Guaranteed Minimum Surrender Benefits in Variable Annuities: The Impact of Regulator-Imposed Guarantees

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Abstract

We analyze the impact of regulator-imposed minimum surrender benefits on variable annuities with a Guaranteed Minimum Accumulation Benefit (GMAB) rider. Based on recent discussions in the German market, we consider different models according to which these Guaranteed Minimum Surrender Benefits (GMSB) are determined: A minimum surrender benefit given by the present value of the GMAB calculated using market interest rates, the present value of the GMAB calculated using some technical rate of interest, and the market-consistent value of the GMAB. We look at the case where the GMSB is introduced before the contract is sold and considered in the pricing of the GMAB rider. We also consider the case when the GMSB is imposed after the contract has been sold and analyze the impact on the technical provisions and capital requirements of already existing contracts. Finally, we analyze how our results change in the presence of a secondary market. Our results show that (if considered in the pricing of the contract) a GMSB can significantly affect the fair guarantee charge of variable annuities. We also find a significant impact on the technical provisions and capital requirements of already existing contracts. Finally, our results indicate that a secondary market adversely affects the insurer's profitability but reduces the impact of the considered GMSBs on the insurers.

Keywords

Variable Annuities, Guaranteed Minimum Accumulation Benefit, Guaranteed Minimum Surrender Benefit, Pricing, Capital Requirements, Secondary Market

1 Introduction

Variable annuities are unit-linked life insurance contracts that often come with investment guarantees. Therefore, they allow policyholders to benefit from the upside potential of the underlying fund and, at the same time, offer protection when the fund loses value (cf. EIOPA, 2011). Such products offer a variety of guarantees. Besides guaranteed minimum death benefits (GMDB), three main types of guaranteed living benefits (GLB) exist: guaranteed minimum accumulation benefits (GMAB), guaranteed minimum income benefits (GMIB), and guaranteed minimum withdrawal benefits (GMWB). GMAB and GMIB offer the policyholder some guaranteed maturity value or some guaranteed annuity benefit, respectively, while GMWB allow policyholders to (temporarily or lifelong) withdraw money from their account, even after its cash value has dropped to zero. Variable annuities have experienced a growth in sales in US and Japan since the 1990s and are also becoming increasingly widespread over Europe (cf. EIOPA, 2011).

The product design of variable annuities usually stipulates that the surrender value of such products coincides with the policyholder's account value (minus surrender charges, if applicable). The "fair value" of the guaranteed benefits or the market value of certain hedge assets is typically not part of the individual policyholder's account value and thus, with the usual product design, not part of the surrender value.

The pricing of the guarantees in variable annuities is usually performed under certain assumptions for future surrender rates. Such assumptions can be, for instance, deterministic surrender or (typically) path-dependent surrender (where assumed surrender rates depend on market parameters and/or the value of the guarantee). However, the pricing is usually not performed under the assumption of "optimal" surrender (in the sense of loss-maximizing behavior from the insurer's perspective, cf. Azimzadeh et al., 2014). This reduces the price of such guarantees since – in simplified terms – future profits the insurer expects from sub-optimal policyholder behavior are given to the client by means of a reduced price for the guarantee. The possibility to allow for sub-optimal policyholder behavior in pricing and hedging of such products is a reason why these (often primarily financial) guarantees can be offered by insurers at competitive prices when compared to similar products offered by banks. This opens opportunities for institutional investors to purchase such policies in a secondary market at a price that exceeds the surrender benefit from policyholders who are willing to surrender their contract. In this situation, selling the contract to the institutional investor instead of surrendering it is beneficial for the policyholder. After acquiring the contract, the institutional investor then maximizes (optimizes) the value of the contract, which typically results in loss-maximizing behavior from the insurer's perspective. Of course, this creates risks for the insurer, most notably the risk that policyholders behave differently than assumed. In Kling et al., 2014, the authors have analyzed the resulting risk in detail.

In this paper, we use their model to additionally analyze a new, regulator-imposed risk that might arise in certain insurance markets: The risk arising from Guaranteed Minimum Surrender Benefits (GMSB). To analyze this risk, we use different exemplary approaches a regulator might choose for including GMSBs in variable annuities. The models result from different approaches that have been discussed in the German insurance industry as described below. Even though the specific approaches are based on discussions in Germany, the basic intuition behind each model could be introduced in any market.

Since its revision in 2008, § 169 of the German Insurance Contract Law (Versicherungsvertragsgesetz) requires guaranteed minimum surrender values for all insurance contracts where, both in case of death and survival, an insurance benefit is paid. For traditional life insurance contracts with an interest rate guarantee, the law even prescribes how this guaranteed minimum surrender value has to be calculated:

it is given by the prospective policy reserve which is the present value of future benefits. As a discount rate, the technical rate used for the calculation of premiums and benefits of the contract has to be applied. Therefore, the surrender value does not allow for any kind of adjustments to changing market conditions. This in consequence means that the surrender value in general is different from a "fair" market value of the contract. In particular, the surrender value will not be reduced if interest rates rise, although both, the assets backing the contract and the "fair value" of the contract, would drop. The resulting risk has been discussed e.g. in Feodoria & Förstemann, 2015. For unit-linked contracts without guarantees, the law requires the surrender value to coincide with the net asset value of the fund. For unit-linked contracts with guarantees, however, the law is not very clear, since it uses the term "time value" of the contract, which is not properly defined.

In particular, the question if and how this law has to be applied to guaranteed minimum benefits in variable annuities is controversially discussed. We consider four different possible interpretations of the law: First, there is a group of legal experts stating that the corresponding section of the insurance contract law defining guaranteed minimum surrender values is not applicable at all for typical "US-style" variable annuities (cf. Grote, 2010). Under this interpretation, the surrender value is given by the policyholder's fund value and neither future guaranteed benefits nor guarantee charges are taken into account. Second, since the law uses the word "time value", a possible interpretation is that a market-consistent value of the contract has to be paid out as surrender benefit, see Deutsche Aktuarvereinigung e.V., 2011. Third, in the same paper, the German Actuarial Association introduced an easy-to-implement method that could serve as an approximation for this market-consistent value if the value of the guarantee has to be considered in the surrender benefit. Note that this paper does not give an opinion on the question whether the value of the guarantee has to be considered at all. It only deals with the question how the value of the guarantee can be considered in case the law is interpreted such that it has to be considered. The paper was published in 2011 as an official internal note of the German Actuarial Association but repealed in 2016. Fourth and finally, based on the interpretation of minimum reserves required in the German Insurance Supervisory Law (Versicherungsaufsichtsgesetz) given in Herde, 1996, for certain other unit-linked insurance products with a maturity guarantee, a minimum reserve for the guaranteed maturity value (which is given by the guaranteed maturity value discounted with some technical interest rate) might also have to be paid out as a minimum surrender benefit. This technical interest rate is set when the contract is concluded and will not change with changing market interest rates – a similar approach as described above for traditional life insurance contracts. We therefore consider this minimum surrender benefit as a further GMSB-model in our analysis.

We analyze the effect of the considered GMSB-models on pricing, profitability, and market as well as behavioral risk. We particularly consider the effect on an insurer's profitability if a GMSB is imposed after a product has been sold. Furthermore, when assessing policyholder behavior and lapse risk, insurers are required to consider activity by institutional investors like hedge funds in a potential secondary market (cf. e.g. Central Bank of Ireland, 2010). Therefore, we also investigate the impact of the different types of GMSB on a secondary market for variable annuities. To our knowledge, such analyses, in particular with respect to variable annuities with regular premium payment, have not yet been performed.

The paper is organized as follows. In Section 2, we present the model framework that we use to conduct our analyses, including the modeling of the pool of policies, the assumed hedging strategy of the insurer, and, of course, the considered models of the GMSB. In Section 3, we present our numerical results regarding the impact of the considered GMSBs on different key figures from the insurer's perspective, such as market risk, sensitivity to changes in surrender rates and the guarantee value of the contract. In Section 4 we introduce institutional investors into our model framework. We first present an extension of the model given in Section 2 and, subsequently, present numerical results for the extended model. Finally, Section 5 concludes.

2 Model

2.1 Product design of the considered variable annuity

We consider a variable annuity contract that offers the policyholder a Guaranteed Minimum Accumulation Benefit (GMAB, see, e.g., Bauer et al., 2008), where the policyholder is entitled to a minimum account value $B^{A,g}$ at maturity of the contract. We assume the duration of the contract to be in whole years and assume all transactions and events within the contract to happen at one of the contract anniversary dates, represented by the set of integers $\mathcal{T} = \{0,1,\dots,N\}$, where N represents the number of years from contract inception until maturity. At these dates, potential premium payments are made by the policyholder or benefits are paid out by the insurer in case the policyholder decided to surrender the contract, the insured person has died or the contract has matured.

