ANALYSIS OF PARTICIPATING LIFE INSURANCE CONTRACTS : A UNIFICATION APPROACH

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ABSTRACT

Fair pricing of embedded options in life insurance contracts is usually conducted by using risk-neutral valuation. This pricing framework assumes a perfect hedging strategy, which insurance companies can hardly pursue in practice. In this paper, we extend the risk-neutral valuation concept with a risk measurement approach. We accomplish this by first calibrating contract parameters that lead to the same market value using risk-neutral valuation. We then measure the resulting risk assuming that insurers do not follow perfect hedging strategies. As the relevant risk measure, we use lower partial moments, comparing shortfall probability, expected shortfall, and downside variance. We show that even when contracts have the same market value, the insurance company's risk can vary widely, a finding that allows us to identify key risk drivers for participating life insurance contracts.

Keywords: participating life insurance, fair valuation, lower partial moments

1 INTRODUCTION

In recent years, interest rate guarantees and other embedded options in life insurance contracts have become a subject of increasing concern for the academic world as well as for practitioners. There are financial and actuarial approaches to handling embedded options. The financial approach is concerned with risk-neutral valuation and fair pricing, and is based on the assumption of a perfect hedging strategy, which insurance companies normally do not or cannot follow. Assuming that an insurer does not invest in a replicating portfolio to meet liabilities, the company remains at risk. The actuarial approach, on the other hand, focuses on shortfall risk under an objective real-world measure, which plays an important role in insurance risk management and practice (e.g., rating agencies, Solvency II). The aim of this paper is to investigate problems that arise under both approaches in order to identify key risk drivers for participating life insurance business.

The field of fair valuation of embedded options in life insurance contracts has been researched by authors such as Garven and Pottier (1995), Briys and de Varenne (1997), Grosen and Jørgensen (2000, 2002), Hansen and Miltersen (2002), Bacinello (2003), and Haberman et al. (2003).

In Briys and de Varenne (1997), the authors use a model with a point-to-point guarantee, that is, the company guarantees only a maturity payment and an optional participation in the terminal surplus at expiration of the contract. The contract's market value in this model is basically a function of the guaranteed interest rate and the terminal surplus participation while only the guaranteed interest rate influences shortfall risk at maturity. Interest rates are thereby modeled stochastically.

Grosen and Jørgensen (2002) expand the model used by Briys and de Varenne (1997) by incorporating a regulatory restriction for the insurer's assets. If the market value of assets drops below a certain threshold at any point in time, it is shared between the policyholders and paid out. They find out that contract values are significantly reduced by the value of this default put option. Though the authors incorporate the company's risk by calculating the value of the default put option, their aim is not to determine the likelihood or extent of a shortfall within the model. Calculating the insolvency put option using risk-neutral valuation is one way to address the life insurer's risk and adequate for evaluating it, but cannot provide or replace a quantification of the likelihood of a future shortfall or the magnitude of such an event, which offers significant additional information value, especially for risk management purposes, ratings, policyholders, investors, and other stakeholders.

The life insurance contract suggested by Grosen and Jørgensen (2000) features some annual surplus participation. In this type of contract, the greater of the guaranteed interest rate or a fraction of the asset return is annually credited to the policy and in turn becomes part of the guarantee, which is why this type is called a cliquet-style guarantee. The insurance contract's market value, as well as the insurance company's risk, depends on the guaranteed interest rate as well as on the amount of ongoing surplus. The authors compute contract values as well as shortfall probabilities for different parameter combinations using Monte Carlo simulation techniques. However, they use the risk-neutral pricing measure $\mathbb{Q}$ for the calculation of shortfall probabilities. By transformation of the real-world measure $\mathbb{P}$ to the pricing measure $\mathbb{Q}$, the probabilities of events with positive probability typically are changed (cf., e.g., Chapter 6.1 in Bingham and Kiesel, 2004). This change of measure highly reduces the suitability of $\mathbb{Q}$-probabilities for interpretation, especially as for risk management or ratings purposes, shortfall probabilities under $\mathbb{P}$ are needed.

Hansen and Miltersen (2002) introduce a model of participating life insurance contracts with practical relevance in Denmark. Besides the interest rate guarantee and a similar smoothing surplus distribution mechanism as in the model suggested by Grosen and Jørgensen (2000), some terminal bonus is provided. Also, the policyholder pays an annual fee to the insurance company.

Common UK cliquet-style contracts with a smoothing mechanism are studied in Haberman et al. (2003). In these contracts, the liabilities annually earn the greater of some guaranteed interest rate and a predetermined fraction of the arithmetic average of the last period returns of some reference portfolio.

