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# Extension, Compression, and Beyond

## A Unique Classification System for Mortality Evolution Patterns

- September, 8th 2015
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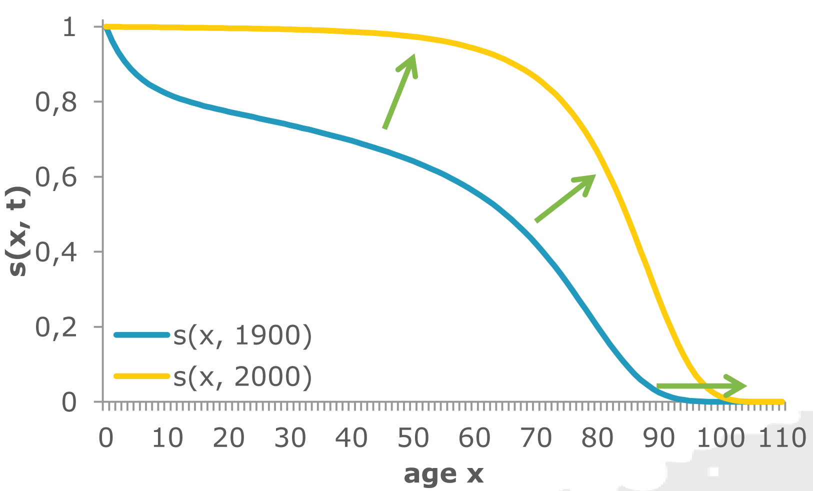
- Joint work with Matthias Börger and Jochen Ruß
- Institute for Finance and Actuarial Sciences and University of Ulm, Germany

# Agenda

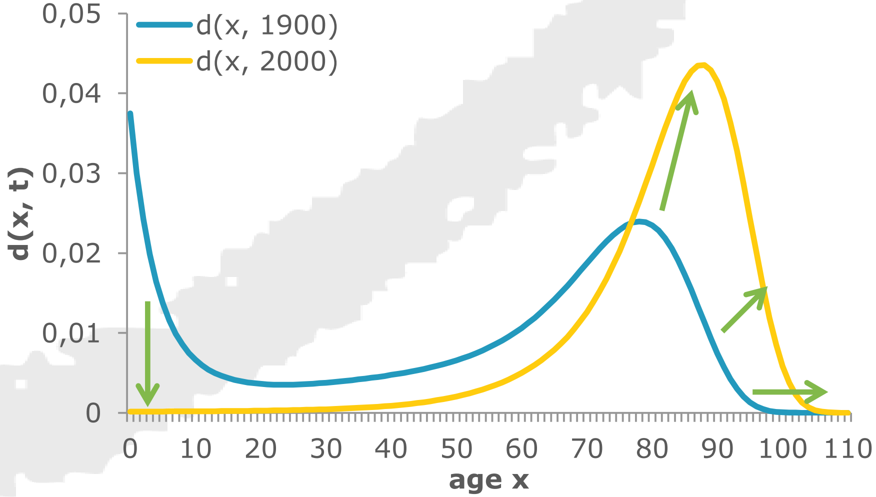
# Key question

- Life expectancy increases in many countries.
- But changes in life expectancy (and other typically used statistics) are only a **consequence of the underlying change of the age distribution of deaths**.