At a contract anniversary i prior to N, the premium P_i is paid by the policyholder, provided the contract is still in force (i.e. the insured person is still alive and the contract has not been surrendered) and the policyholder has not decided to surrender the contract at the contract anniversary i. For a single premium contract, we let $P_0 > 0$ and $P_i = 0$ for all successive anniversaries $i > 0$.

The minimum accumulation benefit $B^{A,g}$ guaranteed at maturity is defined as a percentage γ of the sum of premiums paid by the policyholder, i.e.

$$B^{A,g} := \gamma \cdot \sum_{i=0}^{N-1} P_i$$

and the accumulation benefit B^A is the larger of the account value F_N and the guaranteed minimum accumulation benefit:

$$B^A := \max(F_N, B^{A,g}).$$

In return for this guarantee, the insurer receives an ongoing guarantee charge as a percentage η^g of the policyholder's account value. The ongoing administration charges, also deducted from the account value, are denoted by the percentage η^a. Additionally, acquisition and administration charges are deducted from each premium payment, denoted by the percentage $\eta^{a,u}$.

The account value directly after inception is therefore given by

$$F_0 := P_0 \cdot (1 - \eta^{a,u}).$$

At any contract anniversary $i \in \mathcal{T} \setminus \{0\}$, the account value is calculated as

$$F_i := \left(F_{i-1} + P_{i-1} \cdot (1 - \eta^{a,u})\right) \cdot \frac{S_i}{S_{i-1}} \cdot e^{-(\eta^a + \eta^g)},$$

where S_i denotes the price of one share of the variable annuity's underlying fund at time i.

In case of death of the insured, the policyholder receives the stipulated death benefit at the subsequent contract anniversary and the contract expires. In what follows, B_i^D denotes the death benefit paid at

$i \in \mathcal{T}$ and τ^D denotes the first contract anniversary after the insured's death. If $\tau^D > N$ then the insured is still alive at the contract's maturity. With the considered product design, $B_i^D := F_i$ for all contract anniversaries $i \in \mathcal{T}$.

We assume that the policyholder has the right to (fully) surrender the contract at any time during the contract's lifetime. If the policyholder decides to surrender the contract, the stipulated surrender benefit B_i^S is paid out by the insurer at the subsequent contract anniversary $i \in \mathcal{T}$ and the contract expires.

We assume that the policyholder always waits until a contract anniversary before deciding whether to surrender or continue the contract. This implies that the policyholder knows the exact amount of the potential surrender benefit before deciding on whether to surrender or not. The date at which the policyholder surrenders the contract is denoted by τ^S, with $1 \leq \tau^S \leq N$. If the policyholder does not surrender the contract, we set $\tau^S = N$. We also let $B_N^S := B^A$, i.e. the surrender benefit at the end of the last year is the same as the maturity benefit.

Note that the contract expires at the time $\tilde{\tau} := \min(\tau^D, \tau^S)$ with $1 \leq \tilde{\tau} \leq N$ and the cash flow to the policyholder (or the beneficiaries) is nonzero only at time $\tilde{\tau}$ and equals either $B_{\tilde{\tau}}^S$ or $B_{\tilde{\tau}}^D$.

2.1.1 Guaranteed minimum surrender benefits

We consider four different types of guaranteed minimum surrender benefit (GMSB). As explained in Section 1, these four approaches are based on ideas discussed in Germany. Since these ideas cover a wide range of potential approaches, the results may be of relevance for any market where the introduction of GMSBs is discussed. Even if in some market a different GMSB-model is being considered, qualitatively, the effects will likely be similar to our results.

The GMSB at a contract anniversary i (before deduction of surrender charges) is denoted by $B_i^{S,g,j}$, where the superscript $j \in \{1,2,3,4\}$ indicates the type of the considered GMSB. In all four cases, the surrender benefit at a contract anniversary prior to maturity (i.e. $i \in \mathcal{T} \setminus \{N\}$) is calculated as follows:

$$B_i^S := (1 - \eta^S) \cdot \max\left(F_i, B_i^{S,g,j}\right),$$

with $j \in \{1,2,3,4\}$ and where η^S represents a time-constant surrender charge.

In the first considered case, denoted as "**no GMSB**", there is no guaranteed surrender benefit, i.e. for all $i \in \mathcal{T} \setminus \{N\}$ we set

$$B_i^{S,g,1} \equiv 0.$$

The second case is denoted as "**market-rate GMSB**". This model is similar to the approximation for the fair value given by the German Actuarial Association. Here, the policyholder receives at least the discounted guaranteed accumulation benefit $\hat{B}_i^{A,g}$ that would result if all following premium payments were zero, i.e. only premium payments made prior to t are considered:

$$\hat{B}_i^{A,g} := \gamma \cdot \sum_{k=0}^{i-1} P_k.$$

In order to calculate the guaranteed minimum surrender benefit, this hypothetical guaranteed minimum accumulation benefit $\hat{B}_i^{A,g}$ is discounted with the then-current market rate and multiplied by the probability of the insured to survive until maturity of the contract. Let $Z_i(N-i)$ denote the price of a riskless zero-coupon bond at time i with maturity at time N and let ${}_{N-i}q_{x+i}$ represent the probability that an insured who was x years old when the contract started and whose contract is still in force at time i dies within the time interval $]i, N]$. The guaranteed minimum surrender benefit at a contract anniversary $i \in \mathcal{T} \setminus \{N\}$ is then defined as

$$B_i^{S,g,2} := \hat{B}_i^{A,g} \cdot Z_i(N-i) \cdot (1 - {}_{N-i}q_{x+i}).$$

The third version of the GMSB is denoted as "**technical-rate GMSB**". This is the model that is based on the prospective minimum reserve using a technical interest rate. Here, again the present value of the hypothetical guaranteed minimum accumulation benefit $\hat{B}_i^{A,g}$ is used; however, it is now discounted with a technical, time-constant rate ξ and again weighted with the survival probability. The guaranteed surrender benefit at a contract anniversary $i \in \mathcal{T} \setminus \{N\}$ is then defined as

$$B_i^{S,g,3} := \hat{B}_i^{A,g} \cdot e^{-(N-i)\cdot\xi} \cdot (1 - {}_{N-i}q_{x+i}).$$

In the fourth design, denoted as "**MCV GMSB**", the GMSB is the market-consistent value of the GMAB from the insurer's perspective, i.e. the market-consistent value of the guaranteed minimum accumulation benefit less the market-consistent value of the future guarantee charges to be received by the insurer (not considering the option to surrender at a future date). As with the previous two types of GMSB, the valuation implies that there are no future premium payments.

In our analyses, this GMSB is calculated as the value of a European put option on the account value with strike price $\hat{B}_i^{A,g}$ minus the present value of future guarantee charges. Both, the guarantee at maturity as well as the future guarantee charges, are weighted with the corresponding survival probabilities of the insured. Let $\hat{F}_{i,k}$ denote the account value at time k assuming that since time $i < k$ there were no more premium payments, i.e.

$$\hat{F}_{i,k} := F_i \cdot \frac{S_k}{S_i} \cdot e^{-(\eta^a + \eta^g)(k-i)}.$$

At a contract anniversary $i \in \mathcal{T} \setminus \{N\}$, for the purpose of this GMSB, the value of the guarantee in a contract that is still in force is calculated as

$$\begin{aligned} \hat{V}_i^g &:= \mathbb{E}_Q^i \left[\frac{C_i}{C_N} \max\left(\hat{B}_i^{A,g} - \hat{F}_{i,N}\right) \right] \cdot (1 - {}_{N-i}q_{x+i}) \\ &= \frac{F_i}{S_i} \cdot O_i^P \left(N - i, \frac{\hat{B}_i^{A,g}}{F_i} \cdot S_i, \eta^g + \eta^a \right) \cdot (1 - {}_{N-i}q_{x+i}), \end{aligned}$$

where $\mathbb{E}_Q^i$ denotes the expectation under Q conditional on information relative to financial markets available at time i, C_i denotes the value of the cash account at time i and $O_i^P(s, K, \phi)$ denotes the price of a European put option on the underlying S_i with time to maturity s, strike price K and a drain ϕ due to charges (that are assumed to have the same effect on the option price as a dividend yield).