The primary focus of the literature mentioned so far is on the concept of risk-neutral valuation-and thus pricing the risk of life insurance contracts-but it rarely hardly addresses risk measurement.

Since, e.g., an insurer's asset allocation is subject to regulation in many countries, insurers usually cannot apply optimal hedging strategies. Therefore, a real-world analysis of the resulting risks is important and appropriate. Kling et al. (2006) fill this gap by using an actuarial approach to analyze the interaction of contract parameters, regulatory parameters, and management decisions comparing shortfall probabilities. They present a general framework that can be used for the analysis of cliquet-style guarantees and adapt it to the German market. Boyle and Hardy (1997) compare an actuarial simulation-based approach with a financial option pricing approach for the pricing and reserving of maturity guarantees.

Barbarin and Devolder (2005) propose a model that combines the financial and actuarial approaches. They consider a contract, similar to Briys and de Varenne (1997), with a point-to-point guarantee and terminal surplus participation. To integrate both approaches, they use a two-step method of pricing life insurance contracts: First, they determine a guaranteed interest rate such that certain solvency requirements are satisfied, using value at risk and expected shortfall risk measures. Second, to obtain fair contracts, they use risk-neutral valuation and adjust the participation rate accordingly. This can be done if and only if the surplus participation rate has no impact on risk. Therefore, this procedure is limited to the analysis of contracts featuring parameters without effect on shortfall risk and is not feasible for, e.g., cliquet-style contracts with annual surplus participation only.

The purpose of this paper is to analyze the interaction between the financial and actuarial approaches without merging the two concepts. In particular, we examine the effect of the fair valuation process on the insurer's risk situation. Moreover, the procedure is not restricted to point-to-point guarantees, but also allows an analysis of more complex cliquet-style contracts.

First, we apply the financial concept and calibrate contract parameters that lead to the same market value under the risk-neutral measure. In a second step, we measure the risk associated with fair contracts using lower partial moments, assuming that the insurance company invests in a reference portfolio without following perfect hedging strategies. We compare shortfall probability, expected shortfall, and downside variance, and identify the impact of individual model parameters in fair contracts on the insurer's shortfall risk. This allows the discovery of fair contracts that, at the same time, meet solvency or risk requirements, and the procedure is independent of the individual model structure. Moreover, using lower partial moments as a risk measure provides a reasonable interpretation since they are consistent with maximization of expected utility. To investigate whether the outcomes depend on the type of guarantee or on the surplus distribution mechanisms, we examine a point-to-point model and two cliquet-style models, namely, one general model and one country-specific cliquet guarantee that is representative of contracts commonly offered in Denmark. We also analyzed a UK cliquet-style model following Haberman et al. (2003). The results we found were quite similar to the other models considered and are therefore omitted.

The rest of the paper is organized as follows. The basic model structure is outlined in Section 2.

This model contains all the properties and characteristics common to all the models considered. Then, for each of three models, we specify the individual dynamics and characteristics of the assets and liabilities, find parameter combinations of fair contracts, define a shortfall, and study the risk corresponding to fair contracts. The models thus examined are the point-to-point model, analyzed in Section 3, a cliquet-style model (Section 4), and the Danish cliquet-style model (Section 5). We conclude our analyses in Section 6.

2 MODEL FRAMEWORK

This section presents the framework for the general setting common to all models that will be discussed in this paper. Differences among the various models arise from the development of liabilities due to different types of guarantees and different surplus distribution mechanisms among countries. Individual dynamics will be described in the sections that analyze the respective models.

Model Overview

Assets	Liabilities
$A(t)$	$P(t)$
	$C(t)$
	$B(t)$

Table 1: Insurance company's balance sheet at time t.

Table 1 is a snapshot of the insurance company's balance sheet. It also can be interpreted as the insurance company's financial situation at time t, including the market value of the company's asset base A, the book value of the policy reserves P for one contract or a pool of similar contracts, a company's account C, and the bonus reserve account B that includes reserves for terminal bonus participation, the company's equity, and asset valuation reserves. In what follows, individual account dynamics are described in detail.

Dynamics

The insurance company invests in a reference portfolio A. We assume that the total market value of the portfolio follows a geometric Brownian motion.[1] Under the objective (real-world) measure

[1] Let (W_t), $0 \leq t \leq T$, be a standard Brownian motion on a probability space $(\Omega, \mathscr{F}, \mathbb{P})$ and $(\mathscr{F}_t)$, $0 \leq t \leq T$, be the filtration generated by the Brownian motion.

