Institut für Finanz- und Aktuarwissenschaften

Coherent Projections of Age, Period, and Cohort Dependent Mortality Improvements

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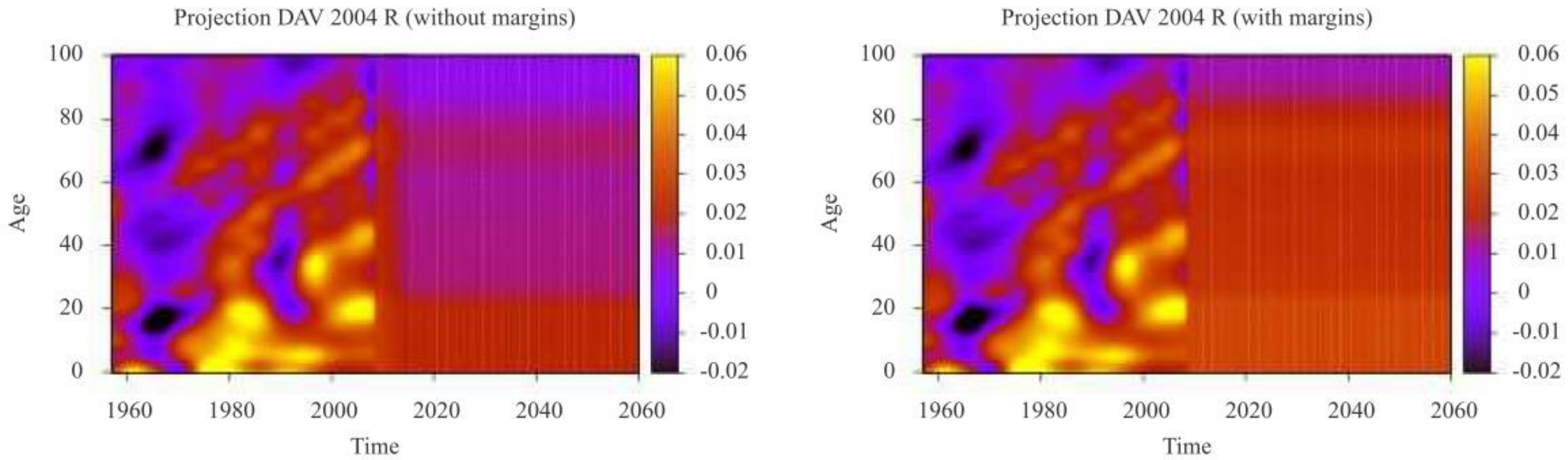
Introduction

Some of the currently used standard projections have significant shortcomings

Example: standard projection for German annuity business



- Structural break between historical and projected improvements
- No cohort effects in projection
- Possibly significant underestimation of future mortality improvements

→ **Space and need for improved projection methodologies**

Introduction

Recent research has focused on stochastic mortality modeling

However, quantification of uncertainty needs to build on realistic best estimate assumptions

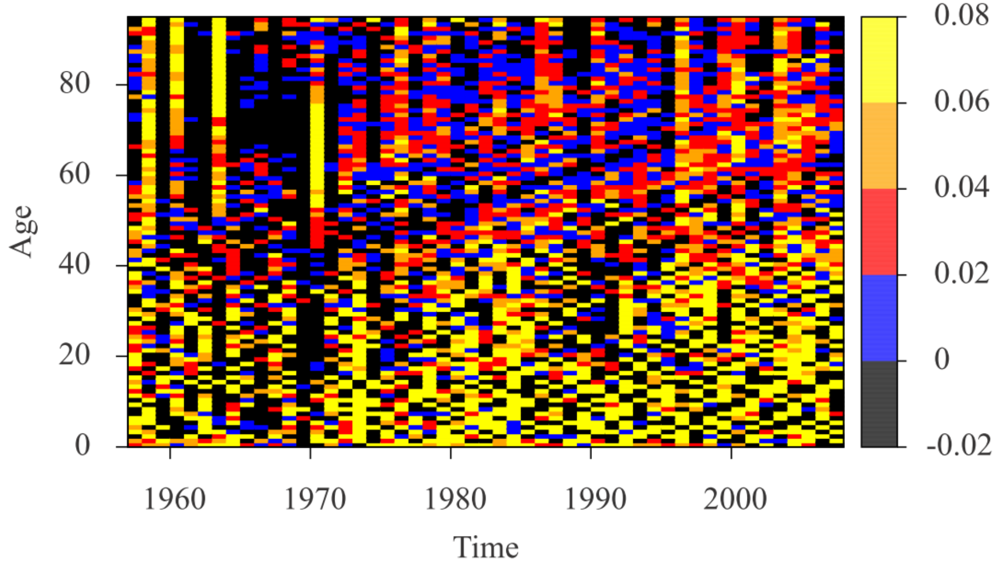
→ We take a step back and focus on best estimate projections