At a contract anniversary $i \in \mathcal{T} \setminus \{N\}$, for the purpose of this GMSB and for a contract that is still in force, the value of the future guarantee charges deducted from the policyholder's account value, denoted by $\hat{V}_i^c$, is defined as[1]

$$\begin{aligned}
\hat{V}_i^c &:= \sum_{k=i}^{N-1} \frac{\eta^g}{\eta^g + \eta^a} \cdot \left(\mathbb{E}_Q^i \left[\frac{C_i}{C_k} \hat{F}_{i,k} \right] - \mathbb{E}_Q^i \left[\frac{C_i}{C_{k+1}} \hat{F}_{i,k+1} \right] \right) \cdot (1 - {}_{k-i}q_{x+i}) \\
&= \frac{\eta^g}{\eta^g + \eta^a} \cdot \sum_{k=i}^{N-1} \left(F_i \cdot e^{-(\eta^a+\eta^g)(k-i)} - F_i \cdot e^{-(\eta^a+\eta^g)(k+1-i)} \right) \cdot (1 - {}_{k-i}q_{x+i}) \\
&= \frac{\eta^g}{\eta^g + \eta^a} \cdot F_i \cdot \left(1 - e^{-(\eta^a+\eta^g)}\right) \cdot \sum_{k=i}^{N-1} e^{-(\eta^a+\eta^g)(k-i)} \cdot (1 - {}_{k-i}q_{x+i}) \,.
\end{aligned}$$

The fourth type of considered GMSB then is calculated by adding the value of the guarantee $\hat{V}_i^g$ minus the value of future guarantee charges $\hat{V}_i^c$ to the account value F_i:

$$B_i^{S,g,4} := F_i + \hat{V}_i^g - \hat{V}_i^c.$$

This GMSB is a representation of an actual (market-consistent) "time value" of the GMAB. However, as the surrender benefit is the maximum of the fund value and GMSB less surrender charges, the GMSB only increases the surrender benefit if the time value is positive, but does not lead to a reduced surrender benefit if the time value is negative. Note also, that the actual liability of the insurer includes future premium payments and the option to surrender the contract in the future, while this GMSB only considers the sum of premiums paid so far and no option to surrender the contract. Also, in our numerical analyses, the European put option is calculated under the assumption of deterministic future interest rates, not considering the effect of future stochastic interest rates on the value of the guarantee.

2.2 Pool of policies

For our analyses on a portfolio level, we assume a pool of policies with identical contract parameters with regard to inception and maturity date, guarantee level, charges, etc. We also assume the pool of insured to be homogeneous and large enough to justify the application of the law of large numbers such that mortality henceforth is only expressed as a percentage of the pool of insured. We denote the number of contracts in the considered pool that are in force at a contract anniversary i by π_i.

The total number of contracts that expire at a contract anniversary i due to death in $]i-1, i]$ is given by

$$\pi_i^D := q_{x+i-1} \cdot \pi_{i-1}.$$

According to a 2016 survey, most variable annuity providers use the so-called "in-the-moneyness" as a key input item to model dynamic lapses in variable annuities with minimum accumulation benefits (see Hartman, 2016). We follow this approach and assume lower surrender rates when the guarantee is "in-the-money" and higher surrender rates when the guarantee is "out-of-the-money". We follow American Academy of Actuaries, 2005, by assuming that "out-of-the-money" surrender rates are twice

[1] The reasoning behind the term $\frac{\eta^g}{\eta^g+\eta^a}$ is the following (cf. DAV 2011): for the points in time t, a and b, with $0 \le t \le a < b \le N$, it holds $\mathbb{E}_Q^t \left[\int_a^b F_s \cdot \eta^g \cdot \frac{C_t}{C_s} ds \right] = F_t \cdot \frac{\eta^g}{\eta^g+\eta^a} \cdot \left(1 - e^{-(\eta^a+\eta^g)(b-a)}\right)$.

as high as “in-the-money” surrender rates and by using a comparable (but simplified) dynamic lapse function.

The policyholders in our model are assumed to surrender according to deterministic base probabilities, which are increased by a factor of 2 if the contract is “out-of-the-money”, i.e. if the contract’s discounted guarantee (assuming no future premium payments) is lower than the current surrender benefit. This represents policyholders who are not able or willing to “fully optimize” their contract, but will have an increasing tendency to surrender their contract if the guarantee appears less valuable.[2]

Therefore, the fraction of policyholders who surrender their contract at the end of the time interval $]i-1, i]$ is given by

$$s_i := \begin{cases} 2 \cdot \tilde{s}_i, & \text{if } B_i^S > \hat{B}_i^{A,g} \cdot Z_i(N-i) \\ \tilde{s}_i, & \text{else} \end{cases},$$

where $\tilde{s}_i$ represents the deterministic base surrender probability of a contract that was still in force at time $i-1$ and where the insured is still alive at time i. The total number of policyholders who surrender their contract at time $i < N$, is then given by

$$\pi_i^S = s_i \cdot \left(\pi_{i-1} - \pi_i^D\right).$$

In line with the approach in Section 2.1, we model the maturity of the contract as all remaining policyholders leaving the pool via “surrender”, i.e. we set $\pi_N^S = \pi_{N-1} - \pi_N^D$.

The number of contracts in force immediately after time i is given by

$$\pi_i = \pi_{i-1} - \pi_i^D - \pi_i^S.$$

2.3 Hedging

We assume the insurer to have a hedging program in place that aims at mitigating the effects the key financial risk drivers have on the insurer’s P&L. Guarantee charges from the pool of policies are used to finance the hedge portfolio. In return, guarantee payments are taken from the hedge portfolio’s funds. At inception, there is no cash flow to or from the hedge portfolio, i.e.

$$\Phi_0^\pi = 0.$$

The cash flow Φ_i^π from the hedge portfolio at subsequent dates $i \in \mathcal{T}, i > 0$ is given by

$$\Phi_i^\pi = -\pi_i \cdot F_i \cdot \frac{\eta^g}{\eta^g + \eta^a} \cdot \left(1 - e^{-(\eta^a + \eta^g)}\right) + \pi_i^S \cdot \left(B_i^S - F_i\right).$$

If the surrender benefit is less than the account value (due to surrender charges), this is also used for financing the hedge portfolio.

Let $\tilde{\Phi}_t^\pi$ denote the cash flow at an arbitrary time t between inception and maturity, i.e. $0 \le t \le N$,

$$\tilde{\Phi}_t^\pi = \begin{cases} \Phi_t^\pi, \text{if } t \in \mathcal{T} \\ 0, \text{else} \end{cases}.$$

[2] See Section 4 for an extension of this modeling approach, where “optimal” (loss-maximizing) behavior by institutional investors is explicitly modeled.

We denote by V_t^π the market-consistent value at time t of the cash flow $\{\Phi_i^\pi,\ i \in \mathcal{T},\ i > t\}$, i.e. the value for which the pool's guarantee-related cash flows at all future dates can be traded.

In order to replicate the changes in the value of V_t^π due to movements in the underlying fund and changing interest rate environment, we assume a hedging strategy using three hedging instruments and the hedge portfolio to be rebalanced on a regular basis. The considered hedging instruments are: cash (overnight lending/borrowing), the underlying fund (long/short exposures) and a zero-coupon paying bond with the same maturity as the variable annuity contract.

The value of the hedge portfolio at time t is denoted by Ψ_t. We assume the hedge portfolio to start with a value of zero, i.e.

$$\Psi_0 = \Phi_0^\pi = 0 .$$

We assume a simple dynamic hedging strategy that aims at offsetting changes in the value of the pool's liability resulting from changes in the underlying's price ("Delta") and shifts in the interest rate structure ("Rho"). For an arbitrary time t, let λ_t^S denote the number of shares of the underlying fund in the hedge portfolio, λ_t^Z denote the corresponding number of zero-coupon bonds with maturity in N and λ_t^C denote the sum invested in a cash account. For a rebalancing date t, let s denote the time when the last rebalancing occurred. The value Ψ_t then calculates as

$$\Psi_t = \Psi_s + \widetilde{\Phi}_t^\pi + \lambda_s^C \cdot \frac{C_t}{C_s} + \lambda_s^S \cdot \frac{S_t}{S_s} + \lambda_s^Z \cdot \frac{Z_t(N-t)}{Z_s(N-s)}.$$

At a rebalancing date t, the weights of the hedge positions in the underlying and the bond, λ_t^S and λ_t^Z, are calculated as follows:

$$\lambda_t^S = \frac{\partial V_t^\pi}{\partial S_t} \ , \lambda_t^Z = \frac{\frac{\partial V_t^\pi}{\partial r_t}}{\frac{\partial Z_t(N-t)}{\partial r_t}} \ .$$

In the simulation, λ_t^S and λ_t^Z are calculated numerically by computing the finite difference with respect to changes in the market-consistent value V_t^π.

The position in cash, λ_t^C, is a function of the other two positions:

$$\lambda_t^C = \Psi_t - \left(\lambda_t^S \cdot S_t + \lambda_t^Z \cdot Z_t(N-t)\right).$$

We assume that the insurer neither injects nor extracts any money from the hedge portfolio, such that the final value of the hedge portfolio, Ψ_N gives an indication of the insurer's profit or loss with regard to pricing and hedging the contracts' guarantees. We assume that acquisition and administration charges and expenses are not part of this consideration.

In the following analyses, we will analyze the probability distribution of Ψ_N and use corresponding (risk) measures as indicators for the market risk of the insurer.