$\mathbb{P}$, we have:

$$dA(t) = \mu(t)A(t)dt + \sigma(t)A(t)dW^{\mathbb{P}}(t), \tag{1}$$

with asset drift $\mu(t)$, volatility $\sigma(t)$, and a $\mathbb{P}$-Brownian motion $W^{\mathbb{P}}$. For all our analyses we assume that $\mu(t) = \mu$, $\sigma(t) = \sigma$ are constant over time and we assume a complete, perfect, and frictionless market. Thus, the solution of the stochastic differential equation[2] (1) is given by:

$$A(t) = A(t-1)e^{\left(\mu-\sigma^2/2+\sigma\left[W^{\mathbb{P}}(t)-W^{\mathbb{P}}(t-1)\right]\right)}.$$

By changing the measure to the (risk-neutral) unique equivalent martingale measure $\mathbb{Q}$, the drift changes to the risk-free interest rate r, and development of the assets is fully described by:

$$dA(t) = rA(t)dt + \sigma A(t)dW^{\mathbb{Q}}(t),$$

where $W^{\mathbb{Q}}$ is a $\mathbb{Q}$-Brownian motion. The solution of this stochastic differential equation under $\mathbb{Q}$ is thus given by:

$$A(t) = A(t-1)e^{\left(r-\sigma^2/2+\sigma\left[W^{\mathbb{Q}}(t)-W^{\mathbb{Q}}(t-1)\right]\right)}.$$

P denotes the policyholder's account, the book value of the so-called policy reserve. To initiate the contract, the policyholder pays an exogenously given up-front premium $P(0) = P_0$ at time $t = 0$. In general, for any $t = 1, 2, \ldots$, the policy reserves P earn some rate of interest $r_P(t)$ every year that depends on the type of guarantee (point-to-point vs. cliquet-style) and the type of surplus distribution provided. It therefore may include a guaranteed interest rate and some surplus that usually depends on the insurance company's financial situation at that time, in particular on the development of the company's assets. Thus, the development of the policy can in general be described by:[3]

$$P(t) = P(t-1)(1 + r_P(t)) = P_0 \prod_{i=1}^{t} (1 + r_P(i)), t = 1, 2, \ldots.$$

Some insurers maintain an account C, $C(0) = 0$, in which fees are collected over time. Since $A(t)$ is the market value of the reference portfolio, but $P(t)$ and $C(t)$ are book values, we introduce the account B on the liability side of the balance sheet to capture the difference as reserves; this is determined by $B(t) = A(t) - P(t) - C(t)$. This account includes reserves for terminal bonuses as well as asset valuation reserves and equity. The bonus reserve serves as a buffer: In times of low investment returns, money is transferred from the bonus account to the policy reserve in order to

[2] For details see, e.g., Bjørk (2004).

[3] In the following analysis, both discrete and continuously compounded rates are used so as to be consistent with Briys and de Varenne (1997) and Hansen and Miltersen (2002).

cover the guaranteed payment; in good times, the reserve is raised. At inception of the contract, the company may have positive initial bonus reserves B_0 that correspond to an initial contribution of the equity holders.

Customer payoff L_T

Common to all models, at maturity, the customer receives the accumulated book value $P(T)$ of the contract, and, for some models, a terminal bonus $S(T)$, e.g., $S(T) = \delta B(T)^+ = \delta \max(B(T), 0)$. Hence, the payoff L_T to the customer is:

$$L_T = P(T) + S(T) = P_0 \prod_{t=1}^{T} (1 + r_P(t)) + S(T),$$

and the equity holders receive the remainder, $E_T = A(T) - L_T$. We discuss the exact form of the terminal bonus $S(T)$ and the policy interest rate $r_P(t)$ in respective sections below.

Fair contracts

To define an equilibrium condition to determine fair contracts we use the risk-neutral valuation formula:

$$V_0(L_T) = e^{-rT} \mathbb{E}^{\mathbb{Q}}(L_T),$$

where $\mathbb{E}^{\mathbb{Q}}(\cdot)$ denotes the expectation under the equivalent martingale measure[4] $\mathbb{Q}$. Thus, we say that a contract is fair if the present value (time zero market value) $V_0(\cdot)$ of the final payoff L_T under the risk-neutral measure $\mathbb{Q}$ is equal to the up-front premium P_0 paid by the policyholder:

$$P_0 = V_0(L_T) = e^{-rT} \mathbb{E}^{\mathbb{Q}}(L_T). \tag{2}$$

We use Equation (2) to find parameter combinations of fair contracts. Models that do not allow for explicit analytical expressions are analyzed using Monte Carlo simulation.[5] As numerical search routines we implemented a modified Newton algorithm and the bisection method.

