IMPLICIT OPTIONS IN LIFE INSURANCE CONTRACTS
Part 1 - The case of lump sum options in deferred annuity contracts

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Abstract

Options in life insurance contracts are being offered more and more frequently. This is for reasons of competitiveness as well as (in particular under German legislation) for tax reasons. Up to now, such options were usually not taken into account when the policy was priced. Two very common options in German life insurance contracts are the lump sum option in deferred annuity contracts and the flexible expiration option in endowment contracts. In Part 1[1], we quantify the value of the lump sum option within a Hull-White model framework and perform extensive sensitivity analysis with respect to parameters of the insurance contract, as well as capital market parameters. The underlying of this option is a portfolio of bonds. In Part 2[2], we will analyze the flexible expiration option.

Keywords

Implicit option, lump sum option, Hull-White model, bond option pricing.

1 Introduction

During the last years, implicit options in life insurance contracts have become more and more interesting. An option is the right to change some product features or to choose

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[1] An earlier version of this paper was presented in 1999 at the 9th International AFIR Colloquium in Tokyo, Japan. A short version was published in [Di/Ru 99].

[2] An earlier version of Part 2 was presented in 2000 at the 4th Conference of the Asia-Pacific Risk and Insurance Association (APRIA) in Perth, Australia, receiving the Outstanding Paper Award.

between some alternatives at some time during the term of the policy. A well known example is the so-called guaranteed insurability option, where the insured person has the right to increase the death benefit at the occurrence of certain events (e.g. marriage, birth of a child, etc), but there are many more kinds of options.[3]

Many of these options can be regarded as interest rate options since the right to change the future cash flow of the policy is often essentially equivalent to a put or call option on (coupon) bonds (which correspond to the different cash flows). Such interest rate sensitive options are of particular interest in the German life insurance market, since most German life insurance companies try to give their policy-holders the same return every year, independent of what they earn on their investment. This smoothening is achieved by accumulating hidden reserves when the markets perform well, and using these reserves to distribute the same profits every year, even if the markets perform bad.

Hence, there are times when the interest earned on life insurance contracts is significantly higher than the rates earned in the bond markets, and vice versa. Policy-holders might use implicit options to profit from these effects. Thus, a detailed analysis of such options is required.

Since these options can have an extreme impact on the cash flow that results from the policy, they can bear significant financial risks that are often not taken into account when the policy is priced. Hence, if some insured persons exercise these options in an advantageous way, other insured persons will implicitly pay for it. This leads to a transfer of risk that is not desired. In spite of these risks, options in life insurance contracts are becoming more and more popular since they make the product more flexible and competitive. Furthermore, in Germany, the taxation of a life insurance policy depends on its term. If the insured person changes the contract later on, the tax authorities tend to regard the altered policy as two separate contracts, one before and one after the change was made. This might lead to a higher taxation. If however, the right to change the policy is included as an option, this is often not the case.[4]

In the literature, there are already several publications dealing with options in life insurance contracts. In Great Britain, Gillespie[5], North and Savill[6], and Lumsden[7] all discuss several options and guarantees offered in Great Britain (i.e. conversion options, renewal options, guaranteed insurability option, maturity guarantees). Gillespie also briefly talks about how to value these options and whether reservations are necessary. North and Savill only give a very general idea of how to price such options but also discuss different rider benefits and the (necessary) conditions for both options and riders in order to minimize anti-selection. Lumsden's main focus is on the terms these options are offered and the

[3]For an overview of implicit options in Germany see [He 99].

[4]Lately, tax authorities changed their view of altered contracts, cf. [N.N. 99].

[5]Cf. [Gi 81].

[6]Cf. [No/Sa 85].

[7]Cf. [Lu 92].

specialties when guarantees are involved. However, they all consider the insurance and underwriting risk and discuss actuarial methods of dealing with these risks, rather than looking at the details of option pricing and actually computing any option price. Also, they do not look at these options from a financial point of view, that is, consider these rights to alter the insurance contract as options with an interest rate sensitive underlying.

All publications concerning options in insurance contracts in the German market are of younger date. While Gebhard[8] discusses the risks associated with surrender close to the expiration of the insurance policy and introduces a new concept for computing surrender values (depending on the actual capital market interest rate), Gerdes[9] looks at different options and computes their values using a rather simple model. In particular, he assumes, that the underlying of each option follows a geometric Brownian motion. This assumption is of course less than satisfactory, especially when the option is an interest rate derivative.

More recently, Herr and Kreer[10] developed a model for the pricing of both the surrender and waiver option, as well as the guaranteed interest rate, using a binomial tree for the forward rates. Their idea is to both price the insurance contract with and without the options and take the difference of the two as the price of the option.

Another approach was used by Haase[11] to price the option to increase the sum assured of a one-period term life assurance. In his model, the price of the option can be found using a duplication strategy similar to the no-arbitrage pricing theory. However, it is not clear so far if this method could also be used to price other (more complex) implicit options.

In our papers, we regard the implicit options as financial options and determine the value of the option as the costs for a perfect hedge, following standard principles of risk neutral valuation.

In what follows, we will use the Hull-White model for our analysis. This is a one-factor no-arbitrage model for the short-rate. In Part 1, we look at a simple example of an implicit option, the so-called lump sum option in a deferred annuity contract. In [Ge 97], Gerdes has examined this option as well, and found that its value can be substantial. The underlying of this option is the expected annuity payment, i.e. a portfolio of zero-coupon bonds. The flexible expiration option in an endowment contract, i.e. the right to terminate the contract before the regular expiration during a specified period of time without surrender charges, will be analyzed in Part 2.

Our paper is organized as follows: In Section 2, we describe the insurance contract with the included lump sum option. In Section 3, we introduce the Hull-White model.[12] We

[8] Cf. [Ge 96].

[9] Cf. [Ge 97].

[10] Cf. [He/Kr 99].

[11] Cf. [Ha 00].

[12] Cf. [Hu/Wh 90].

give closed form solutions for the prices of the included bonds and bond options. In Section 4, we calibrate our model to given market data and derive empirical results for a specific insurance contract. We furthermore perform extensive sensitivity analysis. Section 5 concludes with a summary.

2 The Lump Sum Option

Many deferred annuities include a so-called lump sum option that can be exercised by the insured person at the end of the deferment period. Exercising the option means that the insured person chooses to receive a lump sum rather than a lifelong annuity payment.[13] If the insured person chooses the lump sum, this amount includes surplus that was created during the deferment period.

If we regard the expected annuity payment as a cash flow, the lump sum option is obviously equivalent to a European put option that gives the insured person the right to sell this cash flow for the lump sum (at the end of the deferment period). Usually, this option is not taken into account when the policy is priced. Hence, if some insured persons exercise these options in a profitable way, the resulting costs go to the debit of the remaining insured persons.

In what follows, we will quantify the value of such an option. We let $t = 0$ denote the start of the policy and x the age of the insured person at $t = 0$. We assume the deferment period to be $n > 0$ years, i.e. the first annuity payment is due at time $t = n$, the beginning of year $n + 1$.[14] We assume a lump sum of S_n to be paid at time n if the option is exercised. Also, we let R_j denote the annuity that is paid at time j. Hence, we get

$$v_n S_n = \sum_{j=n}^{\infty} R_j v_j \; {}_{j-n}p_{x+n}.$$

Here, v_k denotes the discount rate from time 0 to k, and ${}_kp_x$ is the probability that an insured person aged x survives the next k years.[15] In case of a constant annuity, we have

[13]The annuity payment includes surplus. Usually, in Germany a guaranteed rate of interest of 3.25% and an additional surplus is earned on the net premiums during the deferment period as well as during the period of annuity payment.

[14]We assume all annuities to be paid in advance. In the case of payment in arrear, our results can be applied analogously by a simple adjustment of the indices. Furthermore, the annuity is of course only paid, if the insured person is still alive at time n.

[15]In our empirical analysis, we use the mortality table of the German Society of Actuaries (DAV).

$R_j \equiv R$, and thus

$$R = \frac{v_n S_n}{\sum_{j=n}^{\infty} v_j \ {}_{j-n}p_{x+n}}.$$

If we furthermore assume $v_j = v^j$, we get

$$\begin{aligned} R &= \frac{S_n}{\sum_{j=n}^{\infty} v^{j-n} \ {}_{j-n}p_{x+n}} \\ &= \frac{S_n}{\sum_{j=0}^{\infty} v^j \ {}_{j}p_{x+n}}. \end{aligned} \tag{1}$$

Hence, for given S_n, we can determine R and thus the expected cash flow. Usually, S_n is not guaranteed. It consists of a guaranteed part and a part resulting from surplus. However, S_n can be predicted rather well, because the surplus rates are very stable in Germany.

Letting L_j denote the expected annuity payment at time j, given that the insured person is still alive at time n, we get

$$L_j = R_j \ {}_{j-n}p_{x+n}, \quad \text{for } j = n, n+1, \dots \ .$$

As mentioned above, the lump sum option is the right to sell the annuity for S_n at $t = n$. Hence, the strike of the option is S_n, the maturity date is $t = n$, and the underlying is the expected cash flow of the annuity.[16] This is equivalent to a coupon bond with annual coupon payment of L_j at time j. In [Hu/Wh 90] and [Ja 89], it was shown how the pricing of a European call option on a coupon bearing bond can be reduced to the pricing of a portfolio of call options on one zero-coupon bond each. We will now apply those ideas to the case of a put option. In what follows, it is crucial that bond prices are decreasing functions in the short-rate r.[17]

We analyze the general case of a European put option with strike X and maturity T on a coupon bond that pays c_k at time $s_k \geq T$, $k = 1, \dots, m$. Let $B(r, t_1, t_2)$ denote the

[16] From the insured person's point of view, the underlying is, of course, the actual annuity payment. However, if the portfolio of policies is not too small, it makes sense from the insurance company's point of view, to regard the expected cash flow as the underlying of the option.

[17] Hence, the following arguments can in particular be applied to all one-factor models for the short-rate.

price at time t_1 of a zero bond maturing at t_2 as a function of the short-rate $r = r(t_1)$. Then, we define r^* by

$$\sum_{k=1}^{m} c_k B(r^*, T, s_k) = X. \tag{2}$$

Hence, the option is exercised, whenever $r(T) > r^*$,[18] since the payoff of the put option is given by

$$\max\left[0, X - \sum_{k=1}^{m} c_k B(r, T, s_k)\right]. \tag{3}$$

This is equal to

$$\sum_{k=1}^{m} c_k \max\left[0, X_{s_k} - B(r, T, s_k)\right], \tag{4}$$

with

$$X_{s_k} = B(r^*, T, s_k). \tag{5}$$

Obviously, (4) is the payoff of a portfolio of m put options, each on a zero bond with strike X_{s_k} and maturity s_k. Hence, the price of a put option on a coupon bond equals the price of a portfolio of put options, each on one zero bond.

We will now apply these ideas to the described lump sum option: The coupons are paid at $t = n, n+1, \ldots$, paying L_j at time j. The maturity of the put option is $t = n$ and the strike is S_n.[19] However, the option can only be exercised, if the insured person is still alive at time n. This is considered in (6).

For $j \geq n$, we now let V_j denote the price at time 0 of a European put option with maturity n and payoff $\max[0, X_j - B(r, n, j)]$. Therefore, the price of the lump sum option is given by

$$p = {}_n p_x \sum_{j=n}^{\infty} L_j V_j. \tag{6}$$

[18] Here, we assume that the investor acts rationally, meaning that he exercises the option if and only if its value is positive. In reality, the decision of an insured person to exercise the lump sum option will additionally depend on other factors. Hence, the values derived within our model are upper bounds for the real value of the lump sum option.

[19] Hence, in (2), (3), (4), and (5) we have to replace c_k by L_{n+k-1}, s_k by $n + k - 1$, T by n, and X by S_n.

To determine the prices V_j, we need a model for the economy. In the next section, we introduce the Hull-White model and derive explicit pricing formulas for our options.

3 The Hull and White Model

In [Hu/Wh 90], Hull and White introduce their model for the term structure. It is a one-factor, no-arbitrage model for the short-rate which is assumed to follow the Itô-process

$$\begin{aligned} dr(t) &= \Big(\theta(t) - ar(t)\Big)dt + \sigma\, dz(t) \\ &= a\left[\frac{\theta(t)}{a} - r(t)\right] dt + \sigma\, dz(t). \end{aligned} \tag{7}$$

with constant $a, \sigma > 0$.[20]

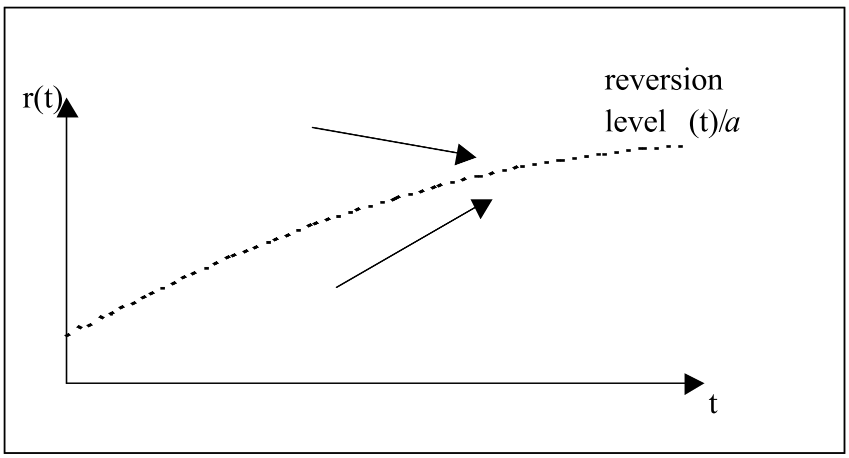

Figure 1: Mean reversion

This model includes mean reversion, that is, at time t, the short-rate $r(t)$ reverts to $\frac{\theta(t)}{a}$ with mean reversion rate a. Hence, for large values of $r(t)$ there is a negative drift of $\theta(t) - ar(t)$ that pulls the short-rate back to the time dependent drift $\frac{\theta(t)}{a}$. Similarly, for small values of $r(t)$ the drift pulls it up as Figure 1 illustrates.

[20] For all that follows, we use standard assumptions, taking in particular a filtered probability space (Ω, Σ, P) with a filtration $\mathcal{F}_t$ as a basis. We furthermore assume a complete and arbitrage-free market. This is essentially equivalent to the unique existence of a so-called equivalent martingale measure Q. We assume the process in (7) to be the so-called risk neutral process, i.e. the process under Q. Here, $z(t)$ denotes an adapted Wiener process under Q. For a detailed overview, cf. e.g. [Du 96]. We furthermore assume the financial markets to be independent of mortality, and the insurance company to be risk neutral with respect to mortality. For a detailed overview of these aspects, cf. e.g. [Aa/Pe 94].

The Hull-White model is a generalization of two other well known term structure models. For $a = 0$ it coincides with the model of Ho and Lee (cf. [Ho/Le 86]). On the other hand, if θ is assumed to be constant over time we get the Vasicek model (cf. [Va 77]). That is, we get the Hull-White model if we include mean reversion into the Ho-Lee model, or if we allow the drift term of Vasicek's model to be time dependent.

Since the model of Hull and White is a no-arbitrage model, it is designed to provide an exact fit to the initial term structure. The function $\theta(t)$ can be calculated from the initial term structure by

$$\theta(t) = F_t(0,t) + aF(0,t) + \frac{\sigma^2}{2a}(1 - e^{-2at})\ .^{21} \tag{8}$$

Usually, the last term of this equation is rather small. Hence the drift of the short-rate process is approximately $F_t(0,t) + a[F(0,t) - r(t)]$, implying that on average, r approximately follows the slope of the initially given curve of instantaneous forward rates. When it moves away from that curve, it reverts back with rate a (i.e. mean reversion).

Within this model, the bond prices $B(r,t,T)$ are given by (cf. [Hu 97]):

$$B(r,t,T) = A(t,T)\, e^{-C(t,T)r(t)} \tag{9}$$

with

$$C(t,T) = \frac{1 - e^{-a(T-t)}}{a} \tag{10}$$

and

$$\begin{aligned} \ln A(t,T) \quad &= \quad \ln \frac{B(r,0,T)}{B(r,0,t)} - C(t,T)\frac{\partial \ln B(r,0,t)}{\partial t} \\ &\quad - \frac{1}{4a^3}\,\sigma^2 (e^{-aT} - e^{-at})^2\,(e^{2at} - 1). \end{aligned} \tag{11}$$

Hence, for given $r(t)$, bond prices at time t can be determined from (9), (10), and (11) using today's bond prices. The partial derivative $\frac{\partial \ln B(0,t)}{\partial t}$ that is needed for the calculation of $A(t,T)$ in (11) can e.g. be approximated by

$$\frac{\ln B(r,0,t+\delta) - \ln B(r,0,t-\delta)}{2\delta}, \tag{12}$$

[21] Here, we denote by $F(t,t_1,t_2)$ the forward rate at time t for the period of time $[t_1,t_2]$. The so-called instantaneous forward rate $F(t,t_1)$ at time t for t_1 is then given by $F(t,t_1) = \lim_{t_2 \to t_1} F(t,t_1,t_2) = \lim_{t_2 \to t_1} \frac{\ln B(t,t_1) - \ln B(t,t_2)}{t_2 - t_1}$. Furthermore, F_t denotes the partial derivative of $F(t_1,t_2)$ with respect to t_2.

for some small $\delta > 0$.

If we assume $\theta(t) \equiv \theta$, $A(t,T)$ is given by

$$A(t,T) = \exp\left[\frac{\left(C(t,T) - T + t\right)\left(a\,\theta - \frac{\sigma^2}{2}\right)}{a^2} - \frac{\sigma^2 C(t,T)^2}{4a}\right].$$

The price at time t of a European call option with maturity T on a zero bond maturing at T_1 ($t \leq T \leq T_1$) is given by[22]

$$B(r,t,T_1)N(h) - X\,B(r,t,T)N(h - \sigma_B), \tag{13}$$

where X denotes the strike price of the option. Furthermore, h and σ_B are given by

$$h = \frac{1}{\sigma_B}\ \ln\frac{B(r,t,T_1)}{X\,B(r,t,T)} + \frac{\sigma_B}{2}$$

and

$$\begin{aligned}\sigma_B &= \left(\int_t^T \sigma^2\left[C(\tau,T_1) - C(\tau,T)\right]^2 d\tau\right)^{\frac{1}{2}} \\ &= \frac{\sigma}{a}\sqrt{\frac{1}{2a}\left[\left(1 - e^{-a(T_1-T)}\right)^2 - \left(e^{-a(T-t)} - e^{-a(T_1-t)}\right)^2\right]}.\end{aligned}$$

In particular, for $t = 0$ we get

$$\begin{aligned}\sigma_B &= \frac{\sigma}{a}\left[1 - e^{-a(T_1-T)}\right]\sqrt{\frac{1 - e^{-2aT}}{2a}} \\ &= \sigma\ C(T,T_1)\sqrt{\frac{1}{2}C(0,T)}\ .\end{aligned}$$

The price of the corresponding put option on that bond is then given by

$$X\,B(r,t,T)N(-h + \sigma_B) - B(r,t,T_1)N(-h). \tag{14}$$

[22]Cf. [Hu/Wh 90]. Here, we only give the result for an option on a bond with face value 1, as this is sufficient for our analysis, cf. (4).

Applying these ideas to the pricing of the lump sum option, we get the V_j of equation (6) as the price at time 0 of a European put option with strike X_j and maturity n on a zero bond maturing at j. Hence, it follows from (14) that

$$V_j = X_j B(r,0,n) N(-h_j + \sigma_B^j) - B(r,0,j) N(-h_j),$$

with

$$h_j = \frac{1}{\sigma_B^j} \ln \frac{B(r,0,j)}{X_j B(r,0,n)} + \frac{\sigma_B^j}{2},$$

and

$$\begin{aligned} \sigma_B^j &= \frac{\sigma}{a}\left[1 - e^{-a(j-n)}\right]\sqrt{\frac{1-e^{-2an}}{2a}} \\ &= \sigma\, C(n,j)\sqrt{\frac{1}{2}C(0,n)}\ . \end{aligned}$$

