (2007) show the (fairly high) variability and numerical instability of estimates derived by above approaches over time. Therefore, we calibrated the interest rate parameters to match level and variability of observed German 1-year and 10-year government bonds[16] and to obtain reasonable (real-world) short rates[17] by setting

κ_r	θ_r	σ_r	λ_r	$r(0)$
20%	4.5%	7.5%	0%	4.5%

Table 4: Interest rate parameters (base case)

Regarding stochastic volatility (more precisely: variance), we use parameters derived by Eraker (2004) and stated in annualized form by Paulsen et al. (2009) as

κ_V	θ_V	σ_V	ρ	$V(0)$
475%	$(22\%)^2$	55%	-56.9%	$(22\%)^2$

Table 5: Volatility parameters (base case)

Finally, we assume risk-neutrality with respect to volatility risk and therefore set the market price of volatility to $\lambda_V = 0\%$.

5 Option pricing

Arbitrage-free derivative prices are usually either calculated by means of Fourier inversion techniques[18] or based on Monte-Carlo simulation[19]. Regarding Fourier inversion, Grzelak and Oosterlee (2010) provide approximations to the necessary characteristic function of the underlying hybrid model which is then used to price vanilla options.

However, the option-based product's derivative (cf. Section 2) depends on the premium payment mode, the included charges and the fund investment chosen and is therefore of rather exotic nature. Further as described in Section 2, the client does not purchase an option on top of his contribution, instead some part of his account value is used to finance the derivative throughout the contract's term. In consequence, a lower (higher) guarantee fee g requires potentially less (more) payments arising from the derivative. Hence, one needs to calibrate the guarantee fee such that the fair values of guarantee fees and the derivative's benefits comply. Therefore, – due to its flexibility – we choose a Monte-Carlo pricing

[16] According time series was obtained from German Federal Bank (http://www.bundesbank.de).

[17] Note, the risky asset's expected daily return is determined by the real-world short rate in our modelling approach.

[18] Cf. e.g. Carr and Madan (1999).

[19] Cf. e.g. Fishman (1996).

approach on the basis of Bauer et al. (2008) using 50,000 trajectories to derive the fair guarantee fee[20].

In the following we will analyze four different model points with regular monthly premium payment of $P = 100$ and single premium payment of $P = 100{,}000$ respectively and distinguish a 12- and 30-year term. Table 6 gives the option based product's fair guarantee fee g for these four model points assuming capital market parameters explained above.

Term	Premium payment mode	
	single premium payment	regular payment
12	1.85% p.a.	3.15% p.a.
30	0.43% p.a.	0.81% p.a.

Table 6: Fair guarantee fee (quoted annually) – base case

First, the fair value of the guarantee rider heavily depends on the contract's term, i.e. the higher the term the lower the required annual guarantee fee. Second, the guarantee is less costly for single-premium contracts than for products with regular premium payment. Considering regular contributions the derivative's payoff can roughly be characterized as forward starting put on future contributions. Therefore, the capital tied up in the contract is generally lower than compared to the single premium payment and hence especially guaranteeing the late contributions is quite costly.

6 Risk-return profiles

We now introduce the concept of risk-return profiles for the considered products. Similar with the approach in Section 5, we perform 50,000 trajectories (but under the probability measure $\mathbf{P}$) to derive estimates of the maturity benefits' distributions.

6.1 Short term-contracts

We start by analyzing the risk-return profile of a 12-year single premium ($P = 100{,}000$) contract. We derive the distribution of maturity benefits and compute their internal rates of return (IRR) accordingly. Figure 7 depicts a graphical illustration of the resulting empirical distributions and further shows results of corresponding sample illustration assuming 6% and 9% p.a. fund performance and 4.5% interest rates flat.

[20] In practice providers do not need to price the derivative fair, but may account for additional risk or profit margins.

