IMPLICIT OPTIONS IN LIFE INSURANCE CONTRACTS
Part 2 - The case of flexible expiration options in endowment contracts

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Abstract

In the present paper we analyze the value of a flexible expiration option in endowment contracts. This option gives the policy-holder the right to terminate his policy before the regular expiration. The option is essentially equivalent to a Bermuda put option on a coupon bond. We evaluate the option within a Hull-White model framework and give empirical results. Furthermore, we perform extensive sensitivity analysis with respect to parameters of the insurance contract, as well as capital market parameters.

Keywords

Implicit option, flexible expiration option, Hull-White model, bond option pricing, Bermuda option.

1 Introduction

During the last years, implicit options in life insurance contracts have become more and more interesting. In Part 1 of this paper we have analyzed the lump sum option in deferred annuity contracts. In this paper, we analyze the so-called flexible expiration option in non-linked with-profit endowment contracts. This option gives the policy-holder the right to terminate his policy before the regular expiration. In Section 2, we describe the insurance contract and the option. In particular, we explain that this option is essentially equivalent to a Bermuda put option on a coupon bond, i.e. an interest rate option. In Section 3, we recall the well known model of Hull and White, introduced in Part 1. This is a one-factor, no-arbitrage model for the short-rate. Of all models that have some desirable features, this is the most tractable. Since – contrary to Part 1 – there are no explicit formulas available for this option, we here focus on numerical procedures for the pricing of options. In Section 4, we calculate the price of the flexible expiration option within this model and perform a detailed sensitivity analysis. We find that the option can be of substantial value. Section 5 closes with a summary and an outlook for further research.

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2 The Flexible Expiration Option

In Germany, many insurance companies include the so-called flexible expiration option in their non-linked endowment policies with profits. This option gives the policy-holder the right to terminate the contract before the regular expiration. It can usually be exercised on every policy birthday during the last couple of years of the contract. This period is called the flexible expiration period and usually lasts between 5 and 10 years. If the option is exercised, the insurance company pays the insurance benefit to the policy-holder. This payment includes any surplus earned so far, and there is no surrender charge. Of course, the benefit is greater, the later the option is exercised.

From a financial point of view, the flexible expiration option is a put option since the policy-holder has the right to ‚sell' the policy for the insurance benefit. At any point during the life of the contract, the remaining cash flow of the policy can be considered as a coupon bond. The coupons correspond to the expected death benefits of the policy, whereas the face value of the coupon bond is the expected payment at expiration.[1] Therefore, the flexible expiration option in an endowment policy can be considered as a so-called Bermuda put option on a coupon bond maturing at the expiration date of the policy. The strike price of the put option is the benefit that the insurance company pays in case the option is exercised. As opposed to a European option, a Bermuda option does not only have a single but several (discrete) possible exercise dates. In our case, these are the policy birthdays within the flexible expiration period.

Let us now introduce the necessary notation for our analysis. First, let $t = 0$ be the time of the start of the policy and x the age (in years) of the insured person (assumed to be male). The endowment policy has a term of n years. That is, if the insured person survives until the end of the policy the final benefit will be paid at time $t = n$. If the insured person dies before then, the insurance company pays a death benefit instead. In general, any such endowment contract is completely characterized by its premiums, its death benefits, and its survival payments[2] at all possible times t.

Let us furthermore assume a contribution period of s years, that is, the premiums are paid at time $t = 0,1,...,s-1$ with $1 \le s \le n$. Also, we assume that the option can be exercised at every policy birthday during the last m years of the contract ($0 < m < n$), i.e. at time $t = n-m, n-m+1,...,n-1$, if the policy-holder is still alive.

In Germany, non-linked insurance policies have a guaranteed rate of interest (currently at 3.25%) that we will denote by g. Thus, if the guaranteed final benefit at time n, denoted by S_n^g, and the guaranteed death benefits D_t^g (for all times t) are given, this rate specifies the premiums P_t. In the simplest case, where all guaranteed death benefits are equal to the guaranteed final benefit ($D_t^g = S_n^g$ for all t) and where there is a constant annual premium

[1] We price the flexible expiration option from the insurance company's point of view. Hence, if the portfolio of policies is not too small, it makes sense to regard the expected cash flow as the underlying of the option. Here, the term ‚expected' is used with respect to mortality. The surplus rates are considered to be deterministic (cf. footnote 16 in Part 1).