Shortfall and risk measurement

As is done in all analyzed models, in pricing insurance contracts we assume that the company is always able to make the guaranteed payments. Hence, there is no default risk for the policyholder. We therefore implicitly assume that there exists some external party that will provide the additional payments needed in case the insurance company's assets are not sufficient to cover the final

[4] For details concerning risk-neutral valuation, see, e.g., Bjørk (2004).

[5] We implemented antithetic variables as a variance reduction technique, cf. Glasserman (2004).

guaranteed payment (e.g., many countries run a protection fund; other insurance companies may offer support). However, shortfall risk should still be of interest to the company and its equity holders, particularly in light of the fact that policyholders tend to be especially risk averse. Risk-neutral valuation of liabilities including a default put option will deliver appropriate prices of risk. However, these values are obtained under a risk-neutral distribution and thus do not reflect information about the real-world shortfall risk, such as the objective probability or extent of a future shortfall. From a risk management point of view, as well as for investors, policyholders, ratings, and other stakeholders, however, this analysis should provide substantial additional information.

We consider it a shortfall if the value of the assets at maturity does not cover the guaranteed book value of the policy, i.e., $A(T) < P(T)$. Thus, we only consider European-style contracts and interpret risk solely as a possible shortfall at maturity, and allow for negative reserves during the term of the contract.[6]

Using this definition, we measure risk under the objective measure $\mathbb{P}$ with lower partial moments[7] (LPM):

$$\begin{aligned} LPM_n &= \mathbb{E}^{\mathbb{P}}\left((P(T) - A(T))^n \mathbf{1}\{A(T) < P(T)\}\right) \\ &= \mathbb{E}^{\mathbb{P}}\left(-(C(T) + B(T))^n \mathbf{1}\{C(T) + B(T) < 0\}\right). \end{aligned}$$

For decision making, the degree of risk aversion can be controlled by varying the power n. LPM with $n = 0, 1, 2$ provides very reasonable interpretations and is consistent with maximization of expected utility for investment decisions and stochastic dominance relationships. For $n = 0$, only the number of shortfall occurrences is counted, for $n = 1$, all deviations are weighted equally, and for $n = 2$, large deviations are weighted more heavily than small deviations:

- shortfall probability

$$LPM_0 = \mathbb{P}\left(C(T) + B(T) < 0\right),$$

- expected shortfall

$$LPM_1 = \mathbb{E}^{\mathbb{P}}\left(\left(-(C(T) + B(T))\right) \mathbf{1}\{C(T) + B(T) < 0\}\right),$$

6 We focus only on financial risk and ignore mortality risk as is done in the underlying models. Hence, there are no surrenders or deaths until maturity and the pool of contracts remains unchanged. Hansen and Miltersen (2002), e.g., show in their appendix that the inclusion of mortality risk does not alter their results in pricing the contracts.

7 Lower partial moments belong to the class of downside-risk measures that describe the lower part of a density function; hence only negative deviations are taken into account. See, e.g., Fishburn (1977), Sortino and van der Meer (1991).

- and downside variance

$$LPM_2 = \mathbb{E}^{\mathbb{P}}\left((C(T)+B(T))^2\,\mathbf{1}\{C(T)+B(T)<0\}\right).$$

We see that shortfall does not depend on an (optional) participation in the terminal surplus $S(T)$. Shortfall occurs only if the value of the reference portfolio at maturity $A(T)$ is less than the accumulated book value of the guaranteed policy reserve $P(T)$.

In the following sections, we first characterize and specify individual development of the liabilities in each model and then calibrate the model parameters so that the contracts are fair. Thereafter, the risk of fair contracts is evaluated under the objective measure $\mathbb{P}$ with lower partial moments of degree $n = 0,1,2$.

3 A POINT-TO-POINT-MODEL

The first model analyzed is a point-to-point model (PTP) based on a version of a model suggested by Briys and de Varenne (1997).[8] Throughout this section, we keep the parameters $T = 10$, $r = 4\%$, and $A_0 = 100$ constant.

Dynamics of the liabilities and customer payoff

At inception of the contract, the policyholder makes an up-front payment $P_0 = \kappa A_0$. Remember that in our basic model, the initial investment made by the equity holders $B_0 = (1-\kappa)A_0$ is credited to the bonus reserve. Grosen and Jørgensen (2002) call κ the 'wealth distribution coefficient'. The PTP model does not incorporate a company's account C, i.e., $C(t) = 0$.