**Survival curve of Swedish females 1900 and 2000**




**Deaths curve of Swedish females 1900 and 2000**




- Key question: **How does the shape of these curves change over time?**

# Classification of mortality evolutions in the past

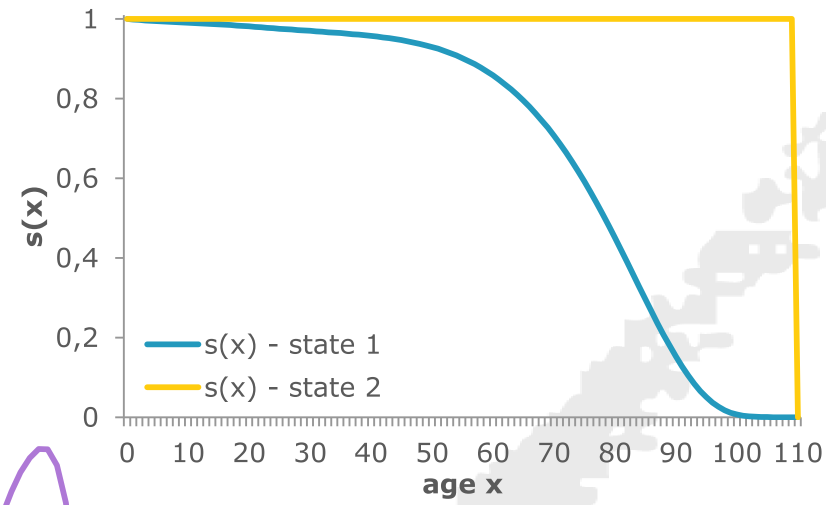
## Shortcomings

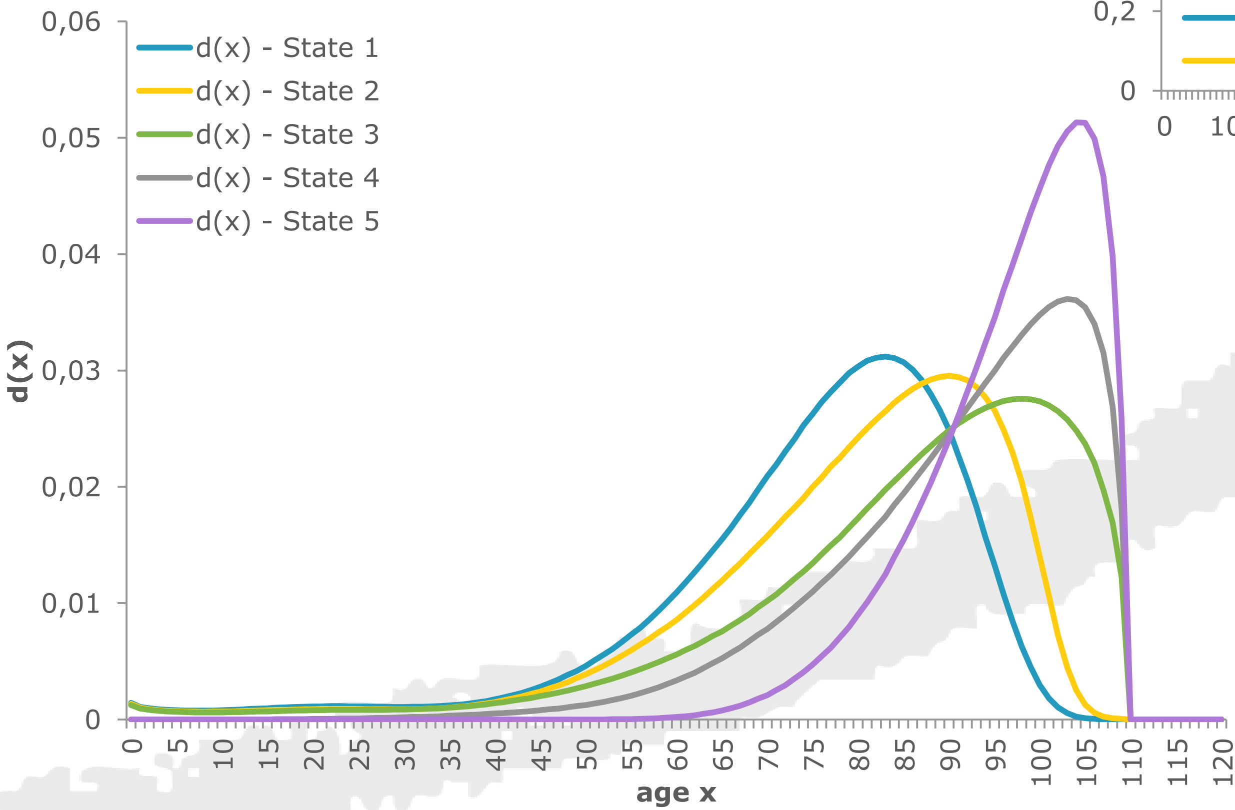
- There exists a variety of literature on the question how the age distribution of deaths changes over time. We have identified some shortcomings there:
  - Different notions for certain observations have been established but often these **scenarios were defined imprecisely**, e.g.:
    - compression (≈ vertical deformation of the deaths curve)
    - extension (≈ horizontal deformation of the deaths curve)
    - rectangularization (≈ survival curve becomes more and more rectangular)
    - …
  - Some of these scenarios were supposed to be **mutually exclusive**, but there are counterexamples.
  - Several often used **statistics are insufficient or even misleading**.
  - Often effects caused by the **choice of a certain age range** under observation were not considered.
- In our paper, we give some examples for each of these shortcomings.