Main goals of new projection methodology:

- Extrapolation of historical mortality patterns, e.g. cohort effects
- Plausible long-term level of mortality improvements
- Coherence between projections for males and females
- Coherence between populations from closely related countries

Model Specification

Raw historical mortality improvements for German males



- Historical data show period and cohort dependent effects
- Mortality improvements have often been shown to be age dependent as well
- Very similar findings for other populations, e.g. German females or US males/females

→ We model one-year mortality improvements according to the **Age-Period-Cohort (APC) model**:

$$v(x,t) = \frac{q(x,t-1) - q(x,t)}{q(x,t-1)} = a_x + p_t + c_{t-x}$$

Model Estimation and Projection

General Procedure

Age and cohort parameters derived from model fit to historical mortality improvements

- Assume that historical parameter values are also valid for the future
- Parameters for new cohorts as reasonable extrapolation of historical cohort parameters

Period parameters projected based on period life expectancy extrapolations

- Chose period parameter such that life expectancy trend is observed
- Life expectancies evolve steadily and often show clear patterns
- Extrapolation of historical trends ensures adequate level of future improvements
- Period life expectancy at any age could be considered, e.g. at birth or at age 65
- Coherence between populations at aggregate level
- Straightforward derivation of stress scenarios and margins

Model Estimation and Projection

General Procedure

General concept: **direct implementation of one`s own expectation of future mortality**

- Standard approach is to choose the statistical model which comes closest to own expectation
- Reconciliation of data driven extrapolations with intuition
- No standard procedure for model estimation and projection
- Case specific assumptions since there is no model which fits in every situation

→ Framework for model estimation only

In the following, introduction of estimation framework

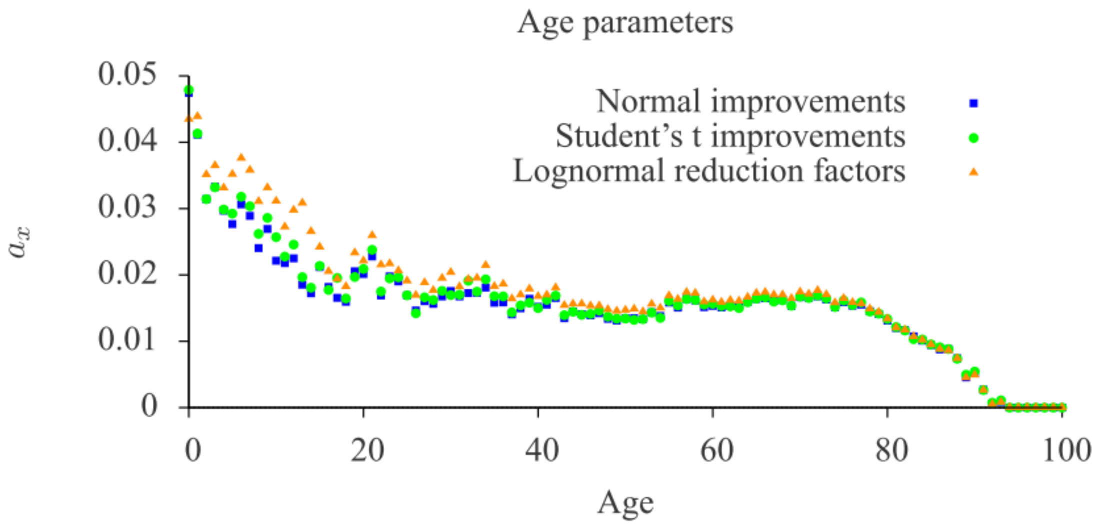
- Illustration by deriving projections for German males and females
- However, model can be applied to any population with sufficient data