During the term of the contract, the premium P_0 is compounded with the guaranteed interest rate g, such that at expiration of the contract the policy reserve accrues to the guaranteed payment:

$$P(T) = P_0 e^{gT}.$$

Additionally, the customer receives some terminal surplus if $\kappa A(T) - P(T) > 0$. This is given by a fraction δ of the difference above, so that the final payoff L_T can be summarized:

$$L_T = P(T) + S(T) = P_0 e^{gT} + \delta\left[\kappa A(T) - P(T)\right]^+.$$

Thus, it can be decomposed into two parts: the first term is a bond with a fixed payoff, whereas

[8] Briys and de Varenne (1997) also include a model of stochastic interest rates. For the purpose of this paper, the risk-free rate is assumed to be constant so that the model can be compared to the other models that are analyzed.

the second term can be written as:

$$\delta\left[\kappa A(T)-P(T)\right]^{+}=\delta\kappa\left[A(T)-\frac{P(T)}{\kappa}\right]^{+},$$

which is the payoff of a European call option on $A(T)$ with Strike $P(T)/\kappa$.

Fair contracts

The closed-form solution for the market value $V_0(L_T)$ of the payoff using European option pricing theory[9] is:

$$\begin{aligned} V_0(L_T) &= e^{-rT}\mathbb{E}^{\mathbb{Q}}(L_T) \\ &= e^{-rT}P(T)+e^{-rT}\mathbb{E}^{\mathbb{Q}}\left(\delta\kappa\left[A(T)-\frac{P(T)}{\kappa}\right]^{+}\right) \\ &= P_0e^{(g-r)T}+\delta\kappa\left(A_0\Phi(d_1)-\frac{P(T)}{\kappa}e^{-rT}\Phi(d_2)\right) \end{aligned}$$

with

$$\begin{aligned} d_1 &= \frac{\ln((\kappa A_0)/P(T))+(r+\sigma^2/2)T}{\sigma\sqrt{T}} \text{ and} \\ d_2 &= d_1-\sigma\sqrt{T}. \end{aligned}$$

For all our numerical analysis, $\kappa = 80\%$. Table 2 contains parameter combinations (g,δ) satisfying the equilibrium condition (2), i.e., $P_0 = V_0(L_T)$. To keep the contract fair, δ decreases in g. This inverse relation between guaranteed interest rate and terminal participation rate is consistent with the results shown in, e.g., Garven and Pottier (1995) and Grosen and Jørgensen (2002). For $\sigma = 10\%$, for instance, $g = 0.5\%$ induces $\delta = 94.3\%$, whereas $g = 3.5\%$ is combined with $\delta = 32.8\%$. Both parameter combinations lead to contracts with the same market value.

	g								
	0	0.5%	1%	1.5%	2%	2.5%	3%	3.5%	4%
$\sigma = 10\%$	96.3%	94.3%	91.3%	86.7%	80.0%	69.9%	55.0%	32.8%	0%
$\sigma = 15\%$	88.6%	84.9%	80.1%	73.9%	65.7%	55.2%	41.4%	23.4%	0%

Table 2: PTP Model. Values of δ for fair contracts with $A_0 = 100$, $\kappa = 80\%$.

Table 2 also shows the effect of asset volatility on parameter combinations of fair contracts:

[9] See, e.g., Bjørk (2004).

With increasing σ, the terminal participation rate δ decreases. For $g = 2\%$ fixed, $\sigma = 10\%$ implies $\delta = 80\%$ and $\sigma = 15\%$ leads to $\delta = 65.7\%$. This pattern can be explained by the impact of volatility on development of the asset base. An increase in σ raises the chances of higher investment returns and, therefore, of a higher terminal bonus. In fair contracts, the participation coefficient thus needs to be decreased.

Having thus calibrated parameters to fair contracts, we now compute the corresponding risk under the objective measure $\mathbb{P}$, assuming that the insurer does not use hedging strategies.