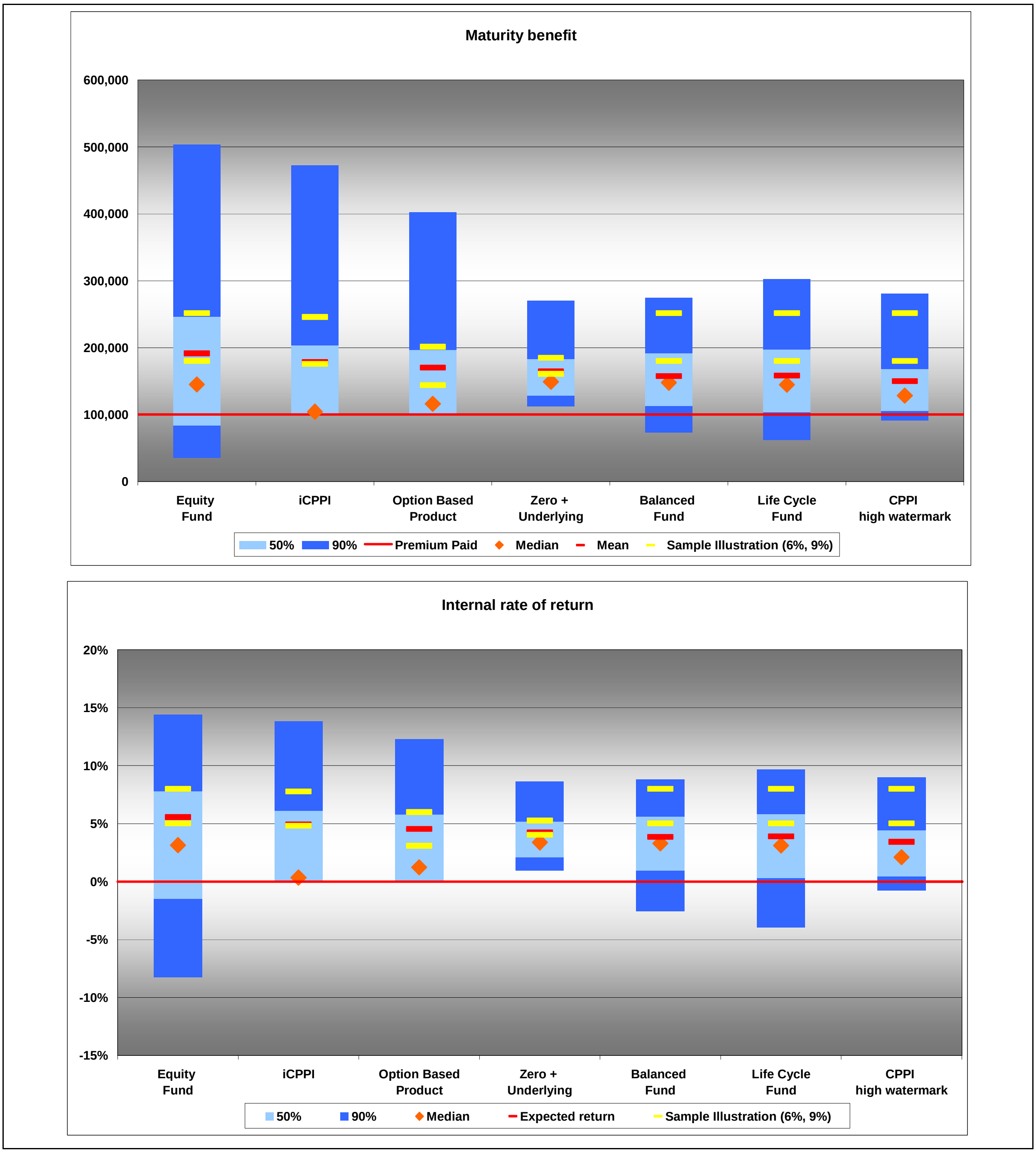


Figure 7: Risk-return profile – 12 year single premium payment

Clearly, using the methodology introduced in Section 3.1 all considered mutual fund products result in exactly the same sample illustration result since the asset allocation within the fund is completely ignored by assuming any constant illustration rate.

However, the stochastic analysis reveals significant differences in the risk-return profile of respective funds. For an investment in a pure equity fund, for example, more than 25% of the scenarios result in negative returns. Even the more conservative balanced or lifecycle funds and also the guaranteed fund (CPPI high watermark) may result in nominal losses. For all fund investments, in more than 50% of observed scenarios the illustrated return is not achieved, i.e. the distribution's median is below the sample illustration of 6%. For the CPPI high watermark fund, even the 75th percentile is below the sample illustration, i.e. the probability to meet the sample illustration result is lower than 25%. At the same time, the CPPI high watermark product de facto shows the least expected return of all considered products, a quite alarming result given that the sample illustration shows one of the highest value implying to financial advisors, that this is a product with a rather high upside potential.

The lifecycle fund and the balanced fund show a very similar risk-return profile. Due to a slightly higher average equity allocation within the lifecycle fund, it shows a slightly higher return potential but also more risk (in terms of downside risk) than the simple balanced fund.

Considering products with embedded guarantees, the iCPPI product shows similar upside potential as the pure equity investment. However, the trade-off between large upside potential and the money back guarantee is reflected in a quite low median of 0.35% p.a., i.e. 50% of the observed scenarios provide a maturity benefit very close to the single premium paid. In contrast, although the option based product's upside potential is slightly less pronounced, its median equals some "moderate" return of 1.24% p.a. and is hence significantly higher than compared to the iCPPI product. The semi-static portfolio insurance provided by the zero + underlying product shows the least variability in returns obtained.