[2] By survival payments we mean the payments of the insurance company in case the insured person survives until then, i.e. the final benefit, as well as the payment at any possible time t if the option is exercised.

($P = P_t$ for all t), for any given value of the guaranteed final benefit, the premium P can be computed by $P = S_n^g \, A_{x:\overline{n|}} / \ddot{a}_{x:\overline{s|}}$.

In general, the insurance company earns more interest on its policy reserves than guaranteed in the contract. Hence, there is a surplus $u > 0$. Each year, this surplus is used to increase the guaranteed values D_t^g and/or S_n^g. We will denote the death benefit including surplus at time t by D_t, and the guaranteed survival payment at time t (that is, including all surplus distributed until time t) by S_t. Since our calculation of the option price is not based on the guaranteed values but on the ones including surplus, it does not matter for our analysis how the profits are distributed, as long as we know the values D_t and S_t for all t. Thus, if the sets $\{D_t\}$, $\{S_t\}$, and $\{P_t\}$ are given (by the insurance company), we neither need to specify the interest rates g and u, nor the relation between regular and terminal bonuses, since they are already included in both D_t and S_t. Note that the values D_t and S_t are of course only estimates, since they include predictions of future surplus rates. In Section 4, however, we will see that in Germany these estimates are very stable.

Summarizing all this we see that the flexible expiration option is a Bermuda put option with maturity n on a coupon bond with face value S_n (i.e. the expected final benefit including surplus), and coupons ${}_{t-1}p_x \, q_{x+t-1} D_t - {}_t p_x \, P_t$ at time t (i.e. the expected death benefit minus expected premium). The option is exercisable at times $t = n-m, n-m+1, ..., n-1$ at a strike price of S_t (i.e. the payment if the flexible expiration option is exercised). Within our model, we can compute the value of the flexible expiration option for any given sets $\{D_t\}$, $\{S_t\}$, and $\{P_t\}$.

3 The Model of Hull and White

We will use the model of Hull and White, introduced in Part 1, as the framework for our analysis.

Since the option analyzed here is a Bermuda option – contrary to Part 1 – there exists no explicit pricing formula. However, a positive aspect of the model of Hull and White is that it can easily be implemented numerically. In [Hu/Wh 94], Hull and White propose the use of a trinomial tree. Its advantage over a binomial tree is that it offers one more degree of freedom that is helpful in realizing mean reversion.

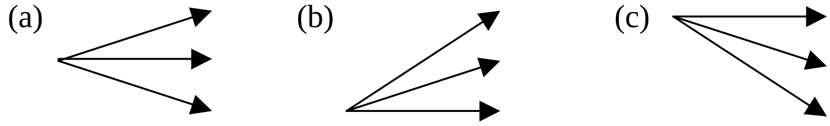

Figure 1: Branching in trinomial tree

Usually, the tree branches as in Figure 1(a). If the short-rate is small, the tree will branch off as in Figure 1(b). That reflects the mean reversion that implies a stronger upward drift. In case the short-rate is rather large, the branching pattern as shown in Figure 1(c) will be used. Therefore, the tree will have a structure as shown in Figure 2.

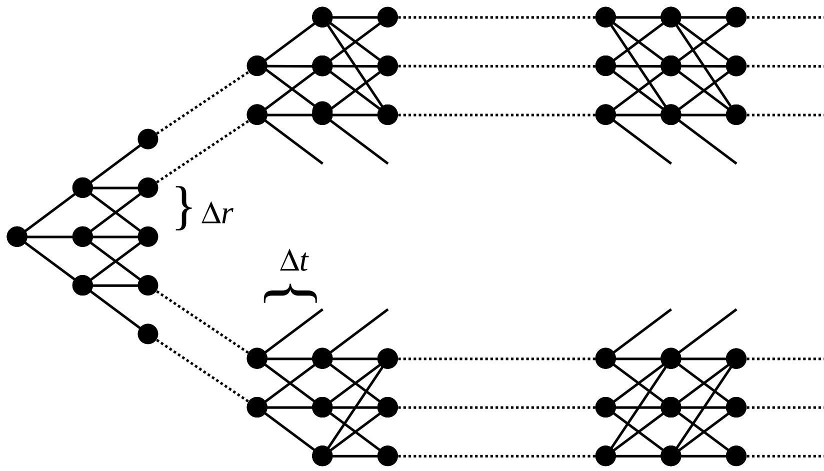

Figure 2: Trinomial tree used for the model of Hull and White

Hull and White give a criterion that specifies the point in time when the branching patterns given in Figure 1(b) and 1(c) will be used first and the tree will stop growing vertically. This not only reflects the mean reversion of the model but also has the advantage that the dimension of the tree is much smaller than that of a binomial tree. We will not go into the details of constructing and calibrating this tree and but only give an idea about the different steps of this procedure. For the technical details we refer the reader to [Hu 97], Chapter 17, or [Hu/Wh 94].

