A Universal Pricing Framework for Guaranteed Minimum Benefits in Variable Annuities

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Agenda

- Introduction
- **Types of guarantees**
  - Guaranteed Minimum Death Benefits
  - Guaranteed Minimum Living Benefits
- **Pricing Framework**
- **Numerical Analysis**
- **Results**
- **Summary & Outlook**

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Types of Guarantees

- **Variable annuities are unit-linked deferred annuities**
  - In the US: Usually single premium contracts
  - Single premium is invested in fund(s)
  - In the 90s, insurance companies started to provide additional guarantees
    - Guaranteed Minimum Death Benefits (GMDB)
    - Guaranteed Minimum Living Benefits (GMLB)
      also called Living Benefit Guarantees (LBG)
  - Fee for the guarantee: annually a certain percentage of the net asset value (NAV)
  - Guarantee provided by the insurance company
  - Risk management
    - "Reinsurance"
    - Internal hedging

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Introduction

- **Variable Annuity Industry Total US Sales (dollars in billions)**

Year	Variable
1985	$5.3
1986	9.3
1987	12.7
1988	11.8
1989	13.5
1990	17.2
1991	21.5
1992	30.7
1993	46.6
1994	50.2
1995	51.3
1996	74.3
1997	88.2
1998	99.8
1999	123.0
2000	137.3
2001	113.3
2002	115.0
2003	126.4
2004	129.7

Source: NAVA, Morningstar, Inc., and LIMRA Int'l

- Variable annuity sales in the US strongly increased over the last years
- During the first half of 2005
  - 28% of VA sales offered a guaranteed minimum accumulation benefit (GMAB)
  - 52% of VA sales offered a guaranteed minimum income benefit (GMIB)
  - 78% of VA sales offered a guaranteed minimum withdrawal benefit (GMWB)
- → These types of guarantees are very popular

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Types of Guarantees: Guaranteed Death Benefits

- **Guaranteed Minimum Death Benefits (GMDB)**
  - Death benefit = max {NAV ; guaranteed benefit base}
  - Typical forms of guaranteed benefit base
    - The premium paid by the policyholder
    - Maximum historical NAV of the fund at certain observation dates
      - e.g. once a year → annual ratchet guarantee
    - Annually increasing death benefit
      - Premium compounded by 5% – 6% p.a.
  - Typical guarantee fees: 0.15% - 0.35% of the NAV p.a.

Types of Guarantees: Guaranteed Living Benefits

- **Guaranteed Minimum Accumulation Benefits (GMAB)**
  - Survival benefit = max {NAV ; guaranteed benefit base}
  - Typical forms of guaranteed benefit base
    - Premium paid
    - Maximum historical NAV of the fund at certain observation dates
      - e.g. once a year → annual ratchet guarantee
  - Typical guarantee fees: 0.25% - 0.75% of the NAV p.a.

Types of Guarantees: Guaranteed Living Benefits

- **Guaranteed Minimum Income Benefits (GMIB)**
  - Guaranteed annuity benefit
    - Guaranteed (lifelong or temporary) annuity in case of annuitization during a certain "annuitization period"
  - During the annuitization period, the policyholder may at any time
    - Annuitize the fund NAV at the current annuity conversion rate $\ddot{a}_{curr}$
    - Receive the fund NAV as a lump sum payment
    - Annuitize the guaranteed benefit base at an annuity conversion rate $\ddot{a}_{guar}$ that has been guaranteed at t=0
  - Typical forms of the guaranteed benefit base
    - Maximum historical NAV of the fund
    - Annually increasing benefit (by 5% - 6% p.a.) (above risk free rate!)
  - Typical guarantee fees: 0.5% - 0.75% of the NAV p.a.