An analysis of the risk-return profiles reveals the misleading information provided by simple approaches such as deterministic sample calculations. In particular, the analyses show (as expected) that there is no "best" product dominating all others (in terms of first order dominance). Instead, there are products that fit a given client's requirements better than others. While deterministic sample illustrations as well as backtestings simply try to rank different concepts and create the impression that the concept with the highest illustrated benefit is superior, risk-return profiles can help advisors and clients understand the upside potential and the risk embedded in a product and match this with the client's preferences. However, the complete distribution of maturity benefits might be too complex for advisors and/or clients to be understood properly. Therefore, key statistics summarizing the up- and downside potential of considered products can and probably should be employed instead. The expected return or certain percentiles such as the 75th or 95th percentile of the internal rate of return may be used to asses the upside potential of the underlying products.

With respect to the expected return, we however note that although, there is a one-to-one relationship between maturity benefits and corresponding internal rates of return, caution is required when deducing further statistics, such as mean or standard deviation, from the distribution of internal rates of return. In our numerical analysis above, the mean of the internal rate of return's distribution is calculated as 3.12% p.a. and 3.37% p.a. regarding the equity fund investment and iCPPI product, respectively. Hence, although the equity fund shows a higher average maturity benefit than the iCPPI managed product, its corresponding IRR's mean is lower. The reason is that the internal rate of return is derived as average annual return on the contributions made such that some maturity benefit is achieved, i.e. determined by a function φ on the maturity benefit. Since for an arbitrary random variable X the equation $\mathbf{E}[\varphi(X)] = \varphi(\mathbf{E}[X])$ does not hold in general, the internal rate of return of the benefit's expectation does not necessarily coincide with the expectation of the IRRs and typically differs due to Jensen's inequality. Therefore, the IRR corresponding to the expected maturity benefit – denominated as "expected return" in the following – is a more meaningful number than the expected value of the probability distribution of the random variable IRR.

Table 7 summarizes a few (downside) risk measures: Besides several shortfall probabilities, we give the expected shortfall as percentage of the premium paid (given a return lower than 0%) and a CTE measure. Analogously to the calculation of the expected return reasoned with above thoughts, the conditional tail expectation at level 95% (CTE 95) gives the internal rate of return corresponding to the average maturity benefit of the lowest 5% of the maturity benefits.

	Equity Fund	iCPPI	Option Based Product	Zero + Underlying	Balanced Fund	Life Cycle Fund	CPPI high watermark
P(IRR<0%)	32.44%	0.00%	0.00%	0.00%	17.16%	22.78%	17.31%
P(IRR<0.01%)	32.49%	20.98%	42.71%	0.00%	17.25%	22.85%	17.47%
P(IRR<2%)	43.37%	62.86%	54.40%	23.11%	35.19%	39.36%	48.66%
Expected shortfall	37.65%	0.00%	0.00%	0.00%	19.82%	24.58%	5.96%
CTE 95	-10.90%	0.00%	0.00%	0.68%	-4.02%	-5.77%	-0.86%

Table 7: Risk measures – 12 year single premium payment

Figure 7 already depicted the rather significant variability and the potential downside risk of the lifecycle managed fund compared to the balanced fund. The lifecycle managed fund indeed shows a higher risk at all considered risk-measures. Out of the guaranteed products, iCPPI and option based products have the largest upside potential (measured by the 95th percentile). However, the probability for low returns (i.e. a benefit at or close to the guaranteed benefit) is pretty large: The probability of getting exactly the guaranteed value (approximated by the shortfall probability at level 0.01%) amounts to 21% for the iCPPI) and 43% for the option based product. Also, the products have a rather high probability of returns below 2% (63 and 54%, respectively).

It is also worth noting that an investment into the CPPI high watermark fund leads to a negative CTE 95 and a relatively high probability for negative returns of about 17%. Finally, the semi-static product shows the lowest risk among the considered products under all considered risk measures.

Next, we investigate the risk-return profiles considering products with monthly premium payment of $P = 100$ summarized by the empirical distribution of their internal rates of return (cf. Figure 8).

