TAX ARBITRAGE IN THE GERMAN INSURANCE MARKET
AN EMPIRICAL ANALYSIS

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ABSTRACT

In this paper we present empirical results concerning the attractiveness of a "mortality swap", which combines an immediate annuity and a whole life insurance contract, in the German insurance market. The analysis follows a methodology introduced by Charupat and Milevsky (1999). Similarly to their results for the Canadian insurance market we find that depending on the level of interest rates there might exist significant arbitrage opportunities in particular for elderly and high income people which can mainly be explained by an inadequate and unsatisfactory method of taxation.

1. INTRODUCTION

In a recent paper, Charupat and Milevsky (1999) analyze arbitrage opportunities in the Canadian insurance market. They compare the rates of return from a risk free investment to those from a certain type of insurance product that has essentially the same cash flow structure. It is shown that this insurance product, which combines an immediate annuity and a life insurance contract, turns out to be more attractive than the risk free investment on an after-tax-basis. Furthermore, Charupat and Milevsky give empirical evidence showing a significant magnitude of differences in the rate of return for certain cases of age and tax rate. Their empirical results also suggest that the discrepancy increases in age and tax rate.

The insurance transaction considered, which is called a "mortality swap", is constructed as follows: The income from an immediate life annuity is partly used to pay the premiums for a life insurance contract. Let, e.g., the annuity investment as well as the sum insured in the life insurance contract be 100 DM. Then, this transaction has the same cash flow structure as a 100 DM bank deposit that yields periodic interest payments and is paid back at the time of death.

In our paper we present empirical results concerning arbitrage opportunities involving mortality swaps offered in the German insurance market under German taxation rules.

The German life insurance market has only been deregulated since 1994. That is why innovative products such as unit-linked insurance contracts still have rather small (but strongly increasing) market shares. For what follows, we focus on traditional (i.e. non-linked) products. Such products have a guaranteed rate of interest (currently at 3.25%). Furthermore, the policies earn some surplus. For single-premium non-deferred annuities, this means that there is some guaranteed annuity and an annuity resulting from surplus. The latter is not guaranteed but calculations based on expected future surplus rates are used when the product is marketed. For this reason insurance companies have a strong interest in keeping surplus rates stable. This is achieved by accumulating hidden reserves in good years and using these reserves to preserve the surplus in years where the insurance companies earn less.

In our analysis, we assume that the promised annuity including surplus will be paid throughout the life of the contract. This will be justified in Section 3.

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Following Charupat and Milevsky, we use the term "arbitrage" in a broad sense meaning the existence of economically equivalent investments that lead to significantly different returns. In competitive markets, alternative investments with exactly the same payoff characteristics should be valued identically. So, the existence of largely equivalent investment opportunities distinctly varying in returns can only be explained by market frictions.

As Charupat and Milevsky argue, their results are mainly due to the way annuities are taxed in Canada which implies that the mortality swap is treated by tax law in a very different way compared to the bank deposit. This, in essence, remains true for our results. The basic principle according to which annuities are taxed in Germany is the same as in Canada: Tax authorities consider an annuity payment as consisting of two portions. One part is used to pay back the invested amount of capital and is therefore not subject to taxation. The second portion is considered as interest and thus taxable.

The taxable portion depends on the age of the annuitant when the annuity payments start (which in our case of an immediate annuity means the age when the annuity is purchased) and is given in a certain table in the tax law.[1] The computation method upon which this table is essentially based assumes that a temporary annuity is paid for the period of a male person's expected remaining lifetime, using an interest rate of 5.5 %.[2, 3] Note that here the German method of calculating the taxable portion differs from what Charupat and Milevsky quote for the Canadian market.

Our empirical results confirm, to a great extent, the results of the paper mentioned above. In the German insurance market, significant arbitrage opportunities can indeed be observed. Possible advantages from purchasing a mortality swap are clearly increasing in the tax rate, and in general also in age. Thus the combined insurance product would be particularly interesting for older people with high income.