2.4 Financial market model

For the valuation as well as the simulation, we need to project the price dynamics of the following assets: the price of one share of the variable annuity's underlying fund, S_t; the price of a risk-free (with regard to default) zero-coupon bond with time to maturity of τ, $Z_t(\tau)$; the price of the cash account, C_t; and the prices of plain-vanilla put options on the underlying fund, $O_t^P(\tau, K, \phi)$, where K denotes the strike level of the respective option and ϕ the drain due to charges (or the dividend yield, respectively).

We use a similar approach and similar model as in Ruez, 2016. However, we use an extension of the Black-Scholes model (Black & Scholes, 1973) with stochastic interest rates via the Cox-Ingersoll-Ross model ("CIR", Cox et al., 1985). Therefore, the dynamics of the market's state variables under the risk-neutral measure Q are given by

$$\begin{aligned} dr_t &= \kappa_r^Q\left(\theta_r^Q - r_t\right)dt + \sigma_r^Q\sqrt{r_t}dW_t^{Q,r}, \\ dS_t &= r_t S_t dt + \sigma_S^Q S_t dW_t^{Q,S} \\ dC_t &= r_t C_t dt \end{aligned}$$

where $W_t^{Q,r}$ and $W_t^{Q,S}$ are two independent Wiener processes under the risk-neutral measure Q. The fair value of a zero-coupon bond can be computed by closed-form formulas given in Cox et al., 1985. In order to avoid "multi-level nested simulations", the value of the plain-vanilla put option is approximated via the Black-Scholes formulas, i.e. under the assumption of deterministic future interest rates.

For the real-world simulation used to project the hedging program of the insurer, we use the same system of stochastic differential equations for the dynamics under the real-world measure P,

$$\begin{aligned} dr_t &= \kappa_r^P(\theta_r^P - r_t)dt + \sigma_r^P\sqrt{r_t}dW_t^{P,r}, \\ dS_t &= (r_t + \mu)S_t dt + \sigma_S^P S_t dW_t^{P,S} \\ dC_t &= r_t C_t dt \end{aligned}$$

where $W_t^{P,r}$ and $W_t^{P,S}$ are two independent Wiener processes under P.

3 Numerical results

In this chapter, we analyze the impact of the considered GMSB on the product's pricing, expected profit and its risk profile with respect to market and lapse risk. The parameters for the interest-rate model used for valuation are taken from Bacinello et al., 2011. We use the same set of parameters for the real-world projection – this implies a market that is risk-neutral with regard to interest-rate risk. For the equity process, we set the volatility to 10% for both, valuation and real-world projection, and we use 3% for the parameter μ in the real-world projection (as in Kling et al., 2014). Table 1 summarizes the parameters used for the market model in the base case (sensitivity analyses follow).

Parameter	Value
$r_0, \theta_r^P, \theta_r^Q$	0.03
κ_r^P, κ_r^Q	0.60
σ_r^P, σ_r^Q	0.03
σ_S^P, σ_S^Q	0.10
μ	0.03

Table 1: Market parameters used in the base case.

The parameters for the variable annuity contract are summarized in Table 2.

Parameter	Value
T	10
γ	100.0%
$\eta^{a,u}$	5.0%
η^{a}	1.0%
η^{s}	1.0%

Table 2: Contract parameters used in the base case.

The rebalancing of the hedge portfolio is assumed to happen on a monthly basis.

We assume the insured person to be 60 years old and male. We use the best-estimate mortality probabilities for annuitants given in the DAV 2004R mortality table published by the German Actuarial Association (DAV). We assume a base surrender rate of 10% in the first year that is subsequently reduced by 1% per year until it reaches 2%, i.e.

$$\tilde{s}_i = \max(2\%, 10\% - (i-1) \cdot 1\%), \qquad i = 1,2,\ldots$$

For the analyses with regard to surrender risk, we also use scenarios with increased and decreased lapse, where the base surrender rates are multiplied by 1.5 and 0.5, respectively, as well as a scenario where no surrender occurs at all.

For the technical-rate GMSB, we use $\xi = 1.25\%$.

We use 25,000 Monte Carlo paths for the valuations and 10,000 paths for the real-world simulation. Within the real-world simulation, we use 1,000 paths to compute the finite differences used in the modeling of the hedging program.

3.1 Impact of the GMSB on contract pricing

We start our analyses with a comparison of the fair guarantee charges for the four considered GMSBs (cf. Section 2.1.1). For the purpose of this analysis, the “fair guarantee charge” is the guarantee charge for which the market-consistent value V_0^{π} of the variable annuity’s guarantee-related cash flow is zero, i.e. at inception of the contract, the value of the guarantee charges coincides with the value of the potential guarantee payments. In order to illustrate the sensitivity with regard to lapse, we use the different lapse assumptions defined above.

Note that in this section, the fair guarantee charge η^{g} is calculated under the assumption that the type of GMSB is already known at the time the insurer prices the variable annuity, i.e. the product is being offered after the GMSB has been required by the regulator.

Figure 1 shows the fair guarantee charges for a single premium product (left) and a regular premium product (right).

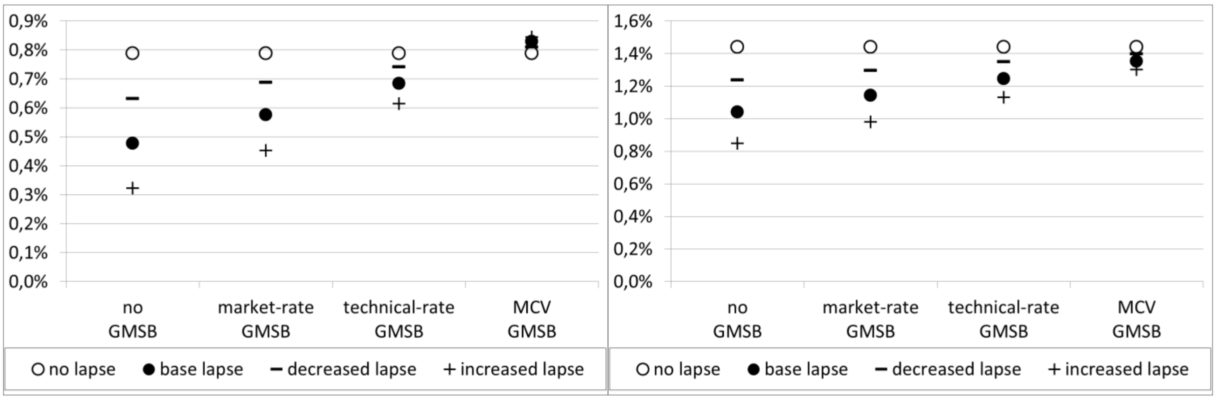

Figure 1 Fair guarantee charges for the single-premium product (left) and the regular-premium product (right).

Even though the fair guarantee charge significantly differs between the single premium and the regular premium contract, the pattern with respect to changes in surrender rates and the different types of GMSB is similar with the exception that for the MCV GMSB, the (rather low) sensitivity to surrender rates changes direction.

As expected, if no GMSB is in place, the fair guarantee charge is the lowest for all considered surrender assumptions. Under the base-lapse assumption it amounts to 0.48% for a single premium contract and 1.04% for regular premium payments. Surrender is on average profitable for the insurer and, thus, the fair guarantee charge decreases if the likelihood of surrender increases.

The addition of a GMSB causes surrender to be less profitable for the insurer. Consequently, the fair charge increases by roughly 10 bp if a market-rate GMSB is enforced and by 20 bp if a technical-rate GMSB is enforced. In case of the MCV GMSB, the fair guarantee charge increases by more than 30 bp. Thus, if the regulator imposes a GMSB in order to treat surrendering customers better, the products will become more expensive for all customers.

While a change in surrender assumptions has a considerable impact on the fair guarantee charge if no GMSB is in place, this sensitivity is reduced if a GMSB is in place. As a consequence, the "potential for mispricing" (by using incorrect surrender assumptions) is the highest if no GMSB is in place. Without GMSB, the fair guarantee charge changes by 15 bp for the single premium case and 20 bp for regular premiums if surrender rates are increased or decreased.

In the case of the MCV GMSB, the fair guarantee charge is the highest out of all considered GMSBs and almost independent of surrender assumptions. In turn, the potential for mispricing with regard to the assumed future surrender rates is fairly low. For the single premium product, the fair guarantee charge with surrender is even higher than without surrender. This means that, on average, surrender with this GMSB means a loss for the insurer, despite the earned surrender charges.

3.2 Impact of the GMSB on the guarantee value of existing contracts

In a next step, we analyze how the value of the variable annuity contract changes if a certain GMSB is enforced by the regulator (immediately) after the contract has been sold. For this and all following analyses, we assume a fixed guarantee charge of $\eta^g = 1.0\%$ p.a. for the single premium contract and $\eta^g = 1.5\%$ p.a. for the regular premium contract has been used by the insurer when the contract was sold.