The transition probabilities from every node to all its possible succeeding nodes have to be computed. They depend on the position of the node within the tree and also on the model parameters θ and σ.[3] They are chosen such that both the expected change and the expected variance of the short-rate for the next time step Δt in the tree fit the corresponding values under the risk neutral measure. As a third condition the sum of all three transition probabilities has to be one. Hull and White also propose a specification for the step length Δr of the short-rate in the tree (depending on the model parameter σ, as well as the length of the time steps Δt).

In a first step, the tree is constructed for the simple process $dr^* = -ar^* dt + \sigma\, dz$. This process is symmetric with respect to $r^* = 0$ and its increments are normally distributed. The resulting tree is shown in Figure 2. In a second step, the difference process $\alpha(t) = r(t) - r^*(t)$, that is, $d\alpha = (\theta(t) - a\alpha(t))dt$, is used to compute the values of *r*(*t*). This means that in every node we compute the interest rate for the next Δt period. If we denote this interest rate by *R,* then we get the instantaneous short-rate *r* from it by (cf. [Hu 97])

$$r = \frac{R\Delta t + \ln A(t, t + \Delta t)}{C(t, t + \Delta t)}.$$

[3] Cf. Part 1.

Using this with the formulas for zero bond prices as given in Part 1, we can compute the prices of zero bonds (and thus the entire term structure) in any node of the tree.

The valuation of any security using this tree is analogous to the valuation in a binomial tree. Starting with the payoff of the security at maturity we use backward induction to compute the price of the security at the beginning. The price of the security at any node is computed from the prices at its succeeding nodes, weighted with the corresponding transition probabilities and discounted with the Δt period interest rate in that particular node (cf. [Hu 97]). In our case, whenever it would be possible to exercise the option at the current node (that is, at any policy birthday within the flexible expiration period) we need to check if exercise is preferable. Suppose we are at node (k, j) in the tree (that is, we are at some integer time $i = k\Delta t$ with $n - m \le i < n$ and at a short-rate level of $j\Delta r$). Then the payoff of the put option can be computed by

$$\max\left[0; S_i - \sum_{l=i+1}^{n} D_l \;{}_{l-i-1}p_{x+i}\; q_{x+l}\; d(i, j, l-i) + \sum_{l=i}^{s-1} P_l \;{}_{l-i}p_{x+i}\; d(i, j, l-i) - S_n \;{}_{n-i}\, p_{x+i} d(i, j, n-i)\right],$$

where D_l denotes the death benefit and P_l the premium in year *l*, ${}_l\, p_x$ denotes the probability of an *x* year old man to survive *l* more years, q_x denotes the probability of an *x* year old to die in the next year, and $d(i, j, l)$ denotes the price of a discount bond in the tree at the node (k, j) with $i = k\Delta t$ and with maturity at time $i + l$. If the option was exercised at time *i* the policy-holder would receive S_i. Since this would also terminate the contract we have to subtract all other expected future payments of the policy, that is, all expected future death benefits, as well as the expected final benefit S_n at expiration of the policy. Of course these future cash flows need to be weighted with their chance of occurring, and they have to be discounted to time *i* (using discount bond prices $d(.)$ computed in the tree). Since the policy-holder has the right to not exercise the option, we take the maximum with 0.

Contrary to our previous work where we also used the Hull-White model, we here use a tree since the involved options are non-European (in Part 1, as well as in [Ru 99] we considered European options and were able to find explicit pricing formulas or used Monte Carlo Simulation). Lately, algorithms for the pricing of non-European options using Monte Carlo Simulation have been developed, cf. e.g. [Ti 93], [Ba/Ma 95], [Br/Gl 97], or [Br/Gl/Ja 97]. Nevertheless, using a tree seems to be more tractable.

4 Empirical Results

In what follows, we will consider a special case of the general situation described in Section 2, and make some more assumptions about the policy to make the computations easier. First, we assume that the contract involves only a single premium that is paid at the beginning and denote it by P_0. It can be computed by $P_0 = S_n^g\, A_{x:\overline{n}|}$.[4] In this case, the coupons of the

[4] For the sake of simplicity, we ignore all costs in our numerical example.

underlying bond are the expected death benefits ${}_{t-1}p_x\, q_{x+t-1} D_t$, while the strike price at time t is still S_t.

Furthermore, we assume that the surplus rate stays constant for the entire term of the endowment policy and denote it by u. Although this seems to be a very strict assumption it is quite reasonable since insurance companies in Germany try to keep the surplus rates fairly constant over time (this can also be seen when looking at historical data). They use their hidden reserves to smoothen the yearly variations of their investment returns. Hence, our analysis quantifies the value of the described option, if the insurance company continues this way of smoothening bonuses. This means, that from a financial point of view, during the flexible expiration period, the policy-holder still receives the interest rates corresponding to a long term investment (the investment return of the insurance company's portfolio) while he can cancel the policy on short notice without surrender charges.

For our empirical calculations we will assume the simplest way of distributing surplus. The return at time t exceeding the guaranteed interest g earned on the policy reserves is used to increase all future benefits by the same factor. Hence, the survival payments and the death benefits annually increase by a factor of $1+u$. Thus, the final benefit is given by $S_n = S_n^g (1+u)^n$, and $S_t = D_t = S_n^g (1+u)^t$ for all t.

For our following analysis, we use the same market data as in Part 1. So again, the parameters for the model are $a = 0.0001$ and $\sigma = 0.6306\,\%$.

If we assume a guaranteed final benefit of $S_n^g = 100{,}000$ DM and a guaranteed rate of interest $g = 3.25\%$, we can compute the single premium for different values of x and n. Here, we use the mortality table DAV94T of the German Society of Actuaries (DAV). The results are given in Table 1.

	$n = 15$	$n = 20$	$n = 25$	$n = 30$	$n = 35$
$x = 20$	55,935.91	46,302.04	38,478.00	32,167.61	27,139.18
$x = 25$	55,948.01	46,358.30	38,623.83	32,460.63	27,648.85
$x = 30$	56,016.96	46,537.04	38,982.99	33,085.32	28,618.33
$x = 35$	56,224.54	46,963.40	39,732.97	34,256.52	30,290.47
$x = 40$	56,622.01	47,735.29	41,004.35	36,129.80	32,852.01
$x = 45$	57,298.31	48,980.21	42,956.22	38,905.52	36,506.27
$x = 50$	58,364.97	50,848.29	45,793.87	42,800.10	41,358.50

Table 1: Single premium P_0 for the described endowment policy
(insured person is male, x and n in years)

Within this framework, we calculated the value of the flexible expiration option for several insurance contracts and analyzed the sensitivity of that value with respect to several input factors. First, we assume the insured person to be male and aged 30. Furthermore, let the sum assured be 100,000 DM and the surplus rate be 3.75%. In this case, the value of the option for different values of the flexible expiration period m and the term of the policy n is shown in Table 2.

	n = 15	n = 20	n = 25	n = 30	n = 35
m = 3	5,278.39	5,482.04	5,301.63	3,586.65	2,309.85
m = 4	6,929.19	7,341.44	7,089.03	4,971.88	3,127.62
m = 5	8,499.59	9,156.49	8,877.06	6,534.56	3,978.55
m = 6	9,707.73	10,944.79	10,657.44	8,250.38	4,874.14
m = 7	11,087.57	12,690.54	12,417.77	9,988.42	6,117.27
m = 8	12,460.24	14,378.81	14,275.14	11,739.52	7,421.68
m = 9	13,710.73	15,994.64	16,108.64	13,495.96	8,754.51
m = 10	15,104.87	17,519.39	17,920.32	15,246.78	10,261.81

Table 2: Value of the flexible expiration option in DM
for different values of n and m (in years)

The results of Table 2 are illustrated in Figure 3. As expected, the option value is increasing in m. The sensitivity with respect to n is more complicated. If we assume that there is no mortality, the option price is of course increasing in n. On the other hand, with increasing values of n the probability that the insured person dies before he can exercise the option also increases. This leads to the effect, that the option price is first increasing and then decreasing in n. An analogous effect can be observed when analyzing the value of the lump sum option in deferred annuities, cf. Part 1.

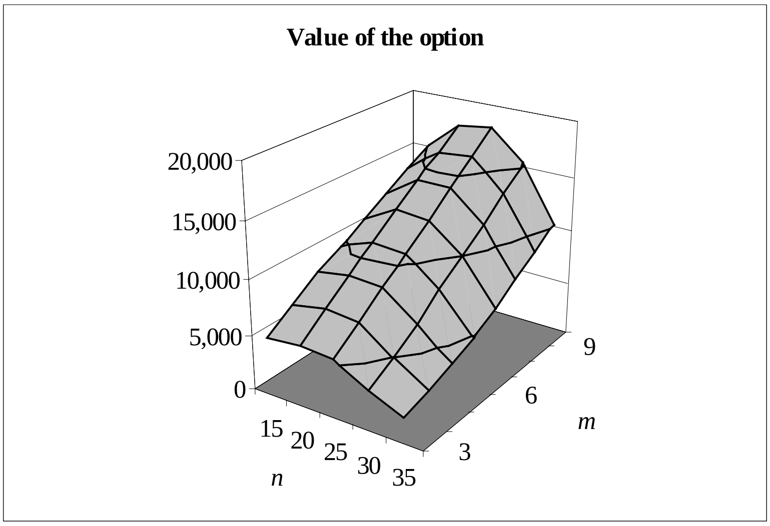

Figure 3: Value of the flexible expiration option as a function of n and m

The value of the option is quite substantial. It is between 2.31% (m=3, n=35) and 17.92% (m=10, n=25) of the sum assured, or between 8.07% (m=3, n=35) and 46.08% (m=10, n=30) of the premium.

We then fixed an expiration period of 10 years and calculated the value of the option for different values of the age x of the insured person, and the term n of the policy. The results are given in Table 3 and Figure 4.

	n = 15	n = 20	n = 25	n = 30	n = 35
x = 20	15,301.40	17,917.13	18,655.12	16,318.18	11,504.78
x = 25	15,236.03	17,783.31	18,402.91	15,916.01	11,016.88
x = 30	15,104.87	17,519.39	17,920.32	15,246.78	10,261.81
x = 35	14,856.97	17,026.19	17,129.79	14,209.32	9,096.82
x = 40	14,431.68	16,267.64	15,947.89	12,642.39	7,473.90
x = 45	13,811.17	15,164.60	14,187.27	10,451.68	5,533.28
x = 50	12,926.88	13,511.38	11,744.05	7,743.78	3,497.16

Table 3: Value of the flexible expiration option in DM
for different values of *x* and *n* (in years)

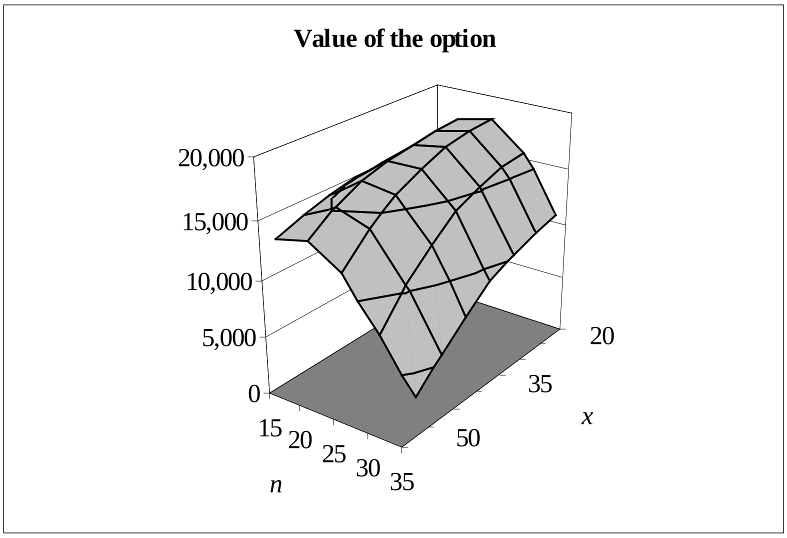

Figure 4: Value of the flexible expiration option as a function of *n* and *x*

The sensitivity with respect to *n* is as described above. Furthermore, the option price is decreasing in *x*. This is due to mortality effects, since the probability that the insured person survives until he can exercise the option is decreasing in his age.

The values of the option vary between 3.5% (*x*=50, *n*=35) and 18.65% (*x*=20, *n*=25) of the sum assured, or 8.46% (*x*=50, *n*=35) and 50.73% (*x*=20, *n*=30) of the premium.

Finally, we analyzed the sensitivity of the option price with respect to the level of interest rates and the volatility of the short-rate process. We assumed shifts in volatility of $\Delta\sigma = \pm 0.2\%$, and $\Delta\sigma = \pm 0.4\%$ as well as shifts in the level of interest rates of $\Delta r = \pm 1\%$, $\Delta r = \pm 2\%$, and $\Delta r = \pm 3\%$. The results for *x*=30, *n*=30, *m*=10, and $u = 0.0375$ are given in Table 4 and Figure 5.

	$\Delta\sigma$ = -0.4	$\Delta\sigma$ = -0.2	$\Delta\sigma$ = 0	$\Delta\sigma$ = 0.2	$\Delta\sigma$ = 0.4
Δr = -3	1,587.24	5,586.21	10,139.58	14,877.70	19,699.92
Δr = -2	4,867.40	8,604.93	12,345.95	16,131.70	19,892.03
Δr = -1	9,295.10	11,462.26	14,107.63	16,886.41	19,774.40
Δr = 0	12,842.18	13,660.41	15,246.78	17,112.79	19,155.67
Δr = 1	14,683.54	14,918.12	15,705.63	16,877.88	18,283.51
Δr = 2	15,145.31	15,196.33	15,533.83	16,209.06	17,100.05
Δr = 3	14,720.35	14,728.69	14,855.22	15,205.84	15,760.34

Table 4: Value of the flexible expiration option in DM
for different market scenarios ($\Delta\sigma$, Δr in %)

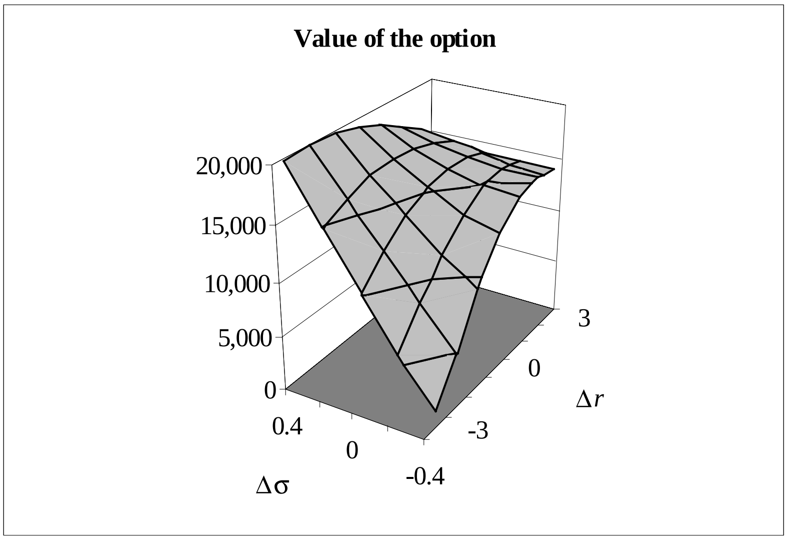

Figure 5: Value of the flexible expiration option for different market scenarios

As expected, the option price is increasing in the volatility. Furthermore, it tends to be increasing in the level of interest rates, as well. For an extremely high level of volatility, however, the value is decreasing in the level of interest rates. The reason for this is, that the value of the put option increasing Δr as the underlying bond becomes cheaper for higher levels of interest rates. However, with increasing Δr, future cash flows will be worth less at time $t = 0$ because the discount rate increases. This effect becomes dominant for high levels of σ (see Figure 5). The values of the option vary between 1.59% ($\Delta\sigma = -0.4\%$, $\Delta r = -3\%$) and 19.89% ($\Delta\sigma = 0.4\%$, $\Delta r = -2\%$) of the sum assured, or 4.8% ($\Delta\sigma = -0.4\%$, $\Delta r = -3\%$) and 60.12% ($\Delta\sigma = 0.4\%$, $\Delta r = -2\%$) of the premium.

5 Summary and Outlook

In the present paper, we quantified the value of the flexible expiration option in endowment policies within a Hull-White model framework. We have shown, that the value of the option can be substantial. Hence, such options should be considered in the pricing of the policy.

Furthermore, as seen in Table 2, the value of the option differs between 8% and 46% of the single premium depending on the term of insurance and the flexible expiration period. Thus it is not fair to charge the total costs to all insured persons.

There are some weaknesses of the applied methods as already mentioned in Part 1:[5]

The model of Hull and White of course has its shortcomings. For example, the short-rate can become negative with positive probability which is not desirable. Nevertheless, this model is still widely used in practice because it is analytically quite tractable and can easily be implemented numerically.

The values calculated in this paper are the costs that arise if the option is fully hedged, that is, if the insurance company would buy financial instruments at the beginning to protect itself against the risks imposed by these options, if all policy holders would exercise at the optimal time. Thus, these values are upper bounds for the real value since most insured persons base their decision about exercising such options on their circumstances of life, rather than only on financial aspects. Nevertheless, we do expect the number of policy-holders considering financial aspects in their decision to increase. Often, policy-holders will not know about the value of their option and use it in their favor. Therefore the insurance companies will realize the theoretical loss of these options only in some cases. However, it is important to know the value of options in the worst case. Evaluating data about the frequency of exercise of such options, depending on market rates, could then be used to account for the real behavior of policy-holders.

In both Part 1 and 2, we assumed a constant surplus rate while in practice the insurance companies can vary these rates. Of course we made this assumption to simplify the computations, but it also models the attempt of insurance companies to keep their surplus rates constant for marketing reasons (cf. Section 4). Nevertheless, the insurance company has the right to adjust the surplus level to their earnings because it is not a guaranteed rate. This corresponds to an option that the insured person has implicitly sold to the insurance company which will reduce the value of the option computed here. It would be interesting to see, how the value of the flexible expiration option is reduced, if surplus rates are adapted to market rates, e.g. surplus rates are reduced whenever interest rates fall below some threshold.

Besides the incorporation of variable surplus rates in the model, there are several other possible extensions to our analysis. Of course we can include costs in our computations and do the same analysis for other kinds of endowment contracts (e.g. regular premium payments), as long as the sets $\{D_t\}$, $\{S_t\}$, and $\{P_t\}$ as described in Section 2 are given.

In practice, the flexible expiration option is also often included in non-linked deferred annuity contracts. In this case, the policy-holder has the right to start the annuity (or receive a lump sum) some time before the date agreed upon in the contract. Again, the flexible expiration period at the end of the deferment period usually lasts between 5 and 10 years. Of course, the annuity will start at some lower level than it would at the end of the deferment period. If this option is not included in the contract and the insured person wants to start the annuity payments earlier he has to surrender his policy and use the money to buy a non-deferred annuity instead. However, in that case he would end up getting a lower annuity because the insurance company would charge surrender fees. Also, he would in general have to pay

[5] Cf. also [Ge 97].

acquisition costs for the new contract.

Similarly, this flexible expiration option is a Bermuda put option, where the underlying is the (deferred) annuity agreed upon at the beginning of the policy. The strike price is the present value of the non-deferred annuity received instead.[6]

Furthermore, it would be interesting to analyze, how the values sum up when several options are combined within the same contract.

Another difficulty arises when we consider unit-linked contracts that include these kind of options, because the value of the option will not only depend on future interest rates but also on future development of the funds or stocks the policy invests in. As a consequence, we need to extend our model by including a second stochastic process that models this other source of uncertainty.

We expect that innovative life insurance products will incorporate more and more flexibility, i.e. options. Furthermore, consumers will become more and more aware of the effect of such options on future cash flows. Thus, it is very likely, that in the future insured persons will more frequently base the decision to exercise an option on financial reasons. In the USA, there already exist financial advisers who analyze the contracts of insured persons and receive a profit share if they find an option that creates a financial benefit for the customer. In order to avoid such risks, insurance companies have to take the value of options into account when policies are priced. The target should be to find a general model, that allows to calculate the value of all options – not only the financial options considered here. Hopefully, our analysis can be a step into that direction.

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[6] Since expected annuities are essentially equivalent to coupon bonds, this case can be analyzed within our model as well. See Part 1, where the case of a lump sum option in a deferred annuity contract is studied.

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