Estimation of Age and Cohort Parameters

Four Steps

1. Model fitting to historical data

- Weighting to account for significantly different variation in numbers of deaths
- Different distributional assumptions may be reasonable → statistical tests
- Extrapolation to very old ages
- Example results: age parameters for German males



Estimation of Age and Cohort Parameters

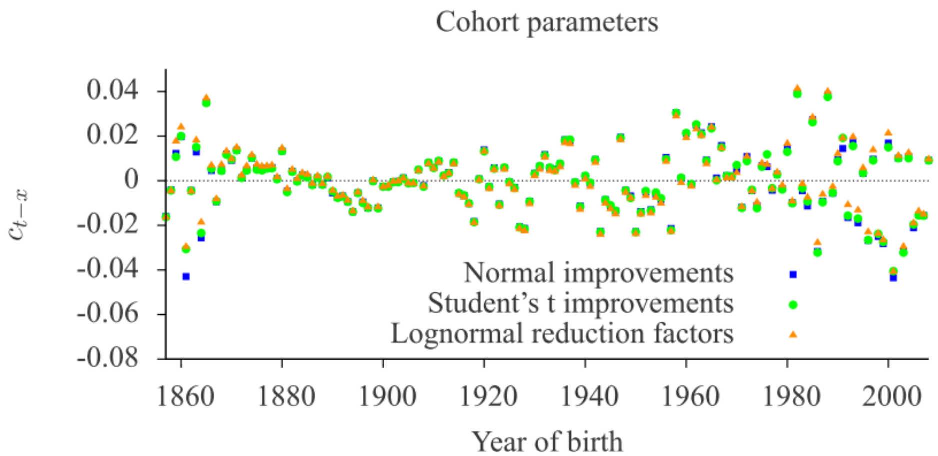
Four Steps

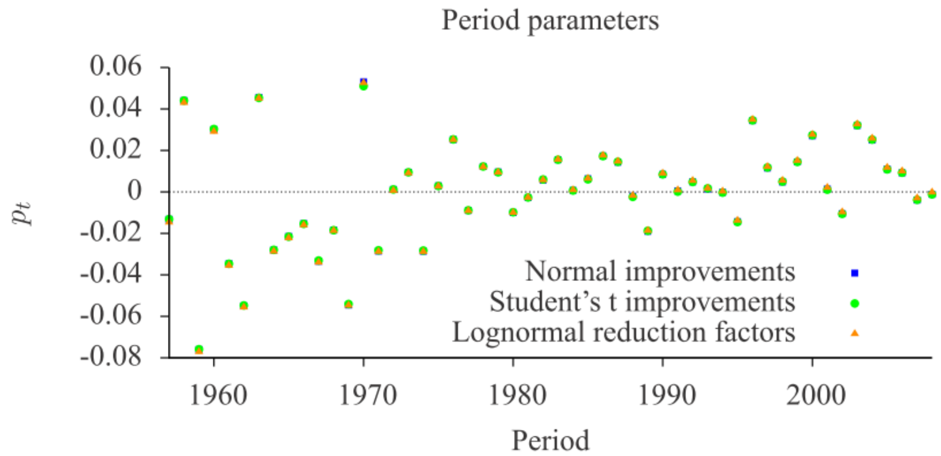
2. Parameter shifting

- APC model parameters are not unique
- Optimal parameters can differ by linear trends in age and time
- This can be used to eliminate any trend in cohort parameters and offset in period parameters

→ Clear interpretation of parameters

- Age parameters: average level of mortality improvements
- Cohort parameters: cohorts with above or below average mortality improvements
- Period parameters: years with above or below average improvements





Estimation of Age and Cohort Parameters

Four Steps

3. Parameter smoothing and significance tests

- Smoothing necessary to eliminate random fluctuations
- Different smoothing methods applicable, e.g. Whittaker-Henderson with smoothing parameter such that generalized cross-validation is minimized
- Significance tests for model components, e.g. likelihood ratio test
  - Tests on smoothed parameters instead of raw parameters as we are interested in significance for projection
- Test results in our example:

Model variant	Females		Males	
	#parameters	p-value	#parameters	p-value
APC model	48.9		51.5	
AP model	21.2	$2.2 \cdot 10^{-15}$	24.0	$8.0 \cdot 10^{-12}$
PC model	31.8	$7.5 \cdot 10^{-10}$	18.8	$4.4 \cdot 10^{-2}$

→ Model cannot be restricted in our example

Estimation of Age and Cohort Parameters

Four Steps

4. Parameter modifications for coherent projections

- Age and cohort parameters could be used for projection as they are
- However, age parameters prevail until infinity
  → Diverging mortality rates for closely related populations if age parameters differ
- Equal long-term age parameters reasonable for males and females
- Most simple approach: projection according to average age parameters for males and females
  - Approach applicable in case age parameters are similar

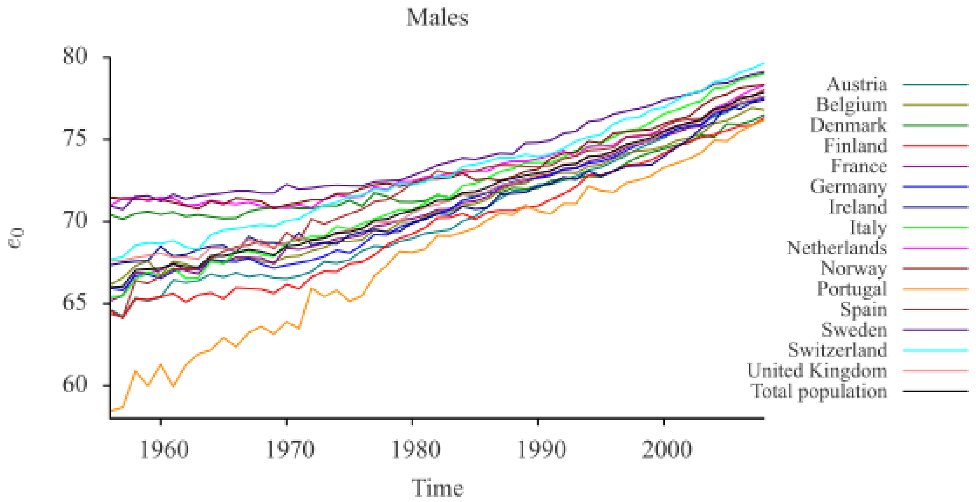
Projection of Period Parameters

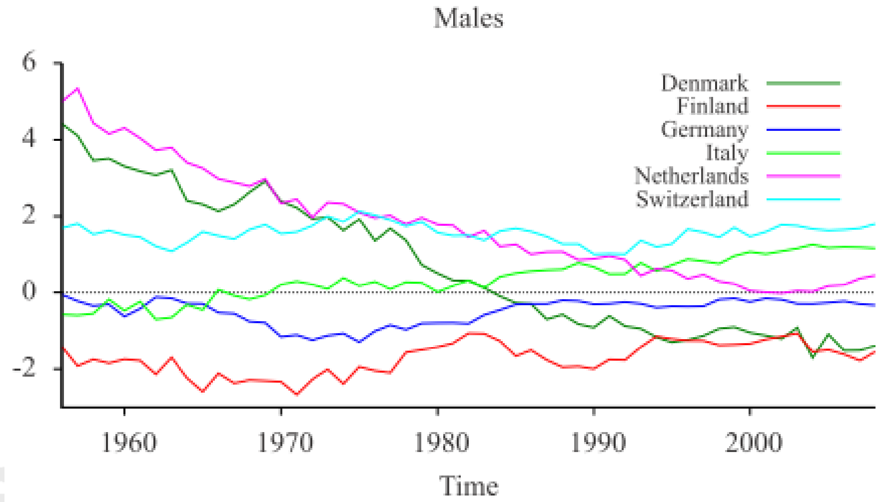
Coherence between Populations

General concept: Derivation of future period parameters based on life expectancy forecasts

Forecasts for any population should be consistent with observations for related populations

- Example: European populations as reference set of populations





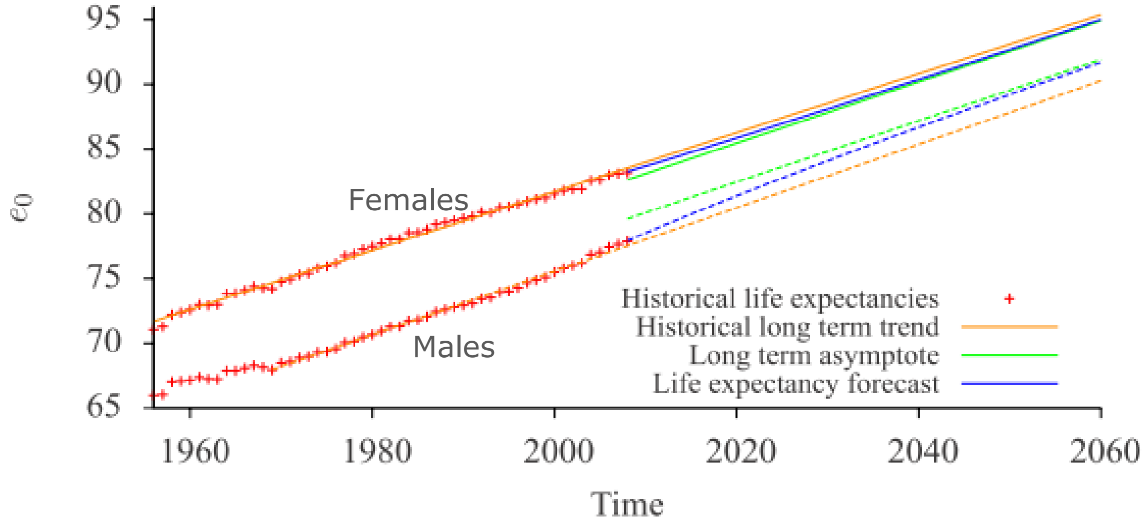
- Clearly a common trend for all populations
- Relation to this trend varies between populations
  - Life expectancies in Switzerland are constantly above average
  - Independent forecasts for Danish and Italian males are likely to be incoherent