We then analyze the change in the value V_0^π caused by the introduction of a GMSB. This can be interpreted as an immediate loss (in case of an increase of V_0^π) or an immediate profit (in case of a decrease of V_0^π) for the insurer caused by the regulatory change.

Figure 2 shows the value of the guarantee for the single-premium product (left) and for the regular-premium product (right) assuming a fixed guarantee charge. Here and in all following figures, values are given as a percentage of the single premium or the sum of premiums, respectively.

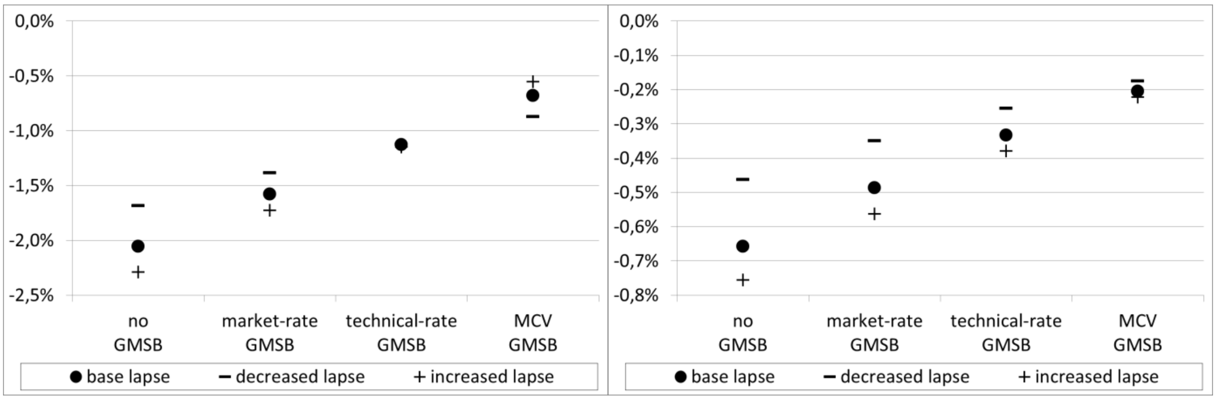

Figure 2 Value of the guarantee for the single-premium product (left) and the regular-premium product (right).

We first have a look at the results for the base lapse rates. The initial value of the guarantee if no GMSB is in place is roughly -2% of the premium for the single-premium contract and roughly -0.65% of the sum of premiums for the regular-premium contract. The introduction of a market-rate GMSB immediately after the start of the contract causes an increase of the value of the guarantee and hence a loss for the insurance company of roughly 0.5% for the single-premium contract and almost 0.2% for the regular-premium contract. The loss caused by the introduction of a technical-rate GMSB is twice as high while that caused by the introduction of a MCV GMSB is roughly three times as high.

Thus, requiring GMSBs for business in force can have a significant impact on the insurer's profitability and can in particular immediately wipe out any safety loadings or profit margins.

The loss caused by the introduction of a GMSB obviously depends on the surrender assumptions used when the contract was priced. The higher the assumed surrender rates, the higher the impact of a GMSB on the value of the guarantee, which is consistent with the results in the previous section.

We finally analyze the sensitivity of the value of the guarantee with respect to changes in surrender rates, i.e. we consider the immediate loss / profit caused by higher / lower surrender rates. As with the fair guarantee charges (cf. Figure 1), the sensitivity to surrender differs significantly between the four GMSB models and also between single and regular premium payment. Without GMSB, surrender on average is profitable for the insurer. For regular premiums surrender is always profitable although any GSMB reduces the profitability. For the single premium product, this changes: if the most valuable GMSB, the MCV GMSB, is considered, additional surrender causes a loss for the insurer, i.e. the additional value of this GMSB outweighs the surrender charges.

The difference between single premium and regular premium payments mostly results from the calculation of the GMSB in the regular premiums case. For the purpose of calculating the GMSBs, the contract is assumed to receive no more premiums after the surrender date. Simply put, if a flat guarantee charge is used, then the guarantee is overpriced for early premiums while it is underpriced for later contributions. If future premiums are not considered and the guarantee charge remains the same, the contract tends to be overpriced and, thus, often becomes less valuable to the policyholder. This is reflected in the value of the MCV GMSB in the regular-premiums case and makes surrender less valuable for the policyholder than in the single-premium case.

3.3 Impact of the GMSB on capital requirements

In market-based solvency regimes, solvency capital requirements are often calculated as an immediate loss resulting from some stress scenario. The above sensitivities can therefore also be interpreted as an indication for solvency capital requirement (SCR) for lapse risk, where only the "direction of change" (increase or decrease of lapse rates) that causes the highest loss for the insurer has to be considered.

Additionally, we will now give an indication for the SCR for market risk via the following approach:

In contrast to e.g. the Solvency II standard formula, where only the risk resulting from an immediate (or one year) shock is being considered, we consider a full lifetime projection of the pool of policies in order to assess the market risk resulting from the total remaining lifetime of the contracts, cf. Section 164 in EIOPA, 2011 in connection with Article 122 of the Directive 2009/138/EC. This includes the risk from accumulated hedging errors, particularly hedging errors that occur close to maturity of the contracts.

We account for the longer time horizon by setting the VaR level at 95% and consider the respective percentile of the real-world distribution of the (discounted) profit/loss $\frac{\Psi_N}{C_N}$ for the insurer after hedging is applied (cf. Section 2.3). We interpret the difference of this 95th percentile and the present value of the guarantee V_0^π as (an indicator for) the SCR for market risk, cf. also Central Bank of Ireland, 2010.

The resulting number shows how much solvency capital needs to be added on the value of the guarantee in order to meet all liabilities until the maturity of the contract with a probability of 95%.

In Figure 3, we show the surrender sensitivities for lapse risk as well as the considered capital requirement for market risk for the single-premium product (left) and for the regular-premium product (right).

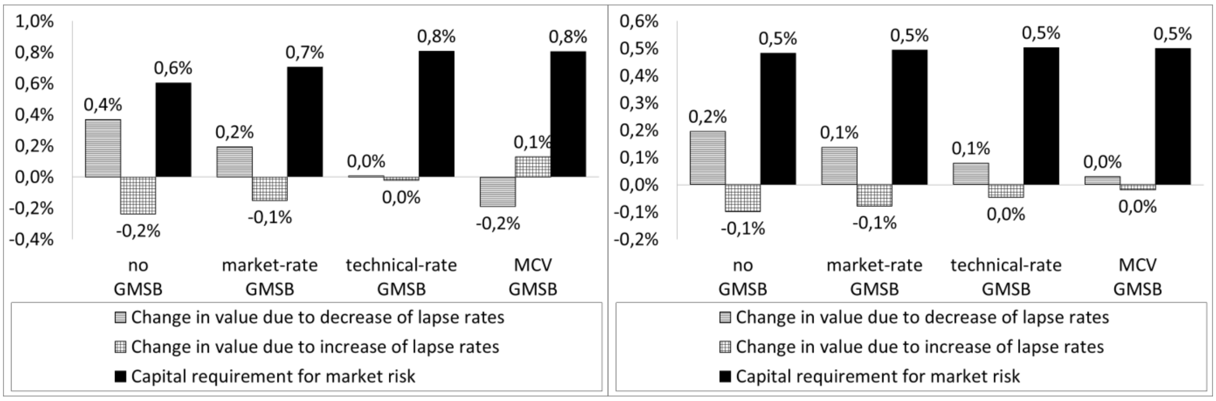

Figure 3 Surrender sensitivities for lapse risk and capital requirement for market risk for the single-premium product (left) and the regular-premium product (right).

For all considered GSMB models, the SCR for market risk remains below 1% of the sum of premiums. While for the regular-premium product market risk is more or less the same for all types of GMSB, it increases with the introduction of GMSBs for the single-premium product. This is in line with the findings regarding the impact on the guarantee value. For regular-premium products the SCR for market risk is roughly 0.5% of the sum of premiums for all GMSBs. For the single premium product, it fluctuates between 0.6% and 0.8% of the single premium.

The considered SCR for surrender risk overall is smaller, but depends more heavily on the GMSB. Again, we can observe that, depending on the GMSB type, the risk of the insurer can lie in either increased or decreased surrender. It is worth noting that the risk arising from the introduction of a certain GMSB after the product has been sold (see Figure 2) is significantly higher than the considered SCR for the risk arising from changes in surrender behavior once a GMSB is in force.