4 Empirical Results

For our following analysis, we use market data from June 24, 1998. In particular, the term structure of interest rates is given by the prices $B(0,t)$ of discount bonds. Some of them are given in Table 1.[23]

t	0.5	1	2	3	4
$B(0,t)$	0.98232	0.96345	0.92316	0.88269	0.84275
t	5	6	7	8	9
$B(0,t)$	0.80251	0.76166	0.72510	0.68908	0.65485
t	10	15	20	25	30
$B(0,t)$	0.62453	0.47465	0.35320	0.25911	0.19563

Table 1: Discount bond prices from June 24, 1998 (t in years)

The parameters of the Hull-White model were given by $a = 0.0001$ and $\sigma = 0.6306\%$.[24] We furthermore assume the policy to be defined as follows: The insured person is male

[23] Any bond price $B(0,t^*)$ that was needed but not given was derived by interpolation: We calculated the corresponding spot rates from the neighboring bond prices $B(0,t_1)$ and $B(0,t_2)$, $t_1 < t^* < t_2$. The spot rate for t^* was then derived by linear interpolation and the bond price $B(0,t^*)$ was calculated from this spot rate. Extrapolations were performed analogously.

[24] The calibration was done using 2-year and 5-year at the money caps.

and aged x years. The deferment period is n years. The guaranteed rate of interest on the net premium is 3.25%. An additional surplus of u_1 and u_2 is paid during the deferment period and during the annuity payment, respectively.[25]

The value of the lump sum option for different values of x, n, and $u = u_1 = u_2$ is given in Table 2. We assumed the insured person to pay a single net premium of 100,000 DM at $t = 0$.[26] For the sake of simplicity, we did not allow for any costs.

	$n = 5$			$n = 10$		
	$x = 20$	$x = 40$	$x = 60$	$x = 20$	$x = 40$	$x = 60$
$u = 2.75$	5,465.95	4,941.43	3,417.46	9,915.84	8,655.20	5,727.00
$u = 3.25$	3,319.85	3,018.98	2,077.96	7,483.25	6,567.24	4,349.76
$u = 3.75$	1,874.09	1,711.46	1,171.94	5,461.76	4,816.04	3,192.86
$u = 4.25$	979.37	897.04	610.64	3,849.83	3,408.96	2,261.45
$u = 4.75$	472.06	433.20	292.91	2,617.00	2,325.86	1,543.52
	$n = 20$			$n = 30$		
	$x = 20$	$x = 40$	$x = 60$	$x = 20$	$x = 40$	$x = 60$
$u = 2.75$	12,787.19	10,170.51	5,294.51	9,645.31	5,927.39	1,397.34
$u = 3.25$	11,205.16	8,965.18	4,697.22	8,962.16	5,518.59	1,300.85
$u = 3.75$	9,661.36	7,777.38	4,102.06	8,222.99	5,075.48	1,196.56
$u = 4.25$	8,194.92	6,636.65	3,524.45	7,449.35	4,609.85	1,087.29
$u = 4.75$	6,835.67	5,568.57	2,977.92	6,662.56	4,134.90	975.88

Table 2: Value of the lump sum option in DM (x, n in years, u in %)

The values of Table 2 for $u = 3.75\%$ are shown in Figure 2.

The option price is higher for younger insured persons. This results from the fact that the probability to survive the deferment period (and hence to be able to exercise the option) is smaller for older persons, cf. (6). A longer deferment period n, in general increases the values V_j in (6). However, the ${}_np_x$ are decreasing in n. The latter effect is the stronger, the older the insured person. Thus, the option value is first increasing and then decreasing in n, cf. Figure 2.

Furthermore, the value of the option depends heavily on u. There are, however, two contrary effects: On the one hand, a high rate of surplus during the deferment period leads to a higher value of S_n, and hence to a higher annuity payment. Thus, the value of the option increases proportionally. On the other hand, a high surplus rate during the time of annuity payment leads to a higher annuity compared to S_n, and therefore decreases the value of the put option since the value of the underlying increases. Table 3 shows the value of the lump sum option for $x = 40$ and $n = 20$ for different values of u_1 and u_2. Here, we can see as expected, that the value is increasing in u_1 and decreasing in u_2. Figure 3 visualizes these effects.

[25]Hence, in (1), we let $v = 1/(1.0325 + u_2)$.

[26]Hence, $S_n = 100,000(1.0325 + u_1)^n$.