Projection of Period Parameters

Life Expectancy Extrapolation

Three step approach for projecting period parameters

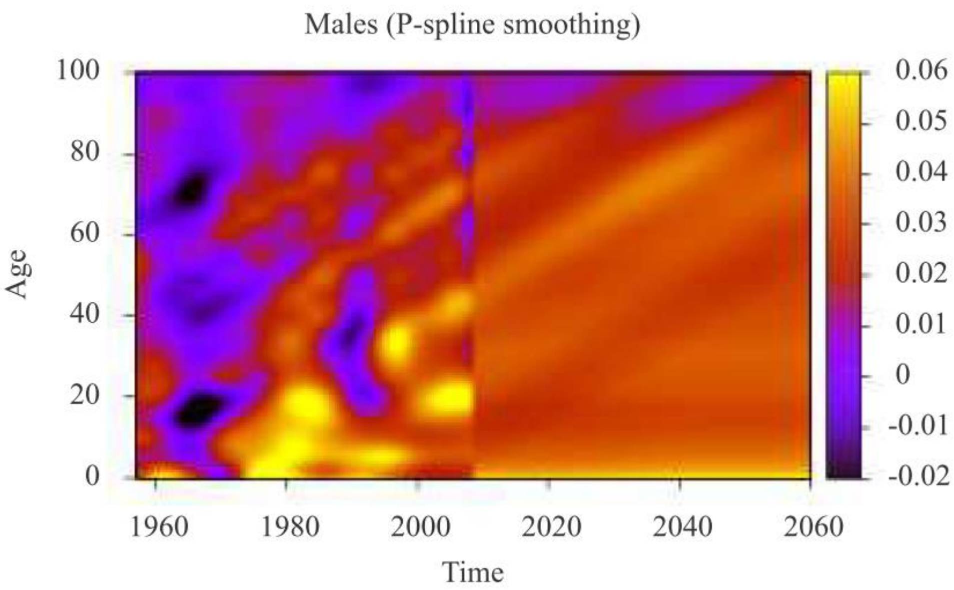
1. Coherent projections of life expectancies for males and females in reference set of populations
   - Example: European reference set of populations

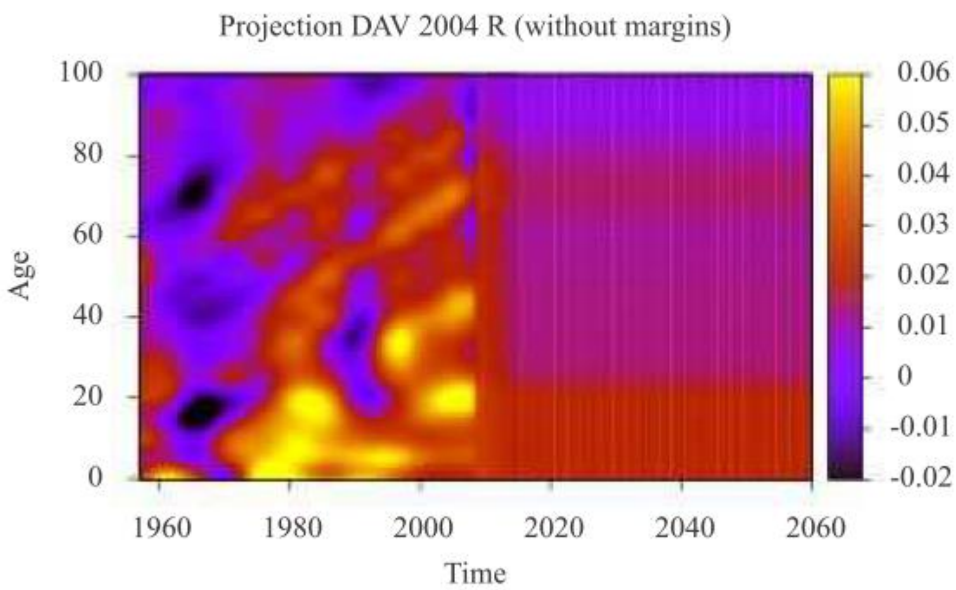


2. Projection of life expectancies in population of interest in relation to reference life expectancies
   - Particularly case specific
   - For Germany, a downward shift of projected European life expectancies appears reasonable
3. Derivation of period parameters from life expectancies from step 2

Projection of Period Parameters
Example Results

Comparison with current standard projection in Germany





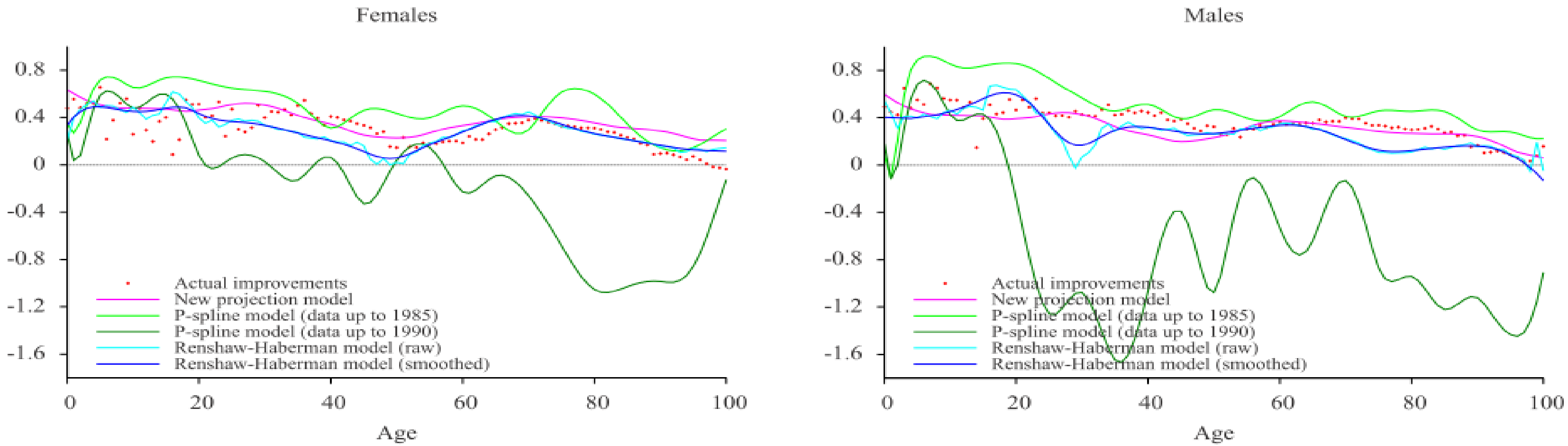
- Cohort structures are preserved
- Magnitude of projected mortality improvements appears plausible
- Structural break mostly due to different degrees of smoothing and averaging of age parameters

→ **New projection appears more plausible than current standard projection**

Back Test

Comparison with P-spline model and Lee-Carter extension of Renshaw and Haberman

- Both models cover full age range and account for cohort effects
- Model fit to data up to 1990 and comparison of aggregated improvements until 2008



Projection method	Females		Males	
	Average error	RMS error	Average error	RMS error
New methodology	0.074	0.126	-0.043	0.099
Renshaw-Haberman (raw)	-0.013	0.140	-0.081	0.150
Renshaw-Haberman (smoothed)	-0.013	0.127	-0.082	0.139
P-splines (data up to 1985)	0.170	0.233	0.149	0.215
P-splines (data up to 1990)	-0.485	0.660	-1.004	1.157

→ New projection methodology provides valuable alternative

Summary

Many currently used mortality projections have significant shortcomings

- Focus on best estimate projections before considering stochastic mortality

The APC model appears very suitable for modeling mortality improvements

Development of a new projection methodology

- Derivation of projections which are coherent between populations
- General idea: implementation of one`s own expectation based on trends in historical data
- Generic framework for model estimation which requires case specific input/implementation

New methodology is able to produce highly plausible projections

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Backup

Quantitative Comparison of Projections

Present values of life annuities for German males in 2008 (constant interest rate of 1.75%)

Deferment period	Age at first payment	New projection	DAV 2004 R best estimate		DAV 2004 R incl. margins	
0 years	65	16.84	15.88	-5.7%	16.45	-2.3%
	75	10.61	10.16	-4.2%	10.42	-1.8%
	85	5.69	5.61	-1.4%	5.68	-0.1%
20 years	65	12.83	10.81	-15.8%	11.97	-6.7%
	75	7.14	6.03	-15.5%	6.87	-3.9%
	85	2.73	2.03	-25.7%	2.44	-10.6%
40 years	65	11.01	8.23	-25.2%	9.61	-12.7%
	75	7.02	4.73	-32.5%	5.90	-15.9%
	85	2.93	1.64	-44.1%	2.35	-19.9%

Present values based on new projection throughout exceed their counterparts based on the DAV 2004 R projection

- The differences are rather small for immediate annuities
- Strong increase with deferment period and thus projection period