Types of Guarantees: Guaranteed Living Benefits

- **Guaranteed Minimum Withdrawal Benefits (GMWB)**
  - Insurer guarantees that
    - The policyholder may withdraw at least the guaranteed withdrawal benefit base over time (even if the account value drops to 0)
    - As long as the annual withdrawal amount is always below some maximum level
  - Example
    - Guaranteed withdrawal benefit base: premium paid by policyholder
    - Maximum annual withdrawal amount: 7% of gross premium paid
  - Huge variety of options on the market
    - Step-up (increase of guarantee under certain conditions)
    - GMWB for life (lifelong guarantees)
  - Typical guarantee fees: 0.4% - 0.65% of the NAV p.a.

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Pricing Framework

- **Define state variables to describe the evolution of the contract and the embedded guarantees**
  - A_t net asset value at time t of the policyholder's account
  - W_t time t value of a hypothetical withdrawals account
  - D_t time t value of a hypothetical death benefit account
  - G_t^D guaranteed minimum death benefit at time t
  - G_t^A guaranteed minimum accumulation benefit at time t
  - G_t^I guaranteed minimum income benefit at time t
  - G_t^W total remaining guaranteed minimum withdrawal amount at time t
  - G_t^E maximum guaranteed withdrawal amount in yeat t
- **State vector**

$$y_t = \left(A_t, W_t, D_t, G_t^A, G_t^I, G_t^D, G_t^W, G_t^E\right)$$

Pricing Framework

- **Describe the evolution of the contract and the state variables**
  - If the asset value of the fund changes or
  - If the policyholder
    - withdraws funds as a guaranteed withdrawal of a GMWB option,
    - performs a partial surrender, i.e. withdraws more than the guaranteed withdrawal amount,
    - fully surrenders the contract, or,
    - passes away
- **Development of the state variables is completely determined by the asset process and the policyholder's actions**
- **Any variable annuity contract with any combination of guarantees can be modeled within this framework**

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Pricing Framework

- **Customer strategy**
  - F_t–measurable process *(X)*, which determines the amount E_t to be withdrawn depending on the state y_t of the system

$$\boldsymbol{X}(t, y_t) = E_t$$

- **"Payoff" of the contract following a given strategy *(X)* is then completely determined by the asset process**
  - Thus, the value $V_0(\boldsymbol{(X)})$ of the contract is given
- **Value of the contract assuming a "rational policyholder" is more complex**

$$V_0 = \sup_{\boldsymbol{(X)} \in \Xi} V_0(\boldsymbol{(X)})$$

where Ξ is the set of all admissible customer strategies

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Numerical Analysis

- **Use Geometric Brownian motion for the underlying assets**

$$\frac{dS_t}{S_t} = r dt + \sigma dZ_t, \; S_0 = 1$$

- **Numéraire process** $\frac{dB_t}{B_t} = r\,dt, \; B_0 > 0$
- **Use Monte-Carlo-Simulation to calculate the contract value $V_0(X)$ for any given strategy (X)**
- **Use a multidimensional discretization approach to calculate the contract value V_0 under rational policyholder behavior**
  - Generalizing Tanskanen and Lukkarinen (2004):
    - determination of a quasi-analytic solution
    - discretization of the problem via a finite mesh
  - Similar to a methodology proposed in "Risk Neutral Valuation of With-Profits Life Insurance Contracts" by Bauer, Kiesel, Kling and Ruß (also presented at this conference)

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Numerical Analysis

- **What's new? compared to Bauer et al.**
  - High dimensionality
    - adequate interpolation scheme
    - complexity / computational time:
      - adequate grids
  - Policyholder's strategy
    - not only: surrender vs. not surrender
    - but also: many possibilities whether, when and how much to withdraw (our algorithm finds optimal strategy)
      ➔ adequate discretization needed, additional level of complexity
  - Details are rather complex
    - see paper