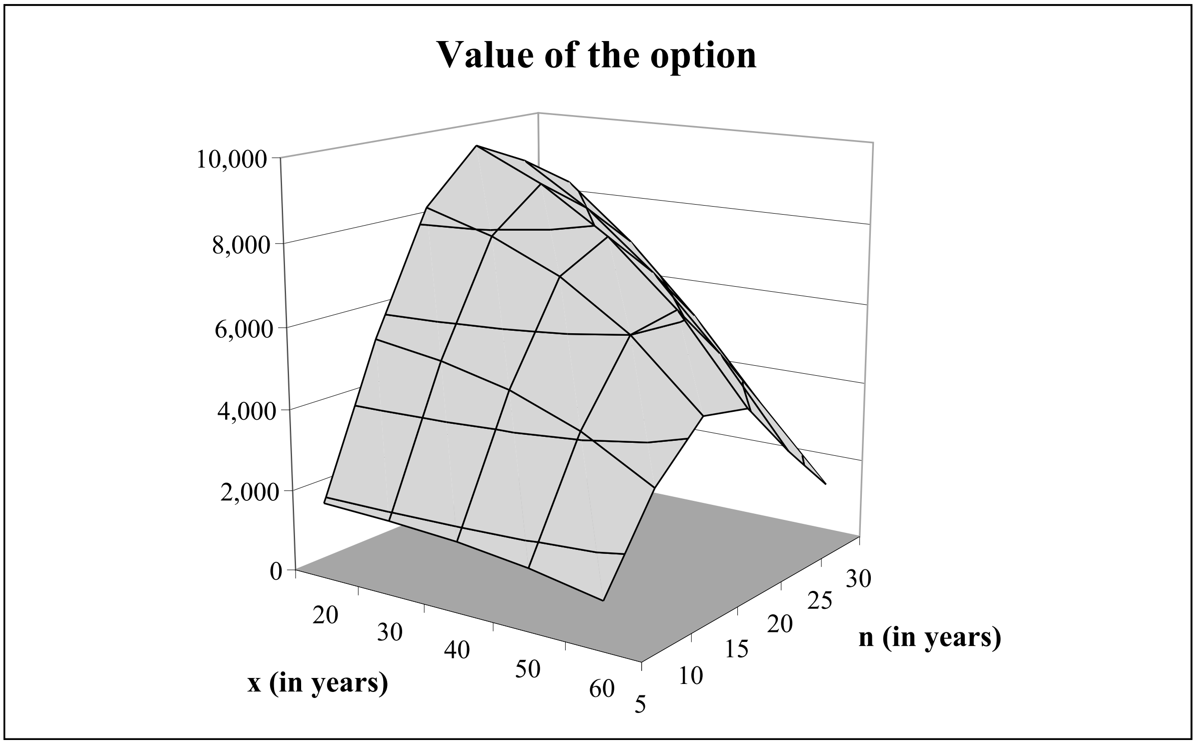

Figure 2: Value of the lump sum option as a function of x and n for $u = 3.75\%$

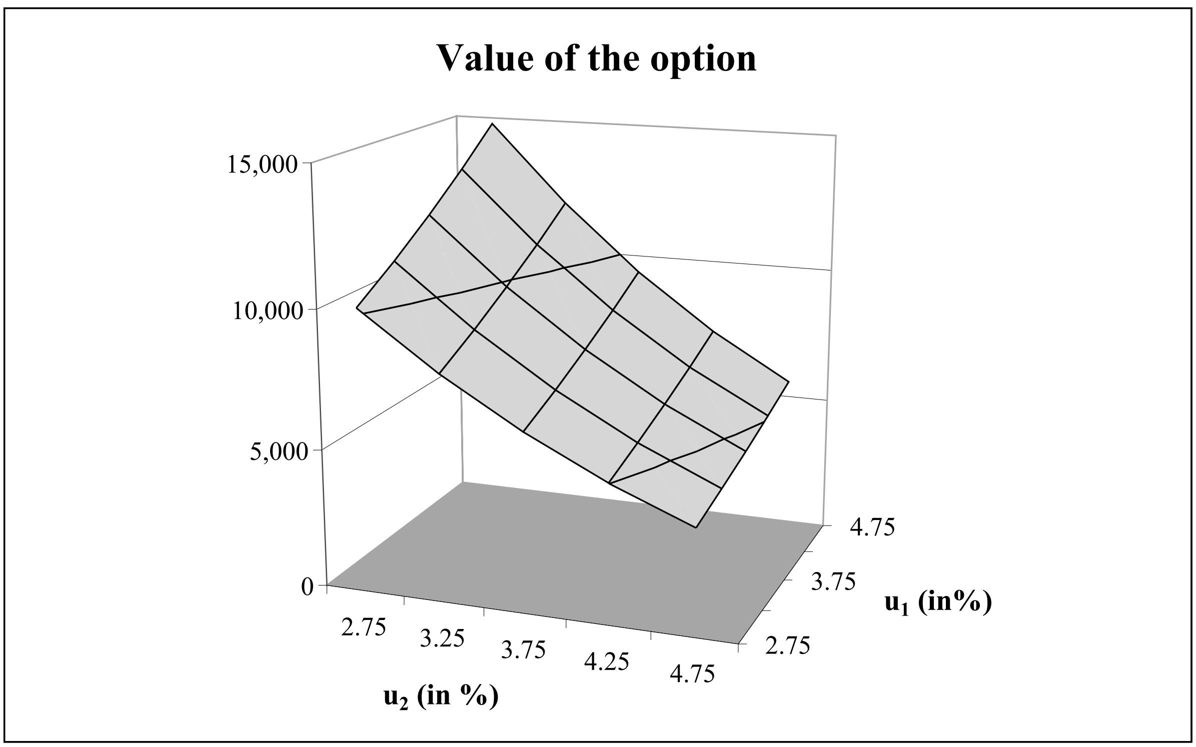

Figure 3: Value of the lump sum option for different values of u_1 and u_2

	$u_2 = 2.75$	$u_2 = 3.25$	$u_2 = 3.75$	$u_2 = 4.25$	$u_2 = 4.75$
$u_1 = 2.75$	10,170.51	8,159.88	6,445.78	5,010.69	3,831.65
$u_1 = 3.25$	11,174.23	8,965.18	7,081.90	5,505.19	4,209.79
$u_1 = 3.75$	12,271.60	9,845.60	7,777.38	6,045.82	4,623.22
$u_1 = 4.25$	13,470.84	10,807.77	8,537.43	6,636.65	5,075.02
$u_1 = 4.75$	14,780.89	11,858.83	9,367.70	7,282.07	5,568.57

Table 3: Value of the lump sum option in DM for different values of u_1 and u_2 (in %)

Table 4 shows the value of the lump sum option for $x = 40, n = 20$, and $u = 3.75\%$ for different market scenarios. We performed interest rate and volatility shifts of Δr[27] and $\Delta\sigma$, respectively. The values are also shown in Figure 4.

	$\Delta\sigma = -0.4$	$\Delta\sigma = -0.2$	$\Delta\sigma = 0$	$\Delta\sigma = 0.2$	$\Delta\sigma = 0.4$
$\Delta r = -3$	0,07	247,18	2.366,50	6.926,60	13.164,36
$\Delta r = -2$	3,40	751,22	3.848,04	8.754,46	14.601,10
$\Delta r = -1$	67,10	1.842,44	5.710,72	10.536,13	15.719,65
$\Delta r = 0$	598,07	3.697,94	7.777,38	12.106,49	16.451,17
$\Delta r = 1$	2.606,81	6.184,01	9.784,20	13.322,57	16.764,26
$\Delta r = 2$	6.277,60	8.805,41	11.452,50	14.085,47	16.661,62
$\Delta r = 3$	9.846,70	10.953,50	12.576,08	14.359,92	16.181,95

Table 4: Value of the lump sum option in DM for different market scenarios (Δr, $\Delta\sigma$ in %)

As expected, the value of the option is increasing in σ. Furthermore, the value of the put option usually increases in Δr, as the underlying becomes cheaper for higher levels of interest rates. However, with increasing Δr, future cash flows will be worth less at time $t = 0$ because the discount rate increases. This effect becomes dominant for high values of σ (see Figure 4).[28]

Our results show, that the value of the option can be substantial. For our standard market scenario with the same surplus rate for both the deferment period and the time of annuity payments, the value of the option varies from 0.29% ($n = 5$, $x = 60$, $u = 4.75$) to 12.79% ($n = 20$, $x = 20$, $u = 2.75$) of the single premium.

5 Summary and Outlook

In the present paper, we quantified the value of the lump sum option in deferred annuity contracts. We have shown that the value of this option can be substantial. Therefore,

[27] All interest rate shifts were performed by shifting the spot rates, cf. footnote 23.

[28] Furthermore, the value of the option is slightly decreasing in a, but as this effect is rather small, we do not quote any values.

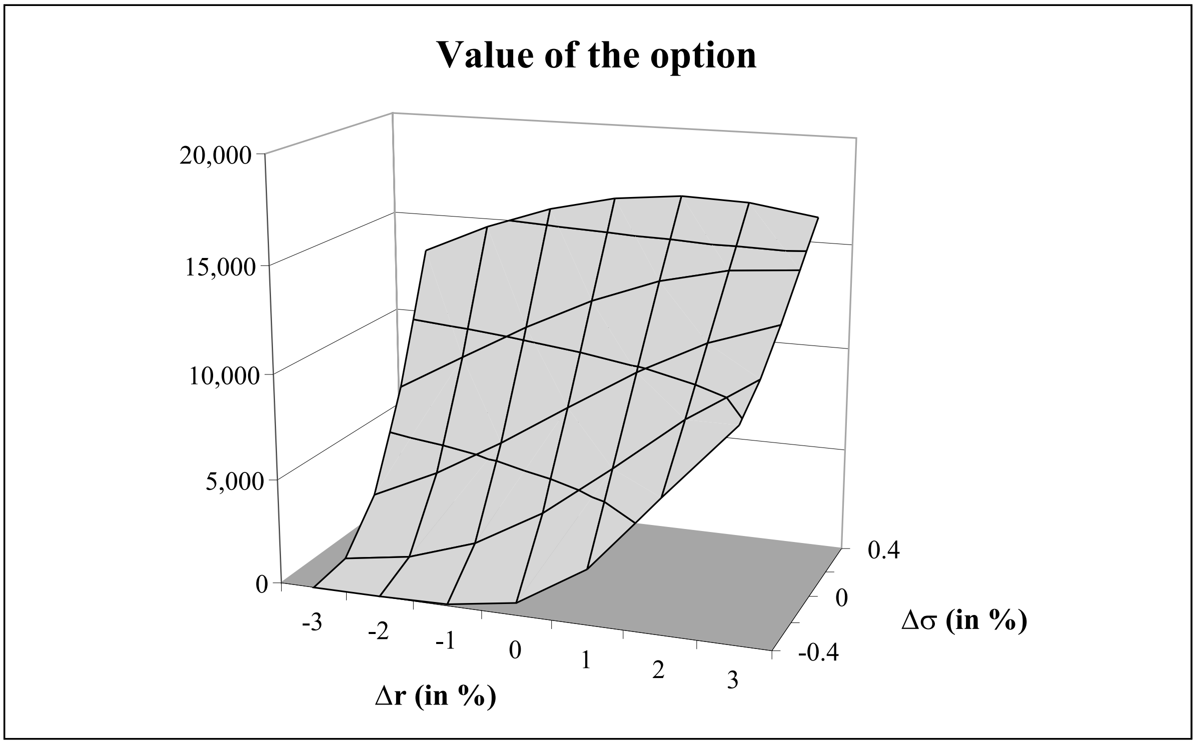

Figure 4: Value of the lump sum option for different market scenarios

the option has to be considered in the pricing of the policy.

The concept that was used in the present paper can be applied to other interest rate sensitive options as well. However, for more complicated options no explicit pricing formulas exist and numerical methods are required.

Another very popular implicit option, for which there is no explicit formula, is the so-called flexible expiration option. We will analyze this option within an endowment contract in Part 2.

Of course, there are some weaknesses of the applied methods which we will discuss in more detail in Part 2. One of them is that the values calculated in this paper are not the real costs the insurance company is faced with. Although these values are the costs that arise if the option is hedged, they are upper bounds for the real value since most insured persons base their decision about exercising such options on their circumstances of life, rather than only on financial aspects.

There are also some obvious extensions to our analysis. Of course every other annuity contract, e.g. with premium refund or a period of guaranteed annuity payments, including the lump sum option can be analyzed in the same way. Furthermore, our model can be applied to the case of regular premium payments and policies including costs. However, that would make the calculations more complicated.

Another possible extension could be to weaken the assumption of a constant surplus rate. The insurance company has the right to adjust the surplus level to their earnings because it is not a guaranteed rate. This corresponds to an option held by the insurance company which will reduce the value of the lump sum option computed here. This could e.g. be taken into account by assuming that whenever the short-rate r falls below some critical level, the insurance company will reduce the surplus rate u.

References

[Aa/Pe 94] Aase, Knut K. and Persson, Svein-Arne 1994: *Pricing of Unit-Linked Insurance Policies.* Scandinavian Actuarial Journal, 1, 1994: 26–52.

[Di/Ru 99] Dillmann, Tobias and Ruß, Jochen 1999: *Implizite Optionen in Lebensversicherungsprodukten. Eine quantitative Analyse am Beispiel des Kapitalwahlrechts.* Versicherungswirtschaft, 12/1999: 847–850.

[Du 96] Duffie, Darrell 1996: *Dynamic Asset Pricing Theory.* Princeton University Press, Princeton, New Jersey.

[Ge 96] Gebhard, Rüdiger 1996: *Risiken aus Rückkaufsoptionen in Lebensversicherungsverträgen.* Zeitschrift für die gesamte Versicherungswissenschaft, 4: 637–649.

[Ge 97] Gerdes, Wolfgang 1997: *Bewertung von Finanzoptionen in Lebensversicherungsprodukten.* Der Aktuar, 3, 3: 117–124.

[Gi 81] Gillespie, Robin G. 1981: *Financial Options in Life Assurance Policies.* Staple Inn Actuarial Society paper, March 1981.

[Ha 00] Haase, Jens 2000: *Optionspreise technisch einperiodiger Versicherungen.* Der Aktuar, 6, 3: 86–88.

[He 99] Held, Wolfgang 1999: *Optionen in Lebensversichungsverträgen.* IFA-Verlag Ulm.

[He/Kr 99] Herr, Hans-Otto and Kreer, Markus 1999: *Zur Bewertung von Optionen und Garantien bei Lebensversicherungen.* Blätter der DGVM, Band XXIV, 2: 179–193.

[Ho/Le 86] Ho, T.S. and Lee, S.-B. 1985: *Term Structure Movements and Pricing Interest Rate Contingent Claims.* Journal of Finance, 41: 1011–1029.

[Hu 97] Hull, John C. 1997: *Options, Futures, and other Derivatives.* Third Edition, Prentice Hall.

[Hu/Wh 90] Hull, John C. and White, Alan 1990: *Pricing Interest-Rate-Derivative Securities.* The Review of Financial Studies, 3, 4: 573–592.

[Ja 89] Jamshidian, Farshid 1989: *An Exact Bond Option Formula.* Journal of Finance, 44: 205–209.

[Lu 92] Lumsden, I. 1992: *Surrenders, alterations and other options.* Life Assurance Monograph, London; Institute of Actuaries and Faculty of Actuaries, London.

[N.N. 99] N.N. 1999: *Bei Novation: Vertrag besteht.* Versicherungswirtschaft, 7/1999: 450.

[No/Sa 85] North, Michael R.T. and Savill, Peter J. 1985: *Rider benefits and Options on Life Assurance Contracts.* Staple Inn Actuarial Society paper, April 1985.

[Va 77] Vasicek, O.A. 1977: *An Equilibrium Characterisation of the Term Structure.* Journal of Financial Economics, 5: 177–188.