Shortfall

A shortfall occurs if $A(T) < P(T)$ or, equivalently, if $B(T) < 0$, since the model does not contain an account C to cover the deficit. Closed-form solutions for risk measures under the objective measure $\mathbb{P}$ are for:

- the shortfall probability

$$LPM_0 = \mathbb{P}(A(T) < P(T)) = \mathbb{P}\left(A(T) < P_0 e^{gT}\right) = \Phi(d) \tag{3}$$

- the expected shortfall

$$\begin{aligned} LPM_1 &= \mathbb{E}^{\mathbb{P}}\left(\left(P_0 e^{gT} - A(T)\right) \mathbf{1}\{A(T) < P_0 e^{gT}\}\right) \\ &= P_0 e^{gT} \Phi(d) - e^{\mu T} A_0 \Phi(d - \sigma\sqrt{T}) \end{aligned}$$

- the downside variance

$$\begin{aligned} LPM_2 &= \mathbb{E}^{\mathbb{P}}\left(\left(P_0 e^{gT} - A(T)\right)^2 \mathbf{1}\{A(T) < P_0 e^{gT}\}\right) \\ &= \left(P_0 e^{gT}\right)^2 \Phi(d) - 2 P_0 e^{gT} A_0 e^{\mu T} \Phi\left(d - \sigma\sqrt{T}\right) \\ &\quad + A_0^2 e^{2\mu T + \sigma^2 T} \Phi\left(d - 2\sigma\sqrt{T}\right) \end{aligned}$$

  where

$$d = \frac{\ln\left(P_0 e^{gT}/A_0\right) - (\mu - \sigma^2/2)T}{\sigma\sqrt{T}}.$$

Φ denotes the cumulative distribution function of a standard normal distributed random variable.

Isoquants

We can now calculate parameter combinations of δ and g that lead to fair contracts, i.e., contracts with the same market value $V_0(L_T(g,\delta)) = P_0$ under $\mathbb{Q}$, as well as parameter combinations of δ

and g that lead to the same given shortfall probability under $\mathbb{P}$ (a so-called iso-shortfall probability curve), e.g., $\mathbb{P}(B(T) < 0) = 3\%$. The trade off between g and δ for fair contracts is shown in Table 2. Since the shortfall probability does not depend on δ, g needs to be adjusted. For LPM_0 in Equation (3), we can solve $\mathbb{P}(A(T) < P(T)) = \alpha$ for g and get:

$$g = \left(\Phi^{-1}(\alpha)\sigma\sqrt{T} - \ln(P_0/A_0) + (\mu - \sigma^2/2)T\right)/T.$$

For $\alpha = 3\%$ we obtain $g = 1.78\%$; for $\alpha = 5\%$ we obtain $g = 2.53\%$.

Figure 1 illustrates the discrepancy between parameter combinations (g, δ) for contracts with the same market value and those with the same shortfall probability. The graphs show that fair contracts with $g > 1.78\%$ have a shortfall probability greater than 3%, whereas contracts with a lower guaranteed interest rate imply a lower shortfall probability.

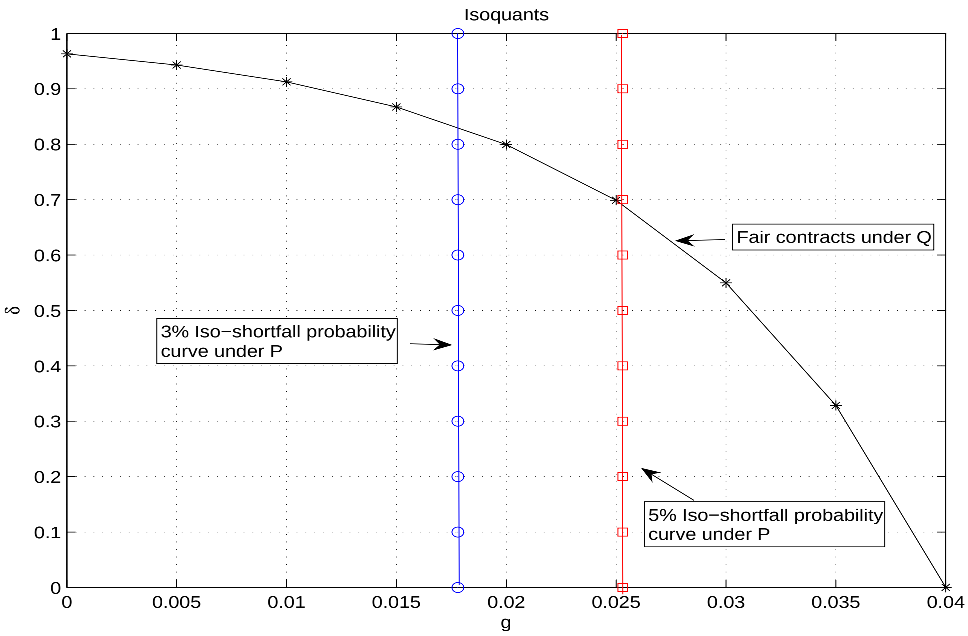

Figure 1: PTP Model. Isoquants for $\sigma = 10\%$.