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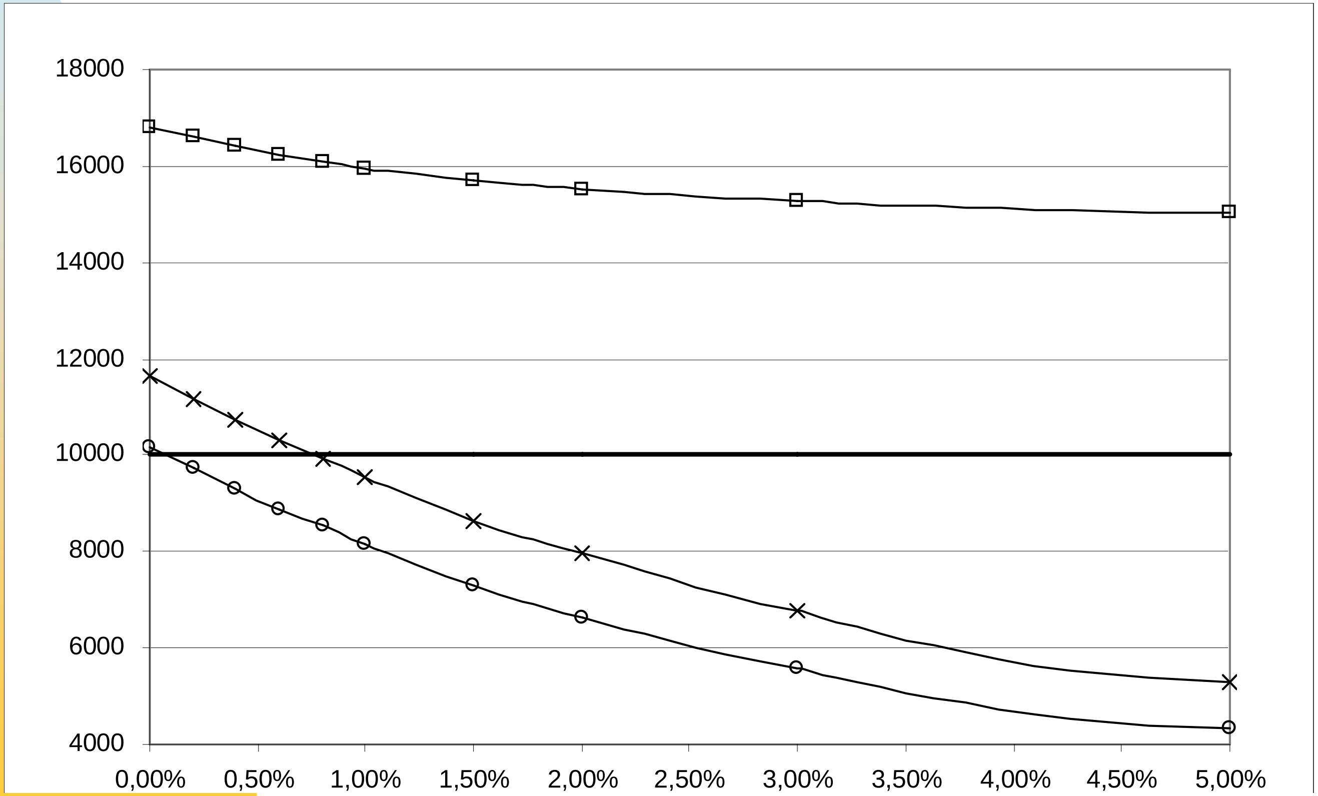
Results

- **How to determine the "fair fee":**
- **Contract value including guarantees = Premium paid**



Results

- **Fair guarantee fee for contracts with GMDB under different customer behavior**

strategy \ contract	Money-back guarantee	Ratchet benefit base	6 % roll-up benefit base
1: no withdrawals or surrenders	0.01%	0.04%	0.14%
2: "typical" deterministic surrender probability	< 0%	< 0%	0.05%

  - fair guarantee fee for all the GMDB contracts analyzed is rather low
  - the fair guarantee fee strongly decreases if a "typical" surrender pattern is assumed
    - customers have paid fees before surrendering but will not receive any benefits from the corresponding options
    - surrender fees can be used to subsidize the value of the guarantees of the clients who do not surrender
  - typical charges in the market exceed the fair guarantee fee