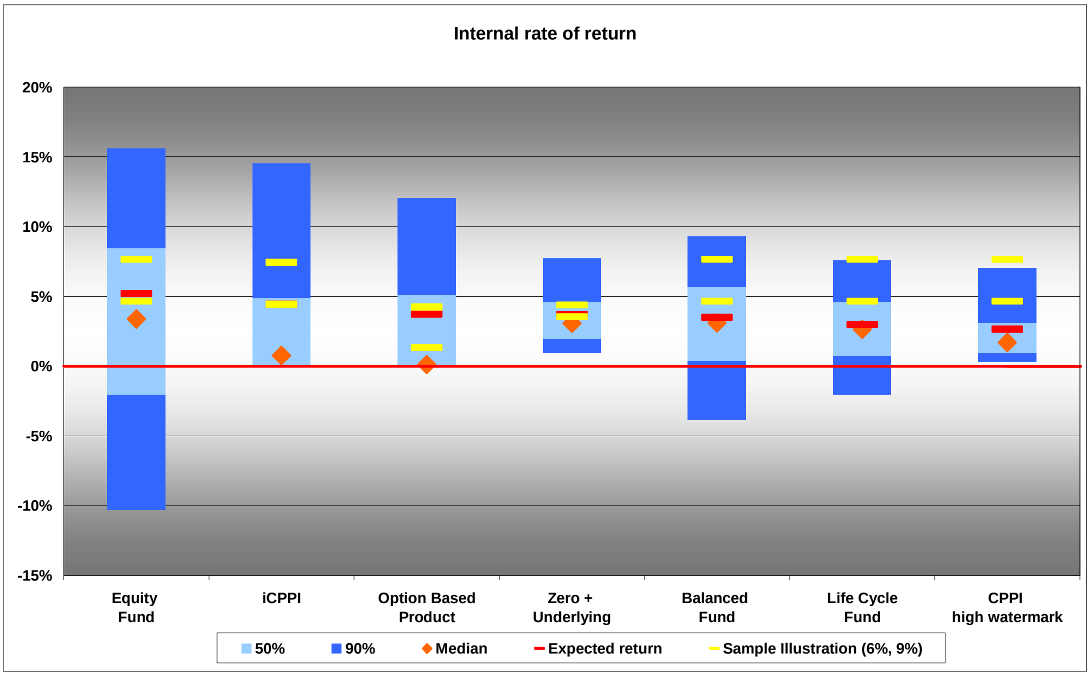


Figure 8: Risk-return profile – 12 year regular payment

The relationship between the different products looks almost the same as in the single premium case. The only relationship that significantly changes is between the lifecycle fund and the balanced fund. While in the single premium case the lifecycle fund showed more upside potential and more downside risk than the corresponding balanced fund investment, in the case of regular contributions the lifecycle fund shows a more "conservative" risk-return profile, i.e. moderate upside potential and less downside risk (cf. Table 8). The reason for this effect obviously is the time-dependent change of the fund's investment strategy (i.e. decreasing equity ratio) combined with the increasing account value of a regular-premium contract: Equity exposure is only high in the early years, where the account value is still low and therefore the average equity exposure over the term of the contract is lower than in the single premium case. For the balanced fund, however, the equity exposure is not time-dependent and thus the average equity exposure of the regular-premium contract is the same as with the single premium contract.

A similar effect can be observed for the CPPI high watermark fund where also the equity ratio is typically decreasing over time and thus the risk-return profile becomes more conservative in the case of regular premium payments.

Considering products with money back guarantee, an essential feature of the iCPPI product is revealed. New contributions made to the product help increasing the amount invested in the risky asset due to leveraging the "new" cushion generated by the new contribution (cf. Section 2).[21] This, of course, increases the return potential of the product. Consequently, the

[21] Thus, a regular-premium iCPPI-product can build new equity exposure after an event that is typically referred to as "cash-lock", i.e. an event where the equity exposure has dropped to zero. This is not the case for single-premium iCPPI products and this is also not the case for a high watermark CPPI fund.

iCPPI's median is increased compared with the single premium case and – in contrast with the single premium's analyses – even higher than that of the option based product, As in the single premium case, among the products with an embedded guarantee, the iCPPI provides the highest upside potential measured by the expected return or the 95th percentile.

Table 8 summarizes the (downside) risk measures for the case of regular premium payments.

	Equity Fund	iCPPI	Option Based Product	Zero + Underlying	Balanced Fund	Life Cycle Fund	CPPI high watermark
P(IRR<0%)	33.44%	0.00%	0.00%	0.00%	22.46%	17.72%	1.02%
P(IRR<0.01%)	33.49%	13.88%	49.44%	0.00%	22.54%	17.81%	1.08%
P(IRR<2%)	42.88%	64.21%	59.70%	25.91%	39.19%	41.32%	57.50%
Expected shortfall	25.19%	0.00%	0.00%	0.00%	13.30%	8.46%	0.76%
CTE 95	-14.04%	0.00%	0.00%	0.72%	-5.78%	-3.18%	0.12%

Table 8: Risk measures – 12 year regular premium payment

Compared to the single premium case, especially the lifecycle fund and the CPPI high watermark fund show a significantly lower shortfall probability and expected shortfall. The CTE 95 now shows a positive number for the CPPI high watermark fund even though there is still the possibility of loosing parts of the contributions. Further, in roughly 50% of the observed scenarios, the option based product just provides the contributions back due to the rather short term and the resulting quite high guarantee fee (cf. Table 6). Further, within more than 64% of observed scenarios an individually managed CPPI strategy does not return more than 2% p.a. and hence although iCPPI provides the highest upside potential among the products with money back guarantee, the probability of getting low returns is rather high.

6.2 Long-term contracts

In this section we focus on the risk-return profiles of the considered products assuming a time to maturity of 30 years.

Figure 9 depicts the resulting risk-return profiles and Table 9 gives according risk measures for the single premium case.