Since the lower partial moments are independent of δ, the risk of fair contracts can be reduced by promising a higher terminal participation rate δ combined with a lower guaranteed interest rate g without changing the market value of the contract. In the next section, we confirm this presumption.

Risk of fair contracts

We now calculate the risk that corresponds to the parameter combinations (g, δ) from Table 2. Figure 2 depicts the risk of these fair contracts measured with lower partial moments of degree

$n = 0, 1, 2$ and plotted in terms of g.

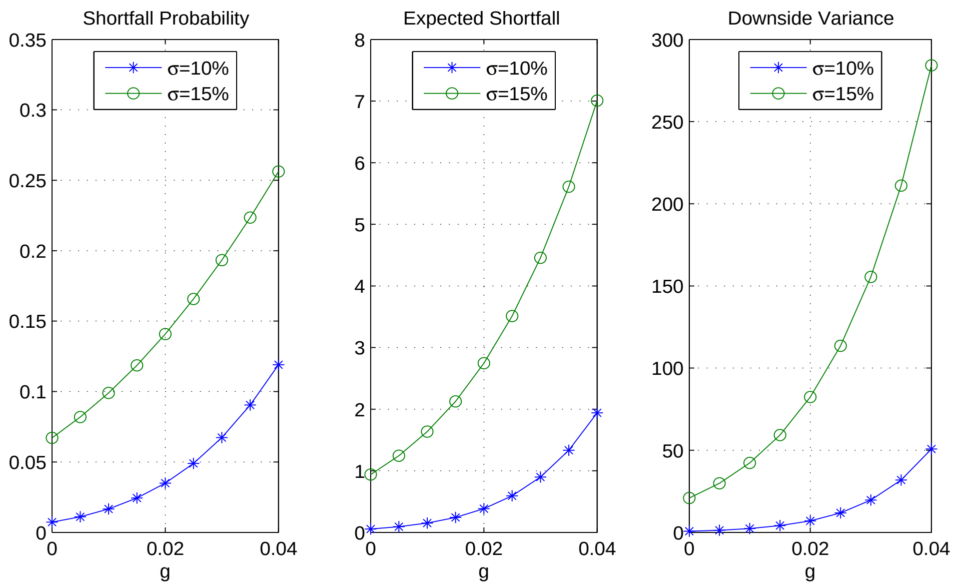

Figure 2: PTP Model. Risk of fair contracts in Table 2 as a function of g.

From left to right, the graphs in Figure 2 illustrate the shortfall probability (LPM_0), the expected shortfall (LPM_1), and the downside variance (LPM_2). It is important to understand that every point on the curves depicts the risk of a fair contract as defined in Equation (2). As a result, for different levels of g, a different terminal bonus δ is provided. For example, where $\sigma = 10\%$, Table 2 shows that $g = 2\%$ implies $\delta = 80\%$. The corresponding shortfall probability depicted in Figure 2 is 3.80%.

The Figure 2 graphs show how the risk of fair contracts varies across the contracts as all three risk measures increase in the guaranteed interest rate g. This occurs because risk does not depend on δ. Nevertheless, every point on the curves in Figure 2 represents a contract with the same market value under the risk-neutral measure $\mathbb{Q}$. By increasing δ, we can lower g and thus significantly decrease the risk of the fair contract. Thus, terminal bonus participation is a key feature for the reduction of risk in an insurance contract. Without regulatory or legal constraints, insurance companies could reduce the risk of new and existing business while still offering fair contracts to insureds by offering lower guarantees combined with higher participation at maturity.

The effect of volatility on risk also can be seen from comparing the curves in Figure 2. As expected, an increase in σ leads to an increase in risk. More precisely, in our example, for $\sigma = 10\%$ the shortfall probability increases from 1% to 12%, whereas for $\sigma = 15\%$, the increase is from 7% to 26%.

The observed tendencies are independent of the choice of risk measure (LPM with $n = 0, 1, 2$). In our analysis, we focused only on g and δ, keeping all other parameters fixed. We did not show results for different wealth distribution parameters κ. The higher κ, the less initial equity $B_0 = (1-\kappa)A_0$ is available and hence the higher is the risk of shortfall. However, changes in κ do not influence the shape of the results shown. We study the effect of the initial bonus reserve B_0 in more detail in Section 4.

To summarize, for contracts with a point-to-point guarantee, surplus distribution does not influence shortfall risk. The key risk driver for such contracts is the guaranteed interest rate. This is an intuitive and unsurprising result. However, if ongoing surplus distribution is included and guarantees are given on a cliquet-style basis, surplus distribution does influence shortfall risk. Interdependencies between surplus distribution and guarantees make the results more complicated for these types of contracts. The following two sections compare different cliquet-style models and identify key risk drivers for the different surplus distribution mechanisms.

4 A CLIQUET-STYLE MODEL

We first analyze a model with cliquet-style guarantee (CS), as suggested by Grosen and Jørgensen (2000).

Dynamics of the liabilities and customer payoff

The life insurance contract guarantees an annual minimum interest rate g and features a smoothing surplus distribution mechanism: annual surplus can be credited to the account P only if the buffer ratio $B(t)/P(t) = (A(t)-P(t))/P(t)$ exceeds a limit, the so-called target buffer ratio γ. This is a management decision in that the company's target is to build up reserves if the reserve quota is below target level and, alternatively, reserves are partly dissolved and distributed to policyholders if the reserve quota is above the target level. Money is transferred to the bonus account B in years of large investment returns or, in less prosperous times, withdrawn from it to cover the guaranteed interest rate g. Instead of maintaining an account C, the company keeps the bonus reserve at maturity as a type of fee paid by the policyholder.

Development of the policy reserve in any year depends on the buffer ratio at the beginning of the year and can be described recursively as follows:[10]

$$P(t) = P(t-1)\,(1+r_P(t)) = P(t-1)\left(1+\max\left\{\tilde{g}, \alpha\left(\frac{B(t-1)}{P(t-1)}-\gamma\right)\right\}\right),$$

where $\alpha \geq 0$ is the annual participation coefficient and $\gamma \geq 0$ is the target buffer ratio. The annual participation in the bonus represents an option element in the contract. Note that the case where

[10] We follow Grosen and Jørgensen (2000) by using discrete compounding and therefore denote the guaranteed interest rate by $\tilde{g}$.

no ongoing surplus is distributed, i.e., $\alpha = 0$ and therefore $r_p(t) = \tilde{g}\ \forall t \in \{1,...,T\}$, results in a point-to-point guarantee, where the policyholder receives exactly the guaranteed floor:

$$P(T) = P_0(1+\tilde{g})^T \text{ for } \alpha = 0.$$

Since the policyholder does not participate in the terminal bonus, the payoff L_T simply consists of the accumulated book value of the policy at expiration of the contract:

$$L_T = P(T) + 0 \cdot S(T) = P_0 \prod_{t=1}^{T} \left(1 + \max\left\{\tilde{g}, \alpha\left(\frac{B(t-1)}{P(t-1)} - \gamma\right)\right\}\right).$$

Due to path dependency, an evaluation of expectations involving $P(T)$ leads to complex integral representations that cannot be transformed into analytical expressions. Thus, Monte Carlo simulation is used for all analyses in this section.

Fair contracts

In our analysis we focus on the interaction between the guaranteed interest rate $\tilde{g}$ and the annual participation coefficient α. We assume $T = 10$, $r = 4\%$, $P_0 = 100$, and $\gamma = 10\%$. We study cases without initial reserves ($B_0 = 0$) and cases with initial reserves of $B_0 = 10$ for different choices of the asset volatility σ. Parameter combinations $(\tilde{g}, \alpha)$ of fair contracts satisfying the equilibrium condition (2) are found in Table 3.

		$\tilde{g}$								
		0%	0.5%	1%	1.5%	2%	2.5%	3%	3.5%	4%
$B_0 = 0$	$\sigma = 10\%$	203%	183%	160%	134%	107%	80%	56%	35%	13%
	$\sigma = 15\%$	90%	78%	66%	55%	45%	35%	27%	18%	7%
$B_0 = 10$	$\sigma = 10\%$	72%	65%	58%	51%	43%	36%	29%	21%	10%
	$\sigma = 15\%$	43%	39%	35%	31%	27%	22%	18%	13%	6%

Table 3: CS Model. Values of α for fair contracts with $\gamma = 10\%$, $T = 10$.

A general pattern observable in Table 3 is the tradeoff between α and $\tilde{g}$, which has also been observed by Grosen and Jørgensen (2000). Since, ceteris paribus, the contract value increases with increasing $\tilde{g}$ as well as with increasing α, for contracts with the same risk-neutral value, α clearly decreases with increasing $\tilde{g}$.

Note that for $B_0 = 0$, asset volatility of $\sigma = 10\%$, and rather low guaranteed interest rates $\tilde{g} \leq 2\%$, