A Universal Pricing Framework for Guaranteed Minimum Benefits in Variable Annuities

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Agenda

- Introduction
- Types of guarantees
  - Guaranteed Minimum Death Benefits
  - Guaranteed Minimum Living Benefits
- Pricing Framework
- Numerical Analysis
- Results
- Summary & Outlook
Types of Guarantees

- Variable annuities are unit-linked deferred annuities
  - In the US: Usually single premium contracts
  - Single premium is invested in fund(s)
  - In the 90s, insurance companies started to provide additional guarantees
  - Guaranteed Minimum Death Benefits (GMDB)
  - Guaranteed Minimum Living Benefits (GMLB)
    also called Living Benefit Guarantees (LBG)
  - Fee for the guarantee: annually a certain percentage of the net asset value (NAV)
  - Guarantee provided by the insurance company
  - Risk management
    - “Reinsurance”
    - Internal hedging
Introduction

Variable Annuity Industry Total US Sales (dollars in billions)

- Variable annuity sales in the US strongly increased over the last years
- During the first half of 2005
  - 28% of VA sales offered a guaranteed minimum accumulation benefit (GMAB)
  - 52% of VA sales offered a guaranteed minimum income benefit (GMIB)
  - 78% of VA sales offered a guaranteed minimum withdrawal benefit (GMWB)

→ These types of guarantees are very popular
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Types of Guarantees: Guaranteed Death Benefits

Guaranteed Minimum Death Benefits (GMDB)

- Death benefit = max \{NAV ; guaranteed benefit base\}

- Typical forms of guaranteed benefit base:
  - The premium paid by the policyholder
  - Maximum historical NAV of the fund at certain observation dates
    - e.g. once a year \(\rightarrow\) annual ratchet guarantee
  - Annually increasing death benefit
    - Premium compounded by 5% – 6% p.a.

- Typical guarantee fees: 0.15% - 0.35% of the NAV p.a.
Guaranteed Minimum Accumulation Benefits (GMAB)

- Survival benefit = max \{NAV ; guaranteed benefit base\}
- Typical forms of guaranteed benefit base
  - Premium paid
  - Maximum historical NAV of the fund at certain observation dates
    - e.g. once a year → annual ratchet guarantee
- Typical guarantee fees: 0.25% - 0.75% of the NAV p.a.
Types of Guarantees: Guaranteed Living Benefits

- **Guaranteed Minimum Income Benefits (GMI B)**
  - Guaranteed annuity benefit
    - Guaranteed (lifelong or temporary) annuity in case of annuitization during a certain “annuitization period”
  - During the annuitization period, the policyholder may at any time
    - Annuitize the fund NAV at the current annuity conversion rate $\tilde{a}_{\text{curt}}$
    - Receive the fund NAV as a lump sum payment
    - Annuitize the guaranteed benefit base at an annuity conversion rate $\tilde{a}_{\text{guar}}$ that has been guaranteed at $t=0$
  - Typical forms of the guaranteed benefit base
    - Maximum historical NAV of the fund
    - Annually increasing benefit (by 5% - 6% p.a.) (above risk free rate!)
  - Typical guarantee fees: 0.5% - 0.75% of the NAV p.a.
Types of Guarantees: Guaranteed Living Benefits

- Guaranteed Minimum Withdrawal Benefits (GMWB)
  - Insurer guarantees that
    - The policyholder may withdraw at least the guaranteed withdrawal benefit base over time (even if the account value drops to 0)
    - As long as the annual withdrawal amount is always below some maximum level
  - Example
    - Guaranteed withdrawal benefit base: premium paid by policyholder
    - Maximum annual withdrawal amount: 7% of gross premium paid
  - Huge variety of options on the market
    - Step-up (increase of guarantee under certain conditions)
    - GMWB for life (lifelong guarantees)
  - Typical guarantee fees: 0.4% - 0.65% of the NAV p.a.
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Define state variables to describe the evolution of the contract and the embedded guarantees:

- \( A_t \) net asset value at time \( t \) of the policyholder’s account
- \( W_t \) time \( t \) value of a hypothetical withdrawals account
- \( D_t \) time \( t \) value of a hypothetical death benefit account
- \( G_t^D \) guaranteed minimum death benefit at time \( t \)
- \( G_t^A \) guaranteed minimum accumulation benefit at time \( t \)
- \( G_t^I \) guaranteed minimum income benefit at time \( t \)
- \( G_t^W \) total remaining guaranteed minimum withdrawal amount at time \( t \)
- \( G_t^E \) maximum guaranteed withdrawal amount in year \( t \)

**State vector**

\[ y_t = (A_t, W_t, D_t, G_t^A, G_t^I, G_t^D, G_t^W, G_t^E) \]
Pricing Framework

- **Describe the evolution of the contract and the state variables**
  - If the asset value of the fund changes or
  - If the policyholder
    - withdraws funds as a guaranteed withdrawal of a GMWB option,
    - performs a partial surrender, i.e. withdraws more than the guaranteed withdrawal amount,
    - fully surrenders the contract, or,
    - passes away

- **Development of the state variables is completely determined by the asset process and the policyholder’s actions**

- **Any variable annuity contract with any combination of guarantees can be modeled within this framework**
Pricing Framework

- **Customer strategy**
  - $F_t$-measurable process $(X)$, which determines the amount $E_t$ to be withdrawn depending on the state $y_t$ of the system
    \[ X(t, y_t) = E_t \]

- “Payoff” of the contract following a given strategy $(X)$ is then completely determined by the asset process
  - Thus, the value $V_0((X))$ of the contract is given

- Value of the contract assuming a “rational policyholder” is more complex
  \[ V_0 = \sup_{(X) \in \Xi} V_0((X)) \]

where $\Xi$ is the set of all admissible customer strategies
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Numerical Analysis

- Use Geometric Brownian motion for the underlying assets
  \[
  \frac{dS_t}{S_t} = r dt + \sigma dZ_t, \quad S_0 = 1
  \]

- Numéraire process
  \[
  \frac{dB_t}{B_t} = r dt, \quad B_0 > 0
  \]

- Use Monte-Carlo-Simulation to calculate the contract value \( V_0(X) \) for any given strategy \((X)\)

- Use a multidimensional discretization approach to calculate the contract value \( V_0 \) under rational policyholder behavior
  - Generalizing Tanskanen and Lukkarinen (2004):
    - determination of a quasi-analytic solution
    - discretization of the problem via a finite mesh
  - Similar to a methodology proposed in “Risk Neutral Valuation of With-Profits Life Insurance Contracts” by Bauer, Kiesel, Kling and Ruß (also presented at this conference)
Numerical Analysis

**What’s new?** compared to Bauer et al.

- High dimensionality
  - adequate interpolation scheme
  - complexity / computational time:
    - adequate grids
- Policyholder’s strategy
  - not only: surrender vs. not surrender
  - but also: many possibilities whether, when and how much to withdraw (our algorithm finds optimal strategy)
    - adequate discretization needed, additional level of complexity
- Details are rather complex
  - see paper
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Results

How to determine the “fair fee”:

Contract value including guarantees = Premium paid
Results

- Fair guarantee fee for contracts with GMDB under different customer behavior

<table>
<thead>
<tr>
<th>strategy</th>
<th>contract</th>
<th>Money-back guarantee</th>
<th>Ratchet benefit base</th>
<th>6% roll-up benefit base</th>
</tr>
</thead>
<tbody>
<tr>
<td>1: no withdrawals or surrenders</td>
<td>0.01%</td>
<td>0.04%</td>
<td>0.14%</td>
<td></td>
</tr>
<tr>
<td>2: “typical” deterministic surrender probability</td>
<td>&lt; 0%</td>
<td>&lt; 0%</td>
<td>0.05%</td>
<td></td>
</tr>
</tbody>
</table>

- fair guarantee fee for all the GMDB contracts analyzed is rather low
- the fair guarantee fee strongly decreases if a “typical” surrender pattern is assumed
  - customers have paid fees before surrendering but will not receive any benefits from the corresponding options
  - surrender fees can be used to subsidize the value of the guarantees of the clients who do not surrender
- typical charges in the market exceed the fair guarantee fee
Results

- Fair guarantee fee for contracts with GMAB under different customer behavior

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Money-back guarantee w/o DB</th>
<th>Money-back guarantee with DB</th>
<th>Ratchet benefit base w/o DB</th>
<th>Ratchet benefit base with DB</th>
<th>6% roll-up benefit base w/o DB</th>
<th>6% roll-up benefit base with DB</th>
</tr>
</thead>
<tbody>
<tr>
<td>1: no withdrawals or surrenders</td>
<td>0.07%</td>
<td>0.23%</td>
<td>0.76%</td>
<td>0.94</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>2: deterministic surrender probability</td>
<td>&lt; 0%</td>
<td>0.12%</td>
<td>0.57%</td>
<td>0.74%</td>
<td>---</td>
<td>---</td>
</tr>
</tbody>
</table>

- fair guarantee fees for the contracts differ significantly
  - no fair guarantee fee for a 6% roll-up benefit base
  - value of the contract for rational customer behavior only slightly above strategy 1
    - mostly due to assumed surrender fee of 5%
### Results

**Fair guarantee fee for contracts with GMIB under different customer behavior ($\bar{a} = \bar{a}_{curr}/\bar{a}_{guar}$)**

<table>
<thead>
<tr>
<th>strategy</th>
<th>contract, $\bar{a}$</th>
<th>Money-back guarantee</th>
<th>Ratchet benefit base</th>
<th>6% roll-up benefit base</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>w/o DB, with DB</td>
<td>w/o DB, with DB</td>
<td>w/o DB, with DB</td>
<td>w/o DB, with DB</td>
</tr>
<tr>
<td>1: no withdrawals or surrenders</td>
<td>$\bar{a}=1.2$</td>
<td>0.14%</td>
<td>0.31%</td>
<td>1.55%</td>
</tr>
<tr>
<td></td>
<td>$\bar{a}=1.0$</td>
<td>0.07%</td>
<td>0.23%</td>
<td>0.76%</td>
</tr>
<tr>
<td></td>
<td>$\bar{a}=0.8$</td>
<td>0.03%</td>
<td>0.18%</td>
<td>0.25%</td>
</tr>
<tr>
<td></td>
<td>$\bar{a}=0.6$</td>
<td>0.01%</td>
<td>0.16%</td>
<td>0.05%</td>
</tr>
<tr>
<td>2: deterministic surrender probability</td>
<td>$\bar{a}=1.2$</td>
<td>0.04%</td>
<td>0.18%</td>
<td>1.24%</td>
</tr>
<tr>
<td></td>
<td>$\bar{a}=1.0$</td>
<td>&lt; 0%</td>
<td>0.12%</td>
<td>0.57%</td>
</tr>
<tr>
<td></td>
<td>$\bar{a}=0.8$</td>
<td>&lt; 0%</td>
<td>0.10%</td>
<td>0.15%</td>
</tr>
<tr>
<td></td>
<td>$\bar{a}=0.6$</td>
<td>&lt; 0%</td>
<td>0.08%</td>
<td>&lt; 0%</td>
</tr>
</tbody>
</table>

- Value of the guarantee depends heavily on $\bar{a}$ (which is not known!)
- Surrender assumption strongly influences the fair guarantee fee
- Value strongly increases for rational policyholder behavior
  - E.g. 6% roll-up benefit, $\bar{a}=0.6$: from 2.32% to > 4%
Results

- **Fair guarantee fee for contracts with GMWB under different customer behavior**

<table>
<thead>
<tr>
<th>strategy</th>
<th>contract</th>
<th>without DB</th>
<th>with DB</th>
</tr>
</thead>
<tbody>
<tr>
<td>1: withdrawals of 700 p.a.</td>
<td>j=1: 0.19%</td>
<td>0.23%</td>
<td></td>
</tr>
<tr>
<td>2: withdrawals of 700 if $A_t &lt; G^w_t$</td>
<td>0.19%</td>
<td>0.28%</td>
<td></td>
</tr>
</tbody>
</table>

- The difference between the two strategies is rather small.
- The additional fee for including a GMDB option is significantly lower than for the GMAB and GMIB contracts.
- Fair guarantee fees are lower than the prices of these guarantees.
- However, the fair guarantee fee under rational customer behavior is extremely higher.
Results

- Influence of the capital market parameters $r$ and $\sigma$ on the fair guarantee fee for a contract with GMI B

<table>
<thead>
<tr>
<th>Volatility</th>
<th>$r=3%$</th>
<th>$r=4%$</th>
<th>$r=5%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma = 10%$</td>
<td>0.46%</td>
<td>0.28%</td>
<td>0.20%</td>
</tr>
<tr>
<td>$\sigma = 15%$</td>
<td>1.09%</td>
<td>0.76%</td>
<td>0.56%</td>
</tr>
<tr>
<td>$\sigma = 20%$</td>
<td>1.94%</td>
<td>1.40%</td>
<td>1.05%</td>
</tr>
</tbody>
</table>

- Fair guarantee fee is a decreasing function of the risk-free rate of interest
  - The risk-neutral value of a guarantee decreases with increasing interest rates

- Fair guarantee fee is an increasing function of the asset volatility
  - For any risk-free rate $r$, the fair guarantee fee for $\sigma = 20\%$ is more than four times as high as the one for $\sigma = 10\%$
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- Some of these guarantees are underpriced
  - insurers assume cross subsidizations from other fees and,
  - insurers assume their customers to not act rational
    - irrational surrender and withdrawal behavior
    - customers not exercising GMIB-annuitization options even when in the money

- Calculation based on irrational policyholder behavior is risky
  - customers may become more educated about their options and might thus exercise these in the most beneficial way
  - market participants might specialize in finding arbitrage possibilities and speculating against insurers
    - strategically buying such policies in the secondary market
    - consulting policyholders about optimal behavior
Summary & Outlook

**Future research**
- Different asset model
  - e.g. of Lévy type
  - including stochastic interest rates
- Analysis of an ongoing risk-management of the considered guarantees
  - implementation of efficient hedging strategies
  - sensitivity of the Delta with respect to different policyholder behavior
- Analyze optimal strategies
- Price new features of GMWB contracts
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