Of course, the taxation rules as described above at first raise the question of why two different investment alternatives which behave fairly similarly on a pre-tax-basis are treated differently by tax law. But even if regulators choose to set up tax laws in favor of the one or the other type of investment the question remains: why do both alternatives then survive in the market? Charupat and Milevsky give several reasons for the existence of this arbitrage opportunity. The most important reason is the fact that not everybody can buy the mortality swap, since the whole life insurance requires a good health condition, which is a particular drawback for older people. Secondly, not many people have enough knowledge of life insurance products to find out how to construct such a mortality swap. Additionally, although the two investment strategies are very similar, there remain certain differences that are explained in detail in Section 2. These differences might cause investors to value the strategies differently from our approach.

2. METHODOLOGY

Our analysis is based on comparing two different investment strategies that yield essentially equivalent return patterns. First, we look at investing an amount N in a coupon bond with a term of T years yielding a constant coupon of c. The cash flow of this product is shown in table 1.

t	0	1	2,...,T-1	T
Cash flow	$-N$	c	c	$c + N$

Table 1: Cash flow of a coupon bond.

[1] See appendix, table 7.

[2] See Schmidt (1998), p. 1776. Before 1994 the law was based upon older mortality tables usually including shorter expected remaining lifetimes and thus implying smaller taxable portions.

[3] It has to be mentioned that the way annuities are treated by the German tax law has been heavily criticized, as it favors people receiving an annuity as a pension as compared to former civil servants whose pensions are more or less entirely taxed. Effectively an old age pensioner who receives nothing but an annuity of about 50000,- DM per year would have to pay no taxes if the annuity starts at the age of 65, since then the taxable portion would be 27% and thus below a level considered a non taxable existential minimum. At the moment a decision from the Federal Constitutional Court is expected in this context.

Secondly, we create a portfolio consisting of two different insurance contracts: The first insurance contract is a single premium lifelong annuity, where the insured person pays a premium P at t=0 and then each year receives a constant annuity payment A until he dies. The second insurance contract is a whole life insurance where the insured person pays an annual premium Q as long as he lives. Upon death, he receives a death benefit D. If we denote the policy birthday after the (stochastic) time of death of the insured person by τ, and assume that all premiums are paid in advance and all benefits are paid in arrear, the cash flow of the portfolio of insurance policies is given in table 2.

t	0	1	2,...,τ–1	τ
Cash flow annuity	-*P*	*A*	*A*	0
Cash flow whole life	-*Q*	-*Q*	-*Q*	*D*
Sum of cash flows	-*P*-*Q*	*A*-*Q*	*A*-*Q*	*D*

Table 2: Cash flow of the investment strategy involving two insurance contracts.

We see that if we let $P+A=D$, this structure is equivalent to the coupon bond described above.

Creating such a portfolio of insurance policies is often referred to as a mortality swap, since the effect of mortality on the effective yield is eliminated by buying one product that pays upon death and another that pays until death. The annuity is often referred to as "pay death/get life", whereas the whole life insurance is called "pay life/get death".

Although the structure of the two payoff patterns is essentially equivalent, there are several significant differences. First, of course, the coupon bond has a deterministic, prefixed term whereas the term of the insurance portfolio is stochastic.[4] We will come back to this issue at the end of this Section. Furthermore, one might argue that the default risk of the different investment strategies is not the same. It should however be possible to find a bond that matches any given default risk. Additionally, an individual's health status may make it impossible for him to buy a whole life policy or may lead to an increasing premium. This implies that our strategy only works for people who are healthy enough to get into a whole life contract.

In our empirical analysis in Section 3, we will look at real insurance products and compare their return after tax with the corresponding return of the coupon bond. The return of the coupon bond depends on the coupon and the tax rate of the investor. The return of the insurance product depends on the amount of the annuity, the premium for the whole life policy, the age of the insured person and his tax rate. Note that age not only influences the premium for the whole life policy and the amount of the annuity but also the taxable portion of the annuity, cf. Section 1.

If the coupon bond has an effective pre-tax yield of $r = \frac{c}{N}$, the effective yield after tax is given by

$$i_{bond} = r(1-t),$$

where t denotes the tax rate of the investor.