3.4 Sensitivity to interest rates

In a next step, we will perform capital market sensitivities. We start with an analysis of the impact of lower interest rates on the value of the guarantee and capital requirements. For this, we set r_0, θ_r^P and θ_r^Q to 1.5% (as opposed to 3.0% in the base case). As with the introduction of the GMSBs, we assume that the change happens after the variable annuity has been sold, i.e. the pricing is not adjusted to the new interest rate level. This represents a scenario in which, after the variable annuity has been sold with the guarantee charge used in the previous sections, the embedded guarantee (and also the modeled hedging portfolio) becomes rather valuable. In such a scenario, without a GMSB, it is highly profitable for the insurer if the policyholder decides to surrender, since in this case the value of the guarantee remains with the insurer. The addition of a GMSB reduces this effect and is thus potentially especially harmful for the insurer in such a scenario.

Figure 4 shows the value of the guarantee for the single-premium product (left) and for the regular-premium product (right) for low interest rates. In Figure 5, we show surrender sensitivities for lapse risk as well as the considered capital requirement for market risk for the single-premium product (left) and for the regular premium product (right) for low interest rates.

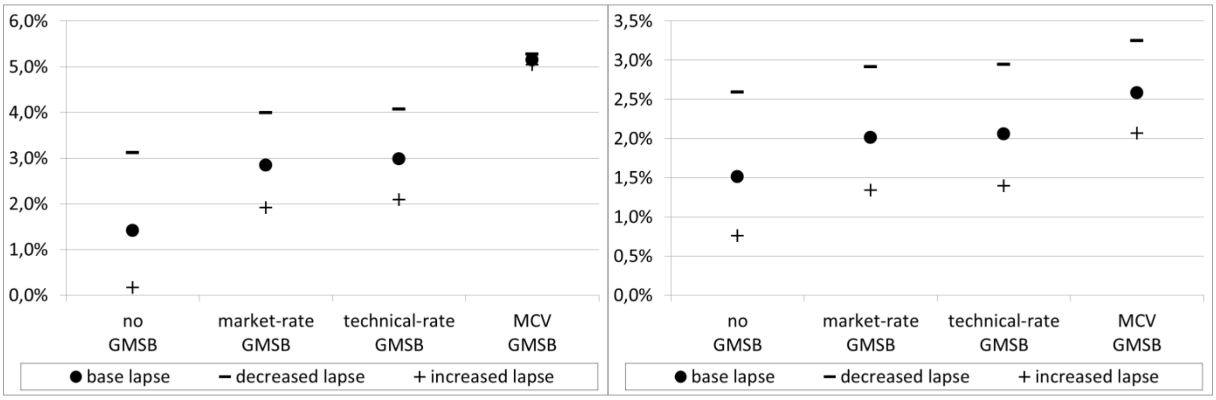

Figure 4 Value of the guarantee for low interest rates, single-premium product (left) and regular-premium product (right).

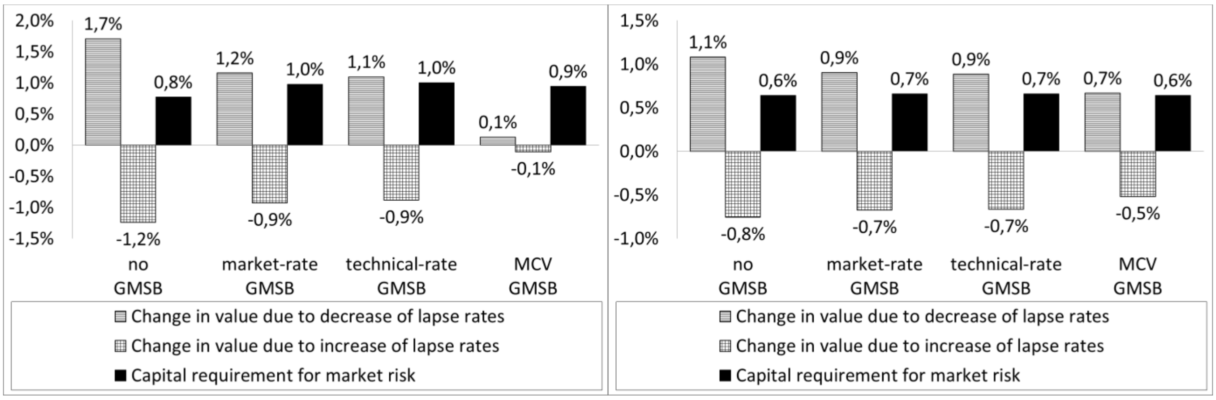

Figure 5 Surrender sensitivities for lapse risk and capital requirement for market risk for low interest rates, single-premium product (left) and regular-premium product (right).

Obviously, a change of interest rates has a tremendous effect on the results. We can see from Figure 4 that the value of the guarantee has increased and is now positive, independent of the assumed surrender behavior and the GMSB model. This means the present value of future expected guarantee payments exceeds the present value of future expected guarantee charges. As such, surrender is highly profitable for the insurer. Overall, without a GMSB, the sensitivity with respect to surrender has strongly increased in comparison to the base case. While market risk has increased only slightly in comparison with the base case and still does not exceed 1% of the sum of premiums, surrender risk can be as high as 1.7% if no GMSB is in place.

As expected, with the higher value of the guarantee, the immediate loss resulting from introducing a GMSB also increases. In the case of the single-premium product, the immediate loss from introducing the MCV GMSB is roughly 4% of the premium. Also, the sensitivity for lapse risk is significantly reduced, i.e. from the insurer's perspective it is now more or less irrelevant if the policyholders do surrender or not. This does not hold for regular premiums, again due to the assumption of a contract with no more future premium payments. With the other, less valuable, GMSB models, surrender is still profitable for the insurer. That means the corresponding guaranteed surrender benefits are below the market value of the contract.

3.5 Sensitivity to equity volatility

Finally, we perform a similar sensitivity analysis with respect to the equity volatility. We assume the equity volatility (i.e. σ_S^P and σ_S^Q) to be 12.5% (as opposed to 10.0% in the base case). As with the scenario of lower interest rates, we assume the change to happen immediately after the variable annuity has been sold with the guarantee charge used in the previous sections.

Figure 6 shows the value of the guarantee for the single-premium product (left) and for the regular-premium product (right) for the increased equity volatility. In Figure 7, we show surrender sensitivities for lapse risk as well as the considered capital requirement for market risk for the single-premium product (left) and for the regular-premium product (right).

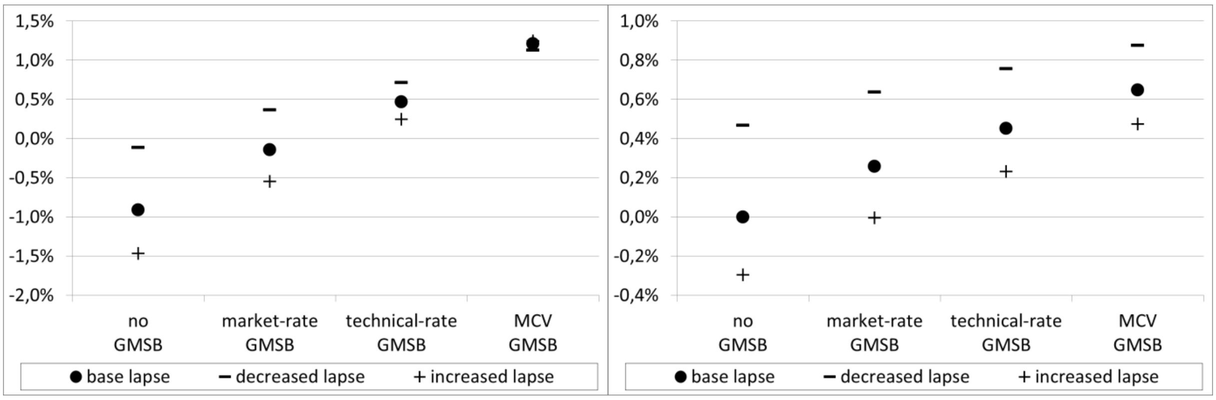

Figure 6 Value of the guarantee for increased equity volatility, single-premium product (left) and regular-premium product (right).

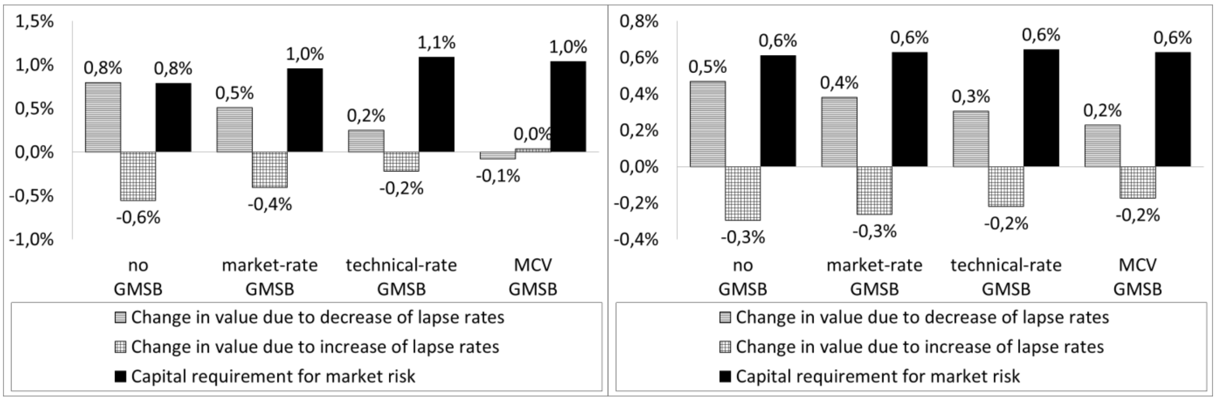

Figure 7 Surrender sensitivities for lapse risk and capital requirement for market risk for increased equity volatility, single-premium product (left) and regular-premium product (right).