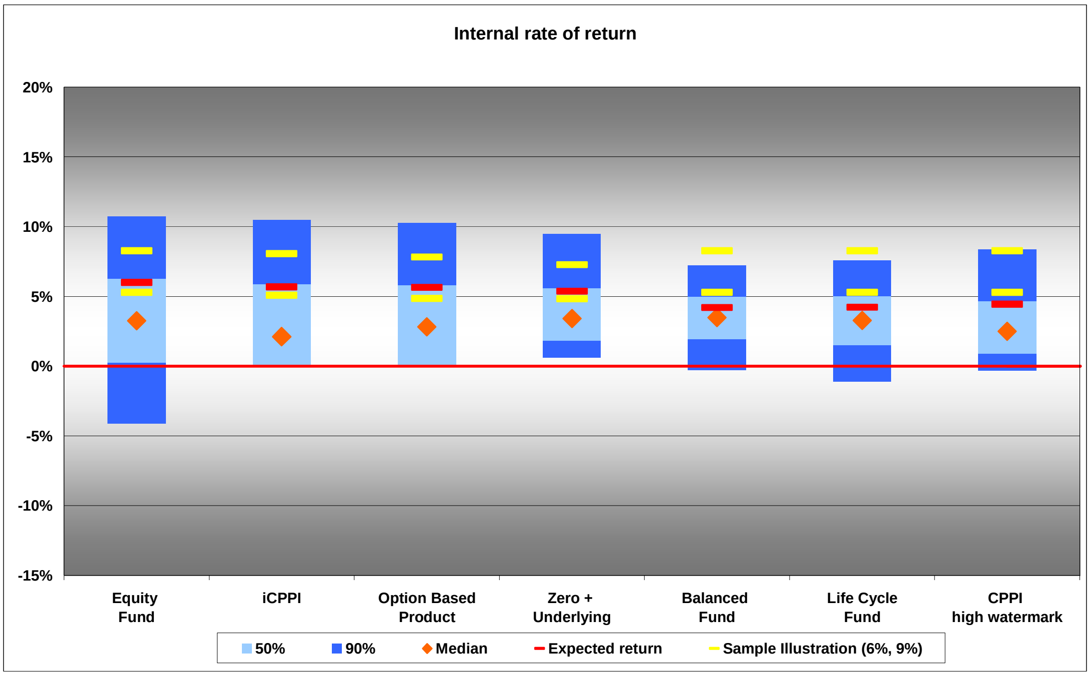


Figure 9: Risk-return profile – 30 year single premium payment

	Equity Fund	iCPPI	Option Based Product	Zero + Underlying	Balanced Fund	Life Cycle Fund	CPPI high watermark
P(IRR<0%)	23.20%	0.00%	0.00%	0.00%	6.40%	11.10%	10.28%
P(IRR<0.01%)	23.26%	30.59%	26.34%	0.00%	6.45%	11.17%	10.43%
P(IRR<2%)	38.92%	49.48%	42.75%	28.12%	25.62%	31.60%	42.93%
Expected shortfall	46.51%	0.00%	0.00%	0.00%	24.36%	29.52%	9.57%
CTE 95	-5.66%	0.00%	0.00%	0.36%	-1.18%	-2.18%	-0.52%

Table 9: Risk measures – 30 year single premium payment

Compared to the results for a short term to maturity (cf. Figure 7 and Table 7), all products show a lower downside risk in terms of reduced shortfall probabilities and generally higher 5th and 25th percentiles. However, the expected shortfall of the products without money back guarantee (equity, balanced, lifecycle and CPPI high watermark fund) is higher than compared to the short term investment. Hence, although losses are less likely for a long term contract, their impact may be higher. Whereas no further significant differences to the short-term investment of equity, balanced and lifecycle managed fund investments are found, the risk-return profile of products with embedded guarantees change quite heavily.

First, in contrast to the short term contract (Figure 7), zero + underlying and the CPPI high watermark fund now provide more upside potential than the balanced and the lifecycle managed fund in terms of expected return and the distribution's 95th percentile. This is due to the long term of 30 years and the resulting lower present value of the guarantee and hence a higher expected equity portion in both concepts. Second, while for the iCPPI product nearly 50% of the observed scenarios in the short term case only provided a return of premium paid and the product's median return was only slightly higher than 0%, the median in the long term case is 2.11% p.a.. However, there is still a 25% chance of only receiving the guarantee.

For the sake of completeness, we finally show the results for regular premium payment of $P = 100$ and a long term contract with a time to maturity of 30 years in Figure 10 and Table 10. The observed effects resulting from extending the maturity are very similar to the single-premium case. Hence, we omit a detailed explanation of the results.

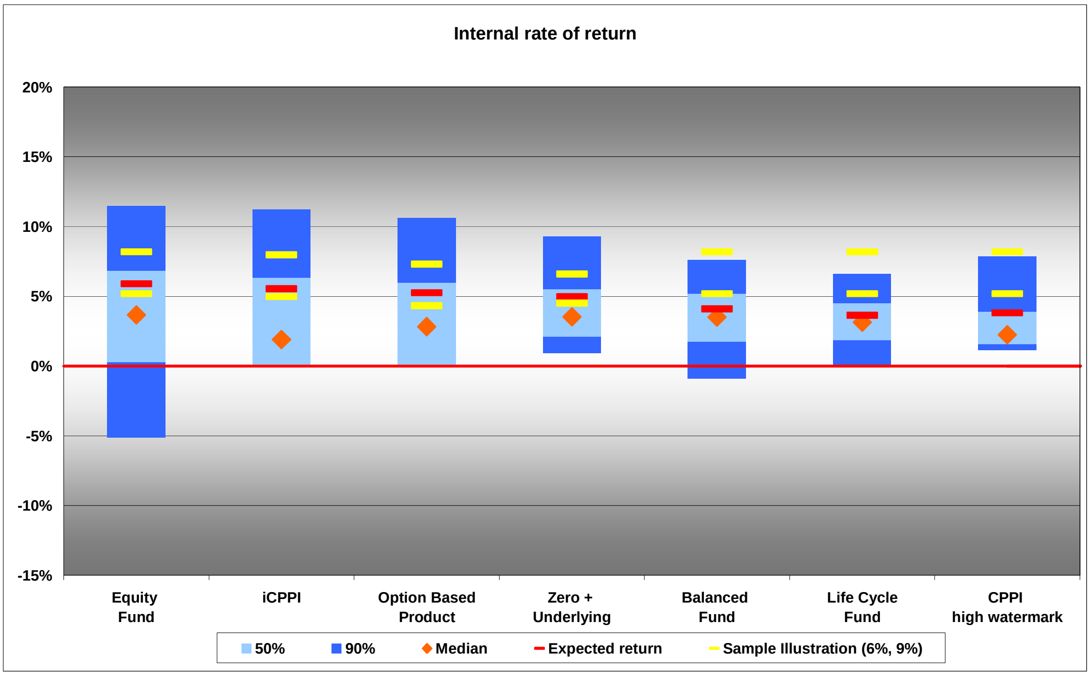


Figure 10: Risk-return profile – 30 year regular payment

	Equity Fund	iCPPI	Option Based Product	Zero + Underlying	Balanced Fund	Life Cycle Fund	CPPI high watermark
P(IRR<0%)	23.41%	0.00%	0.00%	0.00%	9.34%	4.56%	0.00%
P(IRR<0.01%)	23.48%	10.76%	28.74%	0.00%	9.40%	4.59%	0.00%
P(IRR<2%)	37.02%	50.40%	43.40%	23.25%	28.10%	27.54%	42.96%
Expected shortfall	31.95%	0.00%	0.00%	0.00%	16.49%	9.18%	0.00%
CTE 95	-7.53%	0.00%	0.00%	0.63%	-2.07%	-0.58%	0.95%

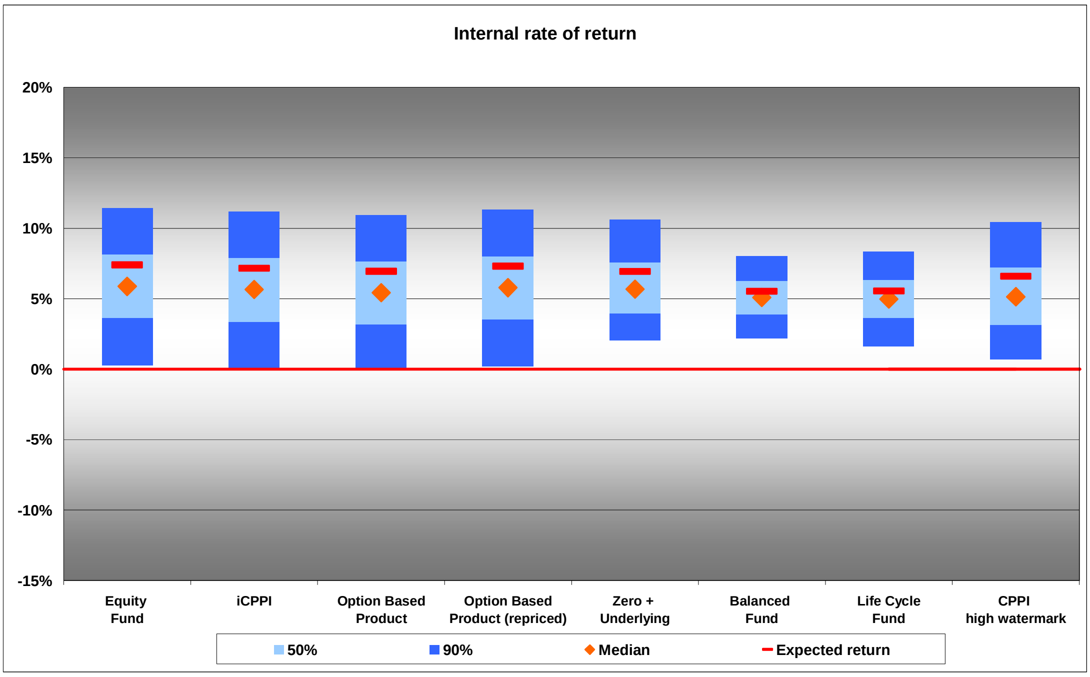
Table 10: Risk measures – 30 year regular payment

6.3 Sensitivity analyses

We now perform some sensitivity analyses with respect to the capital market parameters. We focus on a single premium contract with a time to maturity of 30 years (cf. section 6.2) and consider two sets of capital market parameters: one "good case" parameter set where high long-term interest rates (mean reversion level $\theta_r = 6\%$) are combined with low long-term equity volatilities (mean reversion level $\theta_V = 15\%$) and one "bad case" parameter set where low long-term interest rates (mean reversion level $\theta_r = 3\%$) are combined with high long-term equity volatilities (mean reversion level $\theta_V = 30\%$).

For the option based product, the risk-return profile obviously highly depends on the pricing assumptions for the guarantee. We therefore investigate two option based products in this section: One where the guarantee has been valued given the base case assumptions from Section 4 (i.e. we assume that the product has been sold and hedged under the base case assumptions and the change to the stressed capital market parameters happens immediately after the product has been sold) and one where the guarantee has been priced under the sensitivity assumptions. We refer to the latter as "option based product (repriced)".[22]

Figure 11 shows the resulting risk-return profile and Table 11 the corresponding risk measures for both parameter sets.



[22] We obtain $g = 0.09\% \, p.a.$ (good case) and $g = 1.27\% \, p.a.$ (bad case).

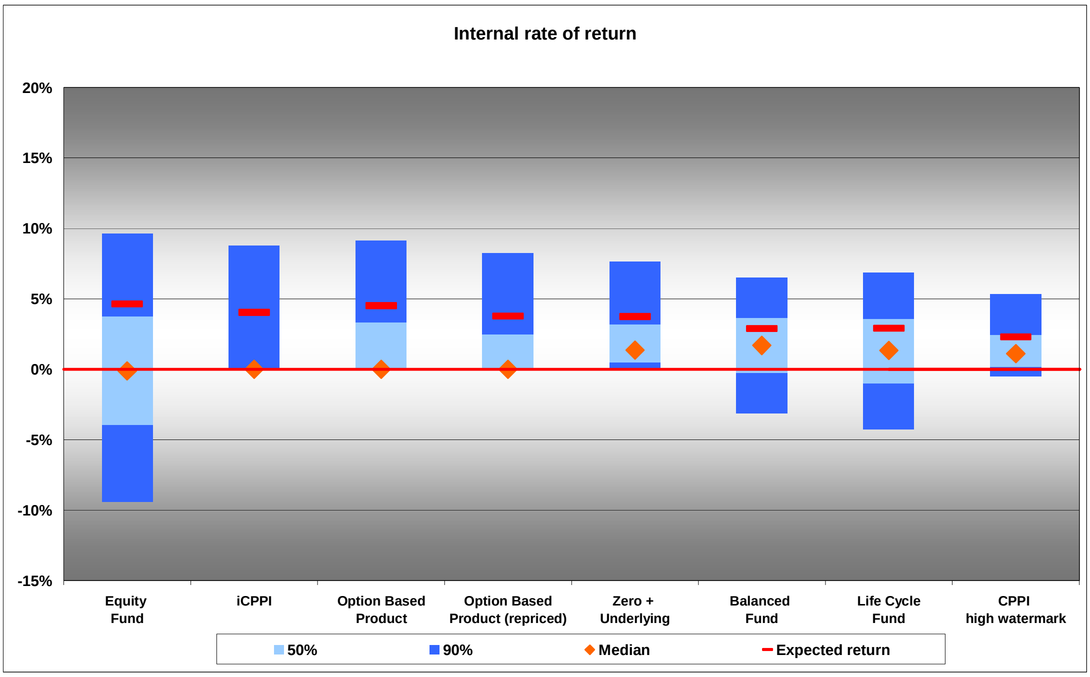


Figure 11: Risk-return profile – 30 year single premium payment considering varying capital markets (upper part: good case, lower part: bad case)

	Equity Fund	iCPPI	Option Based Product	Option Based Product (repriced)	Zero + Underlying	Balanced Fund	Life Cycle Fund	CPPI high watermark
				good case				
P(IRR<0%)	4.18%	0.00%	0.00%	0.00%	0.00%	0.22%	0.93%	2.02%
P(IRR<0.01%)	4.22%	2.72%	5.42%	4.46%	0.00%	0.23%	0.93%	2.07%
P(IRR<2%)	12.19%	15.23%	15.04%	12.70%	4.77%	3.92%	7.32%	13.91%
Expected shortfall	30.67%	0.00%	0.00%	0.00%	0.00%	14.26%	20.98%	8.99%
CTE 95	-0.95%	0.01%	0.00%	0.01%	1.52%	1.51%	0.75%	0.12%
				bad case				
P(IRR<0%)	50.69%	0.00%	0.00%	0.00%	0.00%	28.01%	34.88%	19.99%
P(IRR<0.01%)	50.75%	72.90%	53.64%	59.55%	0.28%	28.12%	34.99%	20.32%
P(IRR<2%)	64.27%	80.39%	67.15%	72.28%	61.11%	54.11%	58.09%	67.96%
Expected shortfall	63.11%	0.00%	0.00%	0.00%	0.00%	37.22%	43.62%	9.36%
CTE 95	-11.21%	0.00%	0.00%	0.00%	0.05%	-4.21%	-5.54%	-0.59%

Table 11: Risk measures – 30 year single premium payment considering varying capital markets (upper part: good case, lower part: bad case)

As expected, a change in capital market parameters does have a significant impact on the results.

In the good case, this impact is quite obvious for the equity fund investment: With a higher expected long-term level of interest rates the expected return on equity also increases leading to much more upside potential. At the same time, the lower expected long-term volatility reduces variability and thus risk. The shortfall probability, for example, is below 5%

in the good case and 5 times as high (23%) in the base case. However, the expected shortfall in the “bad case” is only 50% of that in the “good case”.

In the good case, the iCPPI product and the two option based products show a very similar risk-return profile. Of course, the option based product priced on the base case assumptions is dominated by the repriced one. The latter almost exactly matches the iCPPI product in all percentiles shown.

Due to the rather high long-term expectation in interest rates, even the products with a rather low expected equity ratio provide a decent upside potential represented by a median end expected return of at least 5% p.a. for all products. Furthermore, the low long-term average of volatility appears to be favorable for the CPPI high watermark fund resulting in an upside potential (75th and 95th percentile) close to that of the pure equity investment. Finally, for all products, the resulting risk is significantly reduced.

Opposite results are obtained for the bad case sensitivity, namely generally lower expected returns combined with a higher variability and thus higher risk for all products. Obviously, all products suffer from lower long-term expectations on interest rates and higher volatilities. However, the path-dependent CPPI products suffer most from high volatilities since these concepts “react” to the increased volatility by more frequent reallocations. For the option based products, in contrast, the embedded guarantee is delivered by some kind of hedging and the allocation in the client’s account is not path-dependent .For the iCPPI product, 73% of the scenarios lead to a maturity benefit equal to the guarantee provided. Only the 95th percentile of the empirical distribution shows some upside potential. For the CPPI high watermark fund, the median is only slightly higher than 1% p.a. and the expected return is only slightly higher than 2% showing the rather low upside potential of this product if volatility is rather high.

7 Conclusion and Outlook

In this paper, we have analyzed different practical methodologies of assessing the upside potential and downside risk of old age provision products, in particular different products with embedded guarantees (e.g. money back guarantees). In Section 3, we investigated already existing and wide-spread methodologies such as deterministic sample calculations and historical backtesting and highlighted their weaknesses potentially leading to wrong incentives and misselling by financial advisors. We then introduced a methodology of assessing the risk-return profile of any product by means of stochastic simulation. In our extensive numerical analyses, we analyzed several types of products with and without embedded guarantees and found huge differences between the actual risk-return profiles based on the maturity benefit’s probability distributions and those suggested by existing approaches so far. The introduced methodology can on one hand help matching existing products to clients’ risk appetite and demand for upside potential and on the other hand support future product development.

Although we only analyzed ”product categories”, the methodology can of course be applied to individual products, as well. Our results should therefore be of interest to financial advisors, product providers and regulators, who should all have an interest in more transparency, more information about potential benefits and risks of products sold in the market and a better match of clients' needs and products sold. Discussions how consumer information and transparency can be increased by means of stochastic simulations are currently taking place, e.g. in Germany. We therefore expect that a methodology similar to the one introduced in this paper, will be implemented for practical use, soon.

Of course, the results presented in this paper are only a first step. There are many topics left to further research. The main focus of this work was to assess the risk-return profiles of products common for accumulating money for retirement. Here, the maturity benefit as a one dimensional figure can be analyzed rather handily. However, also for the retirement phase there exists a huge variety of products differing e.g. in the amount of the guaranteed annuity and the potential for increasing annuities. Essentially the same problems – but more complex since one has to analyze a mortality dependent cash-flow stream as opposed to one single payment – as for accumulation phase products arise when trying to assess the risk-return profiles of these products with methodologies used so far. Further research could therefore focus on investigating the risk-return profiles of decumulation products taking into account annuity payments, death benefits and surrender values or other flexibilities. In this context, appropriate ways of communicating the risk-return profiles by means of several key figures is even more of a challenge than in the setting of this paper.

Within our quantitative study, although employing a stochastic volatility model, we skipped modelling any jumps in our financial model. Therefore, the possible impact of jumps – especially on the CPPI products considered – would be of major interest in line with a quantitative assessment of model risk concerning risk-return profiles.

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