For $P+A=D$, the effective yield before tax of the combination of insurance products is given by $\frac{A-Q}{P+Q}$. If we denote the taxable portion of the annuity payment by τ, the effective yield after tax is given by

$$i_{insurance} = \frac{A(1-\tau \cdot t)-Q}{P+Q}.$$

[4] Of course, one could also look at a bank deposit rather than a coupon bond. This would eliminate the problem of the fixed term of the investment, since the money could be deducted from the deposit at the time of death. On the other hand, however, the effective yield would usually be lower.

In the next Section, we will calculate the effective yield of both investment strategies for different combinations of age, tax rate and market interest rates.

Since in Germany insurance companies tend to keep the surplus of their policies very stable, the return of the insurance portfolio will depend only very slightly on the time where the contract is initiated, whereas the return of the bond depends heavily on market interest rates and thus on the time the bond is bought. Hence it is useful to look at the so-called critical level of interest rates i^*. By this we mean the return a bond has to yield before tax, such that the return after tax is the same for both our strategies. Hence, for $r>i^*$, investing in bonds yields a higher return after tax than buying the insurance contracts, whereas for $r<i^*$ the return of the insurance policies is higher. In general, i^* will depend on age and tax rate of the investor.

Some easy calculation shows that for given A, P, Q, t, and τ, i^* is given by

$$i^* = \frac{A(1-\tau t) - Q}{(P+Q)(1-t)}.$$

Since investing in bonds bears a reinvestment risk due to the prefixed term, i^* can also be interpreted as follows: If an investor has the choice of either investing in the mortality-swap strategy or in bonds (reinvesting upon maturity), then the mortality-swap strategy is preferable, if the average return of the bonds bought is below i^*.

3. EMPIRICAL ANALYSIS

3.1 Input Data

The taxable portion of an annuity depends on the age of the insured person and is defined by tax law. The values for a person aged 30, 40, 50, 60, and 70 are given in table 3. Furthermore, the yearly annuity for a male insured person investing a premium of P=100.000 DM is also given in table 3. These values were derived by comparing the offers of all German life insurance companies and using the company that paid the highest annuity. Furthermore, table 3 shows the premium for a whole life insurance paying $P+A$ upon death of the insured. Since pure whole life policies are not sold in the German insurance market,[5] we used products from the UK market instead. The premiums were the cheapest available guaranteed rates. They were provided by a provider for online insurance quotes. Finally, table 3 shows the value of A-Q, which is the "coupon" of our investment strategy.

Age	τ	A	Q	A-Q
30	60	7152	128	7024
40	52	7656	290	7366
50	43	7944	612	7332
60	32	9168	1651	7517
70	21	11544	4280	7264

Table 3: Taxable portions, annuities and life insurance premiums for different ages.

For these input data, we performed the calculations described in Section 2.

3.2 Results

As a point of reference for our further results, we are interested in the effective yield of a coupon bond depending on the market interest rate and the tax rate. Obviously it can easily be calculated and is given in table 4 for certain combinations of market return rate and tax rate. We considered a range of possible interest rates from 4% to 10% in 1% increments and possible tax rates from 0% to 50% in 10% increments.[6]

[5] There are however some similar products where the sum assured is payable upon death or some limiting age (mostly 80 years), whichever occurs first, or where the sum assured is payable upon death but the policy becomes a paid-up policy upon some limiting age of the insured person. Using such policies would still yield similar results, but in this case, the effective yield of the insurance portfolio would depend on the time of death.

[6] The current maximum income tax rate in Germany is 53%. It will probably be changed to below 50% in the near future.

	0%	10%	20%	30%	40%	50%
4%	4,00	3,60	3,20	2,80	2,40	2,00
5%	5,00	4,50	4,00	3,50	3,00	2,50
6%	6,00	5,40	4,80	4,20	3,60	3,00
7%	7,00	6,30	5,60	4,90	4,20	3,50
8%	8,00	7,20	6,40	5,60	4,80	4,00
9%	9,00	8,10	7,20	6,30	5,40	4,50
10%	10,00	9,00	8,00	7,00	6,00	5,00

Table 4: Effective yield of a coupon bond as a function of market return rate and tax rate.