The value of the guarantee as well as both considered risk indicators increase with higher equity volatility. The considered SCR for lapse risk increases more strongly than the SCR for market risk and, for some GMSBs, both reach similar levels.

The immediate loss caused by the introduction of a GMSB increases only slightly but, at the same time, sensitivity with respect to surrender increases. Changing volatility, therefore, not only has an impact on the insurer's market risk, but can have an even higher impact on the insurer's lapse risk.

4 Impact of a secondary market

In this section, we analyze the impact of a secondary market with "rational", i.e. value maximizing, investors. We assume that those policyholders who are willing to surrender their contract have the possibility to alternatively sell their policy to some institutional investor in the secondary market. After purchasing the contract, the investor then acts "rationally".

We assume that the presence of a secondary market has not been considered in the pricing of the contract and therefore use the same contract and guarantee charges as in Chapter 3.

4.1 Model description

We use the model from Section 2 unless stated otherwise in this section.

We assume that the pool of policyholders consists of two groups: Policyholders of "type A" behave according to the model used in Section 2.2. We denote the number of contracts in this group at time i by π_i^A.

The policyholders of type "B" are the investors purchasing policies in the secondary market. They will surrender their contract as soon as the contract's continuation value (assumed to be the market-consistent value of the contract) drops below the surrender benefit (after deduction of surrender charges). The market-consistent value includes the value of all guarantees, i.e. the GMAB as well as the GMSB. From the insurer's perspective, these investors represent a worst-case "loss maximizing" behavior (cf. Azimzadeh et al., 2014). We denote the number of contracts in this group at time i by π_i^B.

The total number of contracts in force at time i is therefore given by $\pi_i := \pi_i^A + \pi_i^B$.

The total number of contracts that expire at time i due to death of the insured person is given by

$$\pi_i^D := \pi_i^{A,D} + \pi_i^{B,D}, \text{where}$$
$$\pi_i^{*,D} = q_{x+i-1} \cdot \pi_{i-1}^{*}\text{, with } * = A, B.$$

Similarly, let s_i^* represent the fraction of policyholders in the sub-pools π_{i-1}^A and π_{i-1}^B, respectively, who want to surrender their contract at the end of the time interval $]i-1, i]$.

The policyholders in sub-pool A are assumed to be "willing to surrender" as explained in Section 2.2. However, we now assume that there exists a secondary market and a certain percentage λ of the policyholders of type A who are willing to surrender the contract will instead sell the contract to an institutional investor (i.e. a policyholder of type B) if such an investor offers to pay more than the surrender benefit. Since an institutional investor would only offer a price exceeding the surrender value if the continuation value exceeds the surrender benefit, the number of transitioning contracts, denoted by $\pi_i^{A\to B}$, then is given by

$$\pi_i^{A\to B} := \begin{cases} \lambda \cdot s_i^A \cdot \left(\pi_{i-1}^A - \pi_i^{A,D}\right), & \text{if } B_i^S < CV_i \\ 0, & \text{else} \end{cases},$$

where CV_i represents the market-consistent continuation value of the contract at time i (assuming loss-maximizing behavior of the policyholder). Note that, with regard to the pool of policies, surrender is assumed to happen right before the next contract anniversary and, thus, only contracts where the insured is still alive at the end of the interval can be surrendered.

The total number of policyholders in sub-pool A that surrender their contract at time $i < N$, denoted by $\pi_i^{A,S}$, then is given by

$$\pi_i^{A,S} = s_i^A \cdot \left(\pi_{i-1}^A - \pi_i^{A,D}\right) - \pi_i^{A\to B}.$$

Policyholders in the sub-pool B surrender their contract if and only if the surrender benefit exceeds the continuation value:

$$s_i^B := \begin{cases} 1, & \text{if } B_i^S > CV_i \\ 0, & \text{else} \end{cases}.$$

The total number of policyholders in sub-pool B that surrender their contract at time $i < N$, denoted by $\pi_i^{B,S}$, then is given by

$$\pi_i^{B,S} = \left(\pi_{i-1}^B - \pi_i^{B,D} + \pi_i^{A\to B}\right) \cdot s_i^B.$$

The total number of contracts that expire at time $i < N$ due to surrender, π_i^S, then is given by

$$\pi_i^S := \pi_i^{A,S} + \pi_i^{B,S}.$$

The number of contracts in the two sub-pools immediately after time i, i.e. after contracts that matured due to surrender or death of the insured have left the respective pool, are given by

$$\pi_i^A = \pi_{i-1}^A - \pi_i^{A,D} - \pi_i^{A,S} - \pi_i^{A\to B} \text{ and}$$
$$\pi_i^B = \pi_{i-1}^B - \pi_i^{B,D} - \pi_i^{B,S} + \pi_i^{A\to B}.$$

4.2 Numerical results

We use the parameters from Section 3 and set $\lambda = 0.5$. This means we assume that half of the policyholders of pool A who are willing to surrender their contract would rather sell it to an institutional investor (if the investor offers a price that exceeds the surrender benefit). For the purpose of calculating the loss-maximizing behavior and the corresponding continuation value, we use the Longstaff-Schwarz method (cf. Longstaff & Schwartz, 2001, and Bacinello et al., 2011).

We first compare the value of the guarantee with and without a secondary market. Figure 8 shows the value of the guarantee for the single-premium contract without secondary market (left) and with a secondary market (right). Figure 9 shows similar charts for the regular-premium contract.

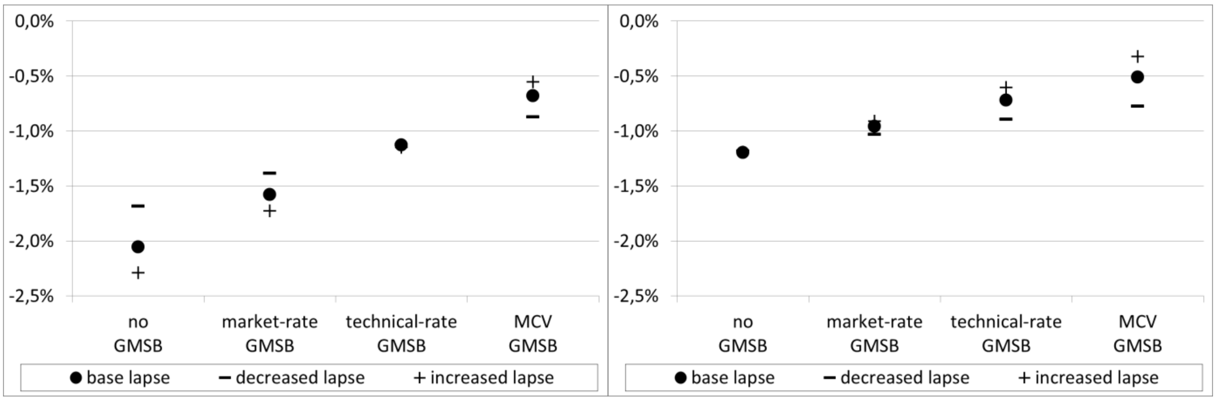

Figure 8 Value of the guarantee for the single-premium contract without secondary market (left) and with secondary market (right)

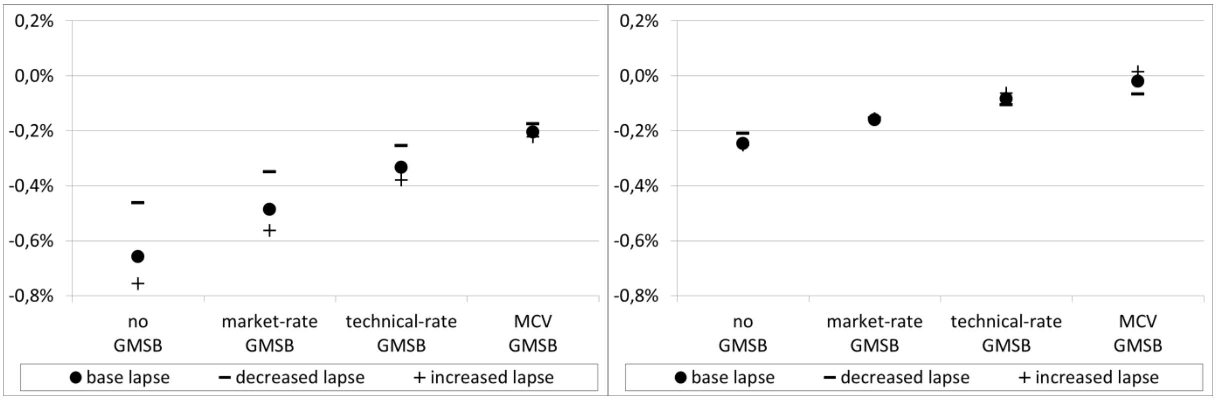

Figure 9 Value of the guarantee for the regular-premium contract without secondary market (left) and with secondary market (right)

For both, the single premium and the regular premium contract, the value of the guarantee is significantly increased by the introduction of a secondary market. This means that – as expected – a secondary market can significantly reduce the insurer’s profitability.

In the single premium case, the introduction of a secondary market leads to an immediate loss of almost 1% of the sum of premiums for the insurance company if no GMSB is in place. For the market-rate GMSB and the technical-rate GMSB, the immediate loss caused by a secondary market is roughly 0.5% of the single premium. For all these GMSB models, there are situations where the insurer can profit from surrender if no secondary market exists. The introduction of the secondary market reduces these profits due to the “rational” behavior of the investor.

A secondary market can only exist if surrender benefits are lower than the continuation value. Otherwise it is not possible to offer the policyholders a price that exceeds the surrender benefit. Since in case of the MCV GMSB the surrender value is rather close to the continuation value, the impact of a secondary market on the value of the guarantee is much lower.

For the regular-premium product, the effects are similar but in general less pronounced. An exception is the MCV GMSB where (in contrast to the single premium case) the introduction of a secondary market has a relatively pronounced effect. This shows that in case of regular-premium payment the MCV GMSB does not fully represent the market-consistent value of the whole contract, which is consistent to the findings in Section 3.2.

In Figure 10 we show surrender sensitivities for lapse risk as well as the considered capital requirement for market risk for the single-premium contract without secondary market (left) and with secondary market (right). Figure 11 shows similar charts for the regular-premium contract.

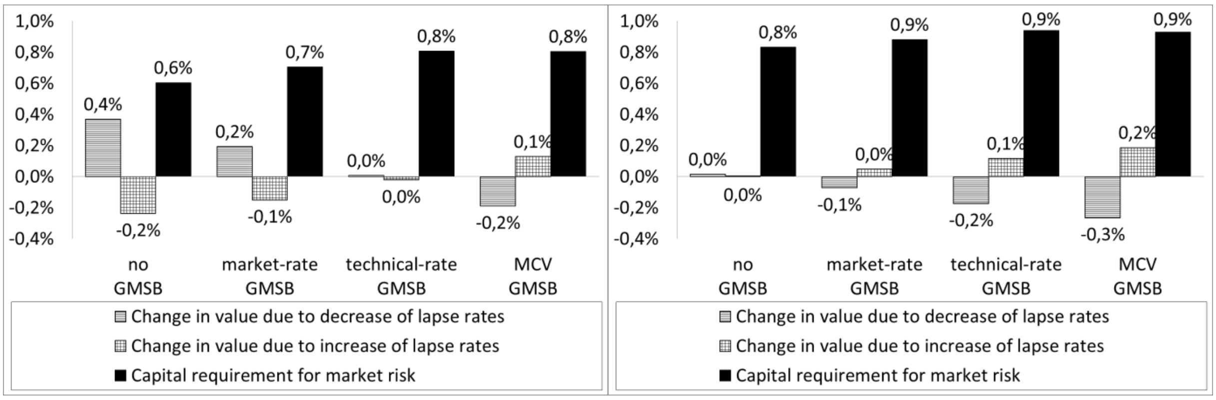

Figure 10 Surrender sensitivities for lapse risk and capital requirement for market risk for the single-premium contract without secondary market (left) and with secondary market (right)

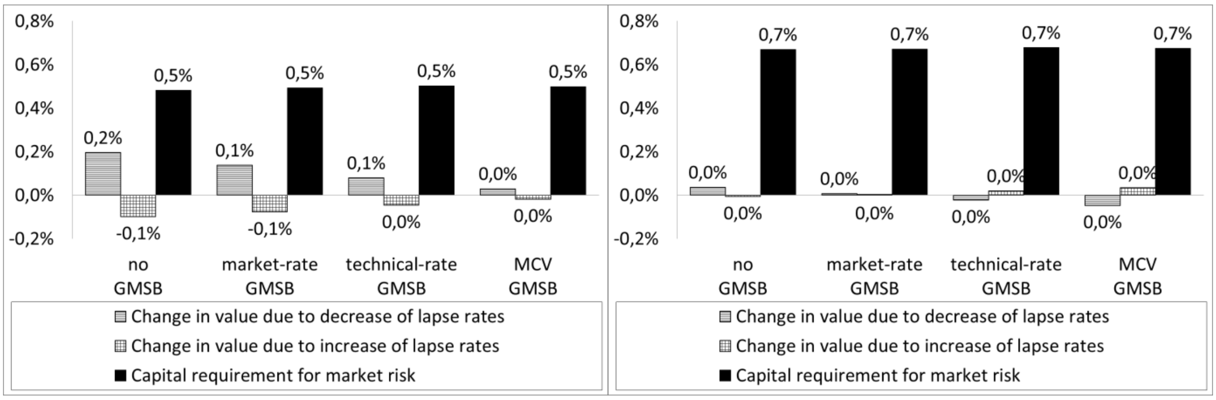

Figure 11 Surrender sensitivities for lapse risk and capital requirement for market risk for the regular-premium contract without secondary market (left) and with secondary market (right)

Figure 10 and Figure 11 confirm the effects described above: lapse risk in general decreases if a secondary market exists with the exception of the MCV GMSB.

Also, the investor typically does not surrender in scenarios where the guarantee is valuable and (absolute) hedging errors tend to be higher and vice versa. This emphasizes hedging errors and, therefore, additionally increases market risk in the presence of a secondary market for both considered contracts and all considered GMSBs.

5 Conclusion

In the present paper, we have analyzed the impact of the introduction of GMSBs to a variable annuity contract with a GMAB. We have considered both scenarios: An introduction of GMSBs after a contract has been sold as well as offering new contracts in a market with mandatory GMSBs. We have analyzed the impact on key figures such as the fair guarantee charge of the contract, the guarantee value of the contract, capital requirements with respect to market risk and the sensitivity with regard to surrender rates. We found that, while the impact of GMSBs on market risk is relatively low, the impact on the fair guarantee charge, the guarantee value and the risk resulting from changes in policyholder behavior is substantial.

If the GMSB model is already known and considered when pricing the contract, the resulting advantage for policyholders who surrender the contract comes at the price of increased guarantee charges for all policyholders, adversely affecting especially those who keep the contract until maturity. The fair charge increases only slightly if a market-rate GMSB is enforced, twice as much if a technical-rate GMSB is enforced and roughly three times as much in case of the MCV GMSB. As a consequence, the same protection level with regard to old-age provision becomes more expensive when GMSBs are in place.

If a GMSB is introduced after inception of the contract, e.g. because of a regulatory change, the insurer will suffer an immediate loss on its market-value balance sheet. This loss is the highest if a MCV GMSB is introduced and the lowest if a market-rate GMSB is introduced. While the value of the contract increases with the value added by the GMSB, the sensitivity with regard to surrender rates decreases, as, from a valuation perspective, it becomes less important whether policyholders decide to surrender or not. As a consequence, the potential for mispricing of the contracts with respect to incorrect surrender assumptions is reduced. The market-rate GMSB shows the lowest potential for mispricing with respect to surrender assumptions.

Our analyses with regard to the impact of a secondary market show that, in a market without GMSBs, the presence of an institutional investor creates a loss for the insurer and also increases market risk. At the same time, the impact of introducing GMSBs is reduced and the specific design of the GMSB is less relevant. On the other hand, if GMSBs are already in place, the potential for a successful secondary market is reduced, since the difference between the surrender benefit of a contract and its continuation value is typically lower.

With a GMSB in place, institutional investors are less likely able to offer prices that exceed the surrender benefit. On the other hand, after the investor has bought a contract, GMSBs offer them additional value that can be exploited by optimized surrender behavior.

Our results strongly indicate that regulators considering the introduction of mandatory GMSBs should carefully analyze the potential impact – also on contracts with different forms of guarantees than the simple GMAB guarantee used in this paper. In particular, the following effects should be considered: For new business, GMSBs will cause increased guarantee charges for the policyholder. This creates redistribution effects from policyholders not surrendering their contract towards surrendering policyholders. Imposing mandatory GMSBs also on already existing contracts increases insurers' risk and can have severe adverse effects on insurers' profitability.

6 Literature

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