Results

- **Fair guarantee fee for contracts with GMAB under different customer behavior**

contract / strategy	Money-back guarantee		Ratchet benefit base		6% roll-up benefit base	
	w/o DB	with DB	w/o DB	with DB	w/o DB	with DB
1: no withdrawals or surrenders	0.07%	0.23%	0.76%	0.94	---	---
2: deterministic surrender probability	< 0%	0.12%	0.57%	0.74%	---	---

- fair guarantee fees for the contracts differ significantly
  - no fair guarantee fee for a 6% roll-up benefit base
- value of the contract for rational customer behavior only slightly above strategy 1
  - mostly due to assumed surrender fee of 5%

Results

- **Fair guarantee fee for contracts with GMIB under different customer behavior ($\ddot{a} = \ddot{a}_{curr} / \ddot{a}_{guar}$)**

strategy \ contract		Money-back guarantee		Ratchet benefit base		6% roll-up benefit base	
		w/o DB	with DB	w/o DB	with DB	w/o DB	with DB
1: no withdrawals or surrenders	**ä=1.2**	0.14%	0.31%	1.55%	1.83%	---	---
	ä=1.0	0.07%	0.23%	0.76%	0.94%	---	---
	ä=0.8	0.03%	0.18%	0.25%	0.40%	---	---
	ä=0.6	0.01%	0.16%	0.05%	0.19%	2.32%	3.76%
2: deterministic surrender probability	**ä=1.2**	0.04%	0.18%	1.24%	1.40%	---	---
	ä=1.0	< 0%	0.12%	0.57%	0.74%	---	---
	ä=0.8	< 0%	0.10%	0.15%	0.29%	> 4%	> 4%
	ä=0.6	< 0%	0.08%	< 0%	0.11%	1.45%	1.88%

- value of the guarantee depends heavily on *ä* (which is not known!)
- surrender assumption strongly influences the fair guarantee fee
- value strongly increases for rational policyholder behavior
  - e.g. 6% roll-up benefit, *ä*=0.6: from 2.32% to > 4%

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Results

- **Fair guarantee fee for contracts with GMWB under different customer behavior**

strategy \ contract	without DB	with DB
1: withdrawals of 700 p.a.	*j*=1: 0.19%	0.23%
2: withdrawals of 700 if $A_t < G_t^W$ **.**	0.19%	0.28%

  - difference between the two strategies is rather small
  - additional fee for including a GMDB option is significantly lower than for the GMAB and GMIB contracts,
  - fair guarantee fees are lower than the prices of these guarantees
  - however, the fair guarantee fee under rational customer behavior is extremely higher

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Results

- **Influence of the capital market parameters *r* and σ on the fair guarantee fee for a contract with GMIB**

risk-free rate / volatility	r=3%	r=4%	r=5%
σ = 10%	0.46%	0.28%	0.20%
σ = 15%	1.09%	0.76%	0.56%
σ = 20%	1.94%	1.40%	1.05%

- fair guarantee fee is a decreasing function of the risk-free rate of interest
  - the risk-neutral value of a guarantee decreases with increasing interest rates
- fair guarantee fee is an increasing function of the asset volatility
  - for any risk-free rate r, the fair guarantee fee for σ = 20% is more than four times as high as the one for σ =10%

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Summary & Outlook

- **Some of these guarantees are underpriced**
  - ➔ insurers assume cross subsidizations from other fees and,
  - ➔ insurers assume their customers to not act rational
    - irrational surrender and withdrawal behavior
    - customers not exercising GMIB-annuitization options even when in the money
- **Calculation based on irrational policyholder behavior is risky**
  - customers may become more educated about their options and might thus exercise these in the most beneficial way
  - market participants might specialize in finding arbitrage possibilities and speculating against insurers
    - strategically buying such policies in the secondary market
    - consulting policyholders about optimal behavior

Summary & Outlook

- **Future research**
  - Different asset model
    - e.g. of Lévy type
    - including stochastic interest rates
  - Analysis of an ongoing risk-management of the considered guarantees
    - implementation of efficient hedging strategies
    - sensitivity of the Delta with respect to different policyholder behavior
  - Analyze optimal strategies
  - Price new features of GMWB contracts

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