The return from a coupon bond is compared to that from a mortality swap as described in Section 2. The effective yield of the combined insurance product depends on the age of the insured and on the tax rate. It is calculated for the tax rates and the ages at entry mentioned above. The results are shown in table 5. Figure 1 visualizes these results.

	30 years	40 years	50 years	60 years	70 years
0%	7,02	7,37	7,33	7,52	7,26
10%	6,60	6,97	6,99	7,22	7,02
20%	6,17	6,57	6,65	6,93	6,78
30%	5,74	6,17	6,31	6,64	6,54
40%	5,31	5,77	5,97	6,34	6,29
50%	4,88	5,38	5,62	6,05	6,05

Table 5: Effective yield of the insurance product as a function of age and tax rate.

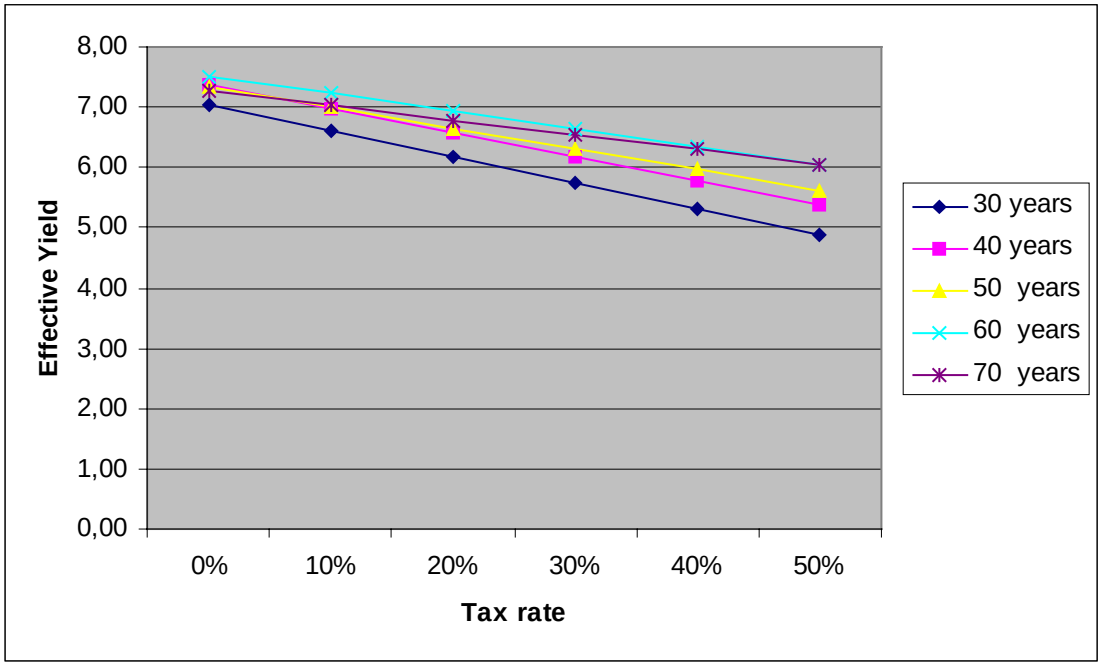

Figure 1: Effective yield of the insurance product as a function of age and tax rate.

Obviously the development of the taxable portion is extremely important for these results (see table 3, Section 3.1). With increasing age the taxable portion decreases. As long as this effect prevails, an increasing advantage from the mortality swap can be observed for each given (positive) tax rate. Up to the age of 60 the decreasing taxable portion seems to level out or is even enhanced by the impact of the development of *A-Q*. The fact that the effective yields slightly decrease when the age is raised from 60 to 70 is due to the sharp increase in the

whole life insurance premium connected to this age increment. At this point the raise of the life insurance premium is not leveled out by the additional tax advantage and the increased annuity payment (the latter, of course, being due to the reduced expected remaining lifetime).

Naturally, the different taxation of the coupon bond on the one hand and the mortality swap on the other gets more important as the tax rate increases. This can easily be seen for any given market return and any given age by comparing the values from tables 4 and 5. But the effect is expressed more accurately by the development of the critical level of interest rate (meaning the rate of return which a bond must offer to be as attractive as the mortality swap). Table 6 gives the critical interest for different parameters. The results are also shown in figure 2.

	30 years	40 years	50 years	60 years	70 years
0%	7,02	7,37	7,33	7,52	7,26
10%	7,33	7,74	7,77	8,03	7,80
20%	7,71	8,21	8,31	8,66	8,47
30%	8,20	8,82	9,01	9,48	9,34
40%	8,85	9,62	9,94	10,57	10,49
50%	9,76	10,75	11,25	12,10	12,10

Table 6: Critical level of interest rate as a function of age and tax rate.

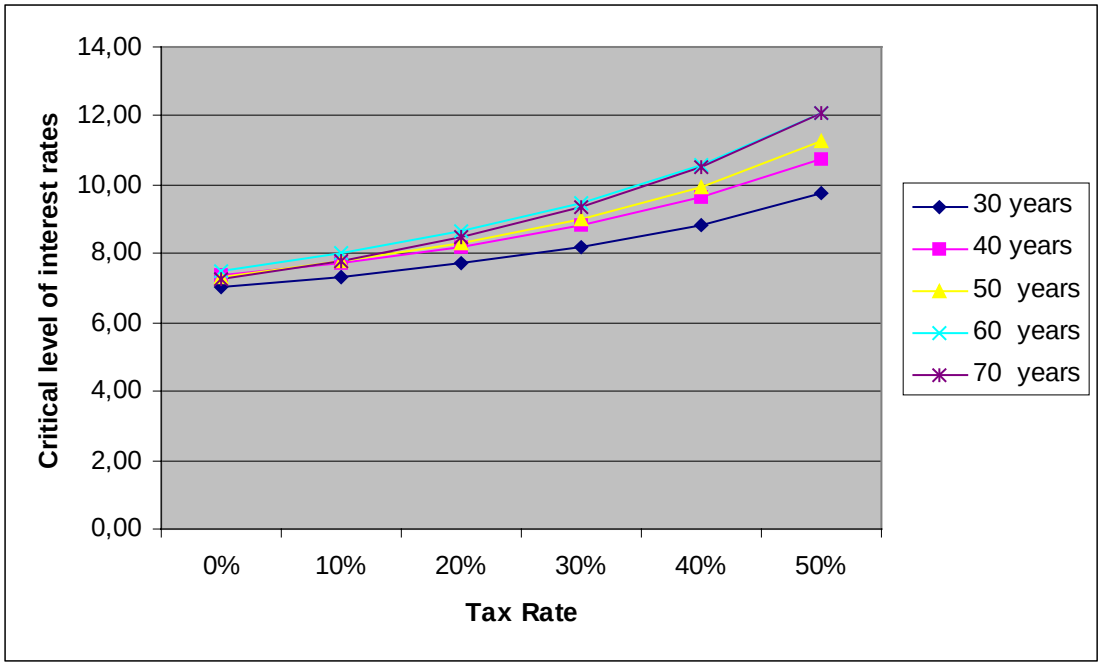

Figure 2: Critical level of interest rate as a function of age and tax rate.

We can conclude that the mortality swap turns out to be an interesting investment opportunity for many cases. It is extremely attractive for people with high tax rates, meaning that it should be taken into consideration especially by high income people.

To summarize these findings, it can be said that our empirical analysis for the German insurance market shows basically the same results as found by Charupat and Milevsky for Canada. Arbitrage opportunities are clearly increasing in the tax rate, and there is also – with the exceptions mentioned above – an enhancing influence of age.

The main reason for the existence of arbitrage opportunities is that the assumptions made by the tax authorities for calculating the taxable portions are incorrect. First, they assume an interest rate of 5.5%. Historically, the rates that were attributed to annuity contracts have always been higher. Secondly, they assume that everybody

lives exactly to his life expectancy. Hence, they systematically misestimate the taxable portion, by calculating the interest part of the annuity based on expected lifetime rather than calculating the expected interest portion. Moreover, the computations are based on values for male persons, ignoring the longer life expectancies of women. The most important flaw in the calculation of the taxable portions, however, is the assumption that the relation between payback of principle and payment of interest is constant over time, whereas it is obvious that – as in a usual payback scheme for debt – the portion of interest should be decreasing in time.[7] This leads to a costless deferment of taxation and thus to tax advantage.

Another reason for these results is that in Germany, insurance companies try to keep their surplus rates very stable. This can also be seen when looking at historical data. They use their hidden reserves to smooth the yearly variations of their investment returns. Thus, the insured person not only invests in a well-diversified portfolio consisting of shares, bonds and other investments, he also receives the same return every year. This return is – roughly speaking – the average return of this portfolio. Our calculation assumes that the surplus rates will continue to be that stable. This seems to be a reasonable assumption – though not for all life insurers. There are life insurers whose hidden reserves amount more than 20% of the book value of their assets. It is very likely that these companies will be able to continue to attribute the "promised" return to their life insurance policies.

4. SUMMARY AND OUTLOOK

In the present paper we have performed empirical analysis on the existence of arbitrage opportunities in the German insurance market. We analysed a so-called mortality swap versus a risk free investment. For the combined insurance products we considered the critical level of interest rate a coupon bond would have to provide to be similarly attractive on an after-tax basis. It could be shown, that this critical level is basically increasing in the insured's age and tax rate and thus advantages exist in particular for elderly and high income people.

The attractiveness of the mortality swap can mainly be explained by an inadequate and unsatisfactory method of taxation: Obviously a multitude of flaws entered the formula for the determination of the taxable portion of an annuity, the most serious being the simplification according to which the interest portion of an annuity remains constant over time. Quantifying the error resulting from these flaws and determining "fair" taxable portions is an interesting topic for further research.

Furthermore, analysing the combination of a temporary annuity and an endowment policy could be of particular interest since this "temporary mortality swap" has an upper bound for the maturity and is hence more comparable to a coupon bond.

REFERENCES

Schmidt, L. (1998): EStG Einkommensteuergesetz Kommentar, 17th ed., Munich.

Charupat, N., and M.A. Milevsky (1999): Mortality Swaps and Tax Arbitrage in the Annuity Market, paper presented at the 1999 Annual Meeting of the American Risk and Insurance Association.

[7] Of course, the taxation could be held constant over the life of the annuity. The calculation of this constant value should, however, take into account that the relation between payback of principle and payment of interest varies over time.

APPENDIX

Annuitants age when annuity payments start	*Taxable portion in %*	*Annuitants age when annuity payments start*	*Taxable portion in %.*	*Annuitants age when annuity payments start*	*Taxable portion in %.*
0 – 3	73	44	49	68	23
4 – 5	72	45	48	69	22
6 – 8	71	46	47	70	21
9 – 11	70	47	46	71	20
12 – 13	69	48	45	72	19
14 – 15	68	49	44	73	18
16 – 17	67	50	43	74	17
18 – 19	66	51	42	75	16
20 – 21	65	52	41	76	15
22 – 23	64	53	40	77	14
24 – 25	63	54	39	78	13
26 – 27	62	55	38	79	12
28	61	56	37	80 – 81	11
29 – 30	60	57	36	82	10
31	59	58	35	83	9
32 – 33	58	59	34	84 – 85	8
34	57	60	32	86 – 87	7
35	56	61	31	88	6
36 – 37	55	62	30	89 – 91	5
38	54	63	29	92 – 93	4
39	53	64	28	94 – 96	3
40	52	65	27	≥ 97	2
41 – 42	51	66	26		
43	50	67	25		

Table 7: Taxable percentage of an annuity according to Par. 22 *Einkommensteuergesetz* (income tax law).