# Classification of mortality evolutions in the past

## Shortcomings

- **Imprecise scenario definitions:**
  - E.g., **rectangularization** is defined by a final state.

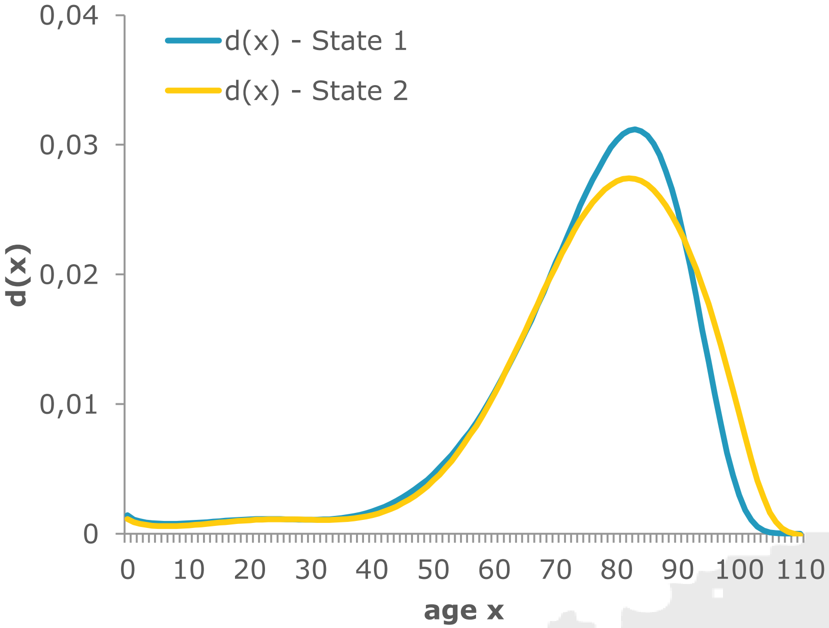


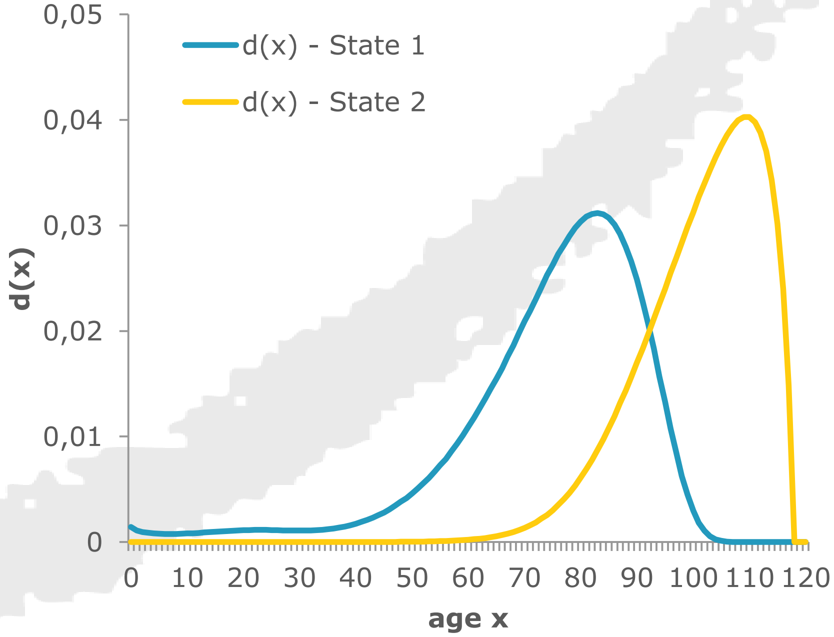
# Classification of mortality evolutions in the past

## Shortcomings

- **Exclusiveness of scenarios:**
  - E.g., **compression** and **shifting mortality** are assumed to be opposing scenarios.




Neither compression nor shifting mortality prevail.

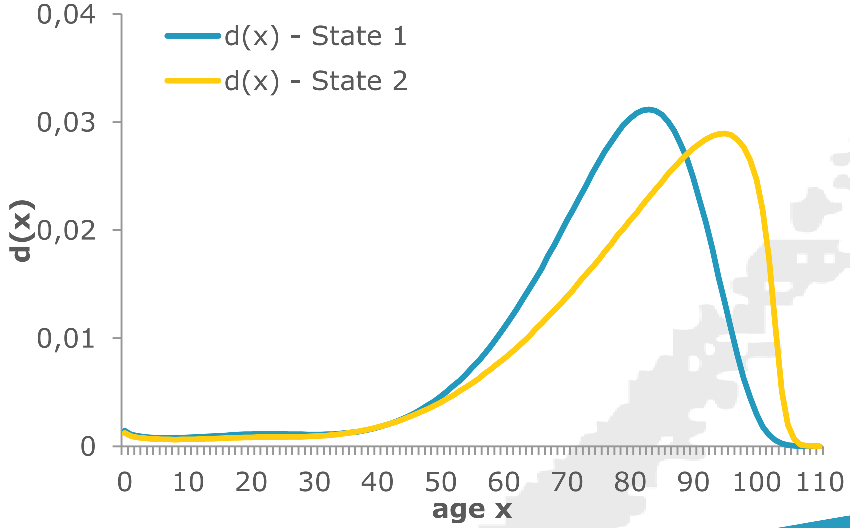



Compression and shifting mortality coexist.

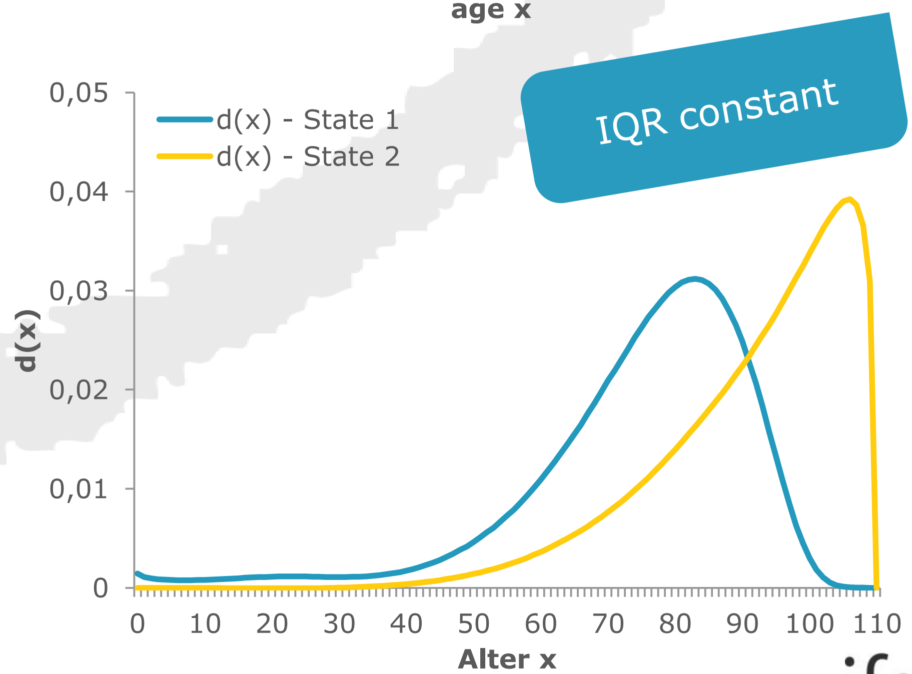
# Classification of mortality evolutions in the past

## Shortcomings

- **Insufficient or misleading statistics:**
  - Example 1: compression cannot always be detected by an exclusive analysis of **M and SD(M+).**



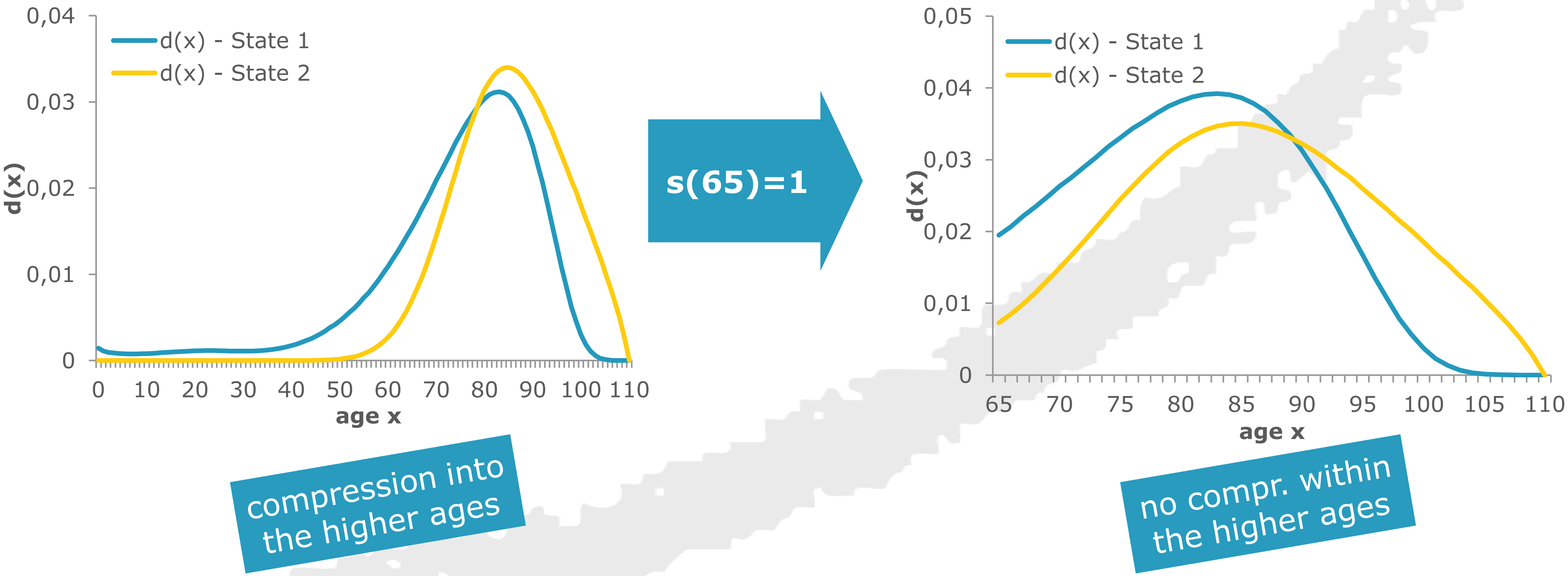

  - Example 2: compression cannot always be detected with **IQR**



# Classification of mortality evolutions in the past

## Shortcomings

- The choice of the **age range matters:**
  - The age range should be chosen depending on the question at hand.



# A new classification framework

## Requirements

In light of these shortcomings of previous approaches, we postulate that a new classification system should…

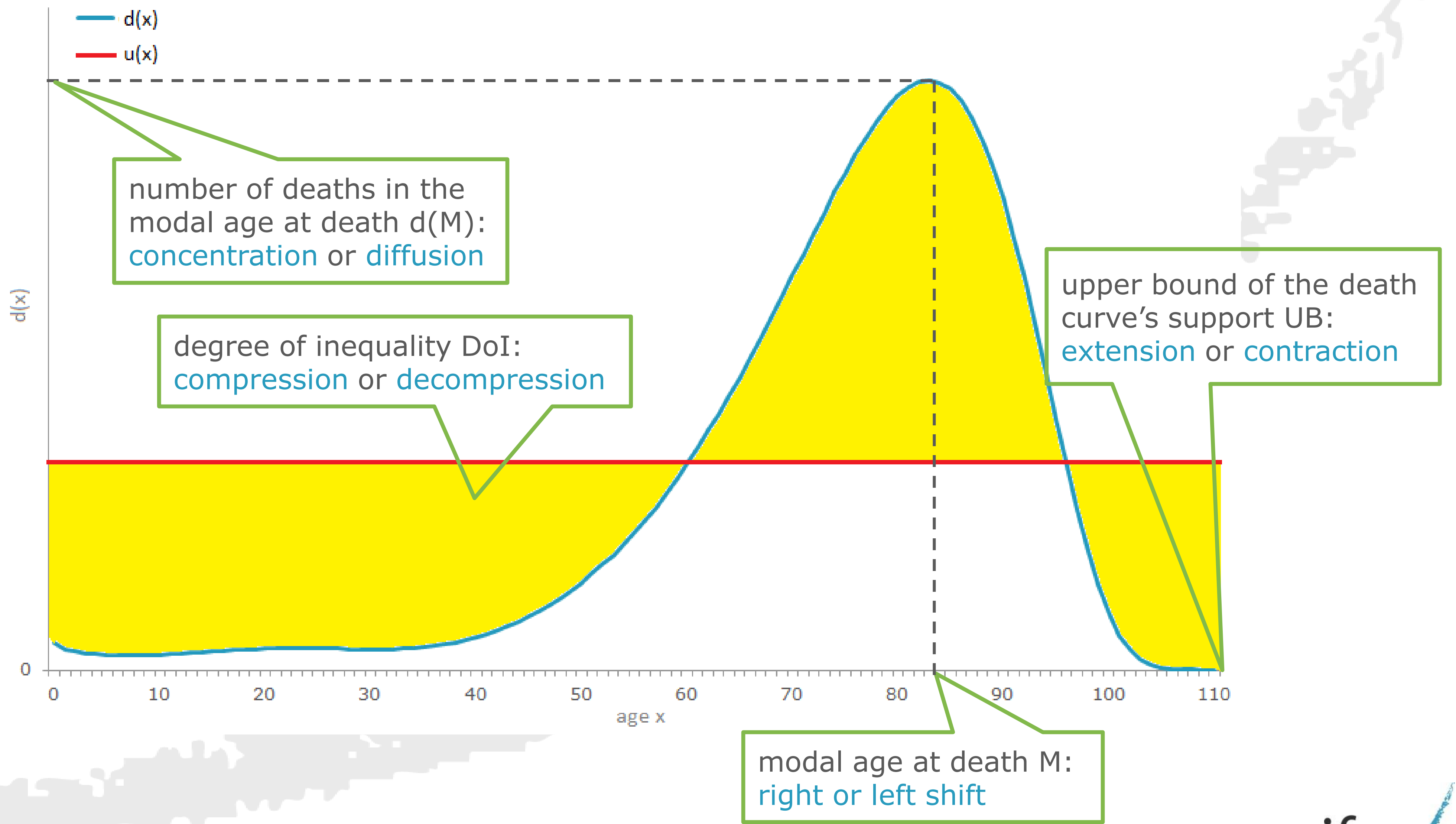
- … capture every observed mortality evolution,
- … allow for mixed scenarios,
- … be applicable to different age ranges,
- … build on statistics that can be feasibly calculated and easily interpreted,
- … be extendable by additional components if needed.

Our new approach:

- We use the deaths curve as basis for the framework.
- We define 4 characteristics of the deaths curve for a unique classification of observed mortality evolutions.

# A new classification framework

## Details



# A new classification framework

## Details

Each scenario is defined by a **4-dimensional vector** where each component can have three specifications:

| component | attainable states |
|---|---|
| **M** | right shift / neutral / left shift |
| **UB** | extension / neutral / contraction |
| **DoI** | compression / neutral / decompression |
| **d(M)** | concentration / neutral / diffusion |

- This allows for $3^4$=81 different scenarios (some of which might not be relevant in practice)
- The framework satisfies the requirements:
  - Each observed mortality evolution can uniquely be classified in one of those scenarios.
  - Pure and mixed scenarios are included.
  - The framework can be applied to age ranges starting at any given age up to UB.
  - Feasible and easily interpretable statistics are used.
  - The framework is extendable by additional statistics if needed.

In the paper, we discuss different issues in estimating these statistics, e.g. how to estimate UB.

# A new classification framework

## Application: The mortality evolution of Swedish females

Example

age range 10 to UB:

<table>
<tr><th>Scenario Component</th><th>Statistic Used</th><th>1860s</th><th>1870s</th><th>1880s</th><th>1890s</th><th>1900s</th><th>1910s</th><th>1920s</th><th>1930s</th><th>1940s</th><th>1950s</th><th>1960s</th><th>1970s</th><th>1980s</th><th>1990s</th><th>2000s</th></tr>
<tr><td>1</td><td>M</td><td colspan="4">right-shift</td><td colspan="5">neutral</td><td colspan="6">right-shift</td></tr>
<tr><td>2</td><td>UB</td><td colspan="6">extension</td><td colspan="3">neutral</td><td colspan="6">extension</td></tr>
<tr><td>3</td><td>DoI</td><td colspan="4">neutral</td><td colspan="6">compression</td><td colspan="2">neutral</td><td colspan="3">compression</td></tr>
<tr><td>4</td><td>d(M)</td><td colspan="2">neutral</td><td colspan="8">concentration</td><td colspan="2">neutral</td><td colspan="3">concentration</td></tr>
</table>

- Each component of the vector develops independently from the others (no redundant information).
- We observe mixed scenarios (rather the rule than an exception).

age range 60 to UB:

<table>
<tr><th>Scenario Component</th><th>Statistic Used</th><th>1860s</th><th>1870s</th><th>1880s</th><th>1890s</th><th>1900s</th><th>1910s</th><th>1920s</th><th>1930s</th><th>1940s</th><th>1950s</th><th>1960s</th><th>1970s</th><th>1980s</th><th>1990s</th><th>2000s</th></tr>
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<tr><td>3</td><td>DoI</td><td colspan="4">decompression</td><td colspan="11">compression</td></tr>
<tr><td>4</td><td>d(M)</td><td colspan="6">neutral</td><td colspan="4">concentration</td><td colspan="2">diffusion</td><td colspan="3">concentration</td></tr>
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- We observe different scenarios for different age ranges (age range matters).

In the paper, we analyze this application in more detail.

# Summary

In the paper, we have…

- … identified **shortcomings** of previous approaches for classification of mortality scenarios,
- … derived **requirements** for a new framework,
- … identified 4 central **characteristics** of the deaths curve,
- … derived a **new classification framework** based on these characteristics, which
  - … builds on clear scenario definitions,
  - … provides a unique classification for each mortality evolution,
  - … allows for mixed scenarios,
  - … is applicable for different age ranges,
- … **applied the framework** to concrete data.

# Thank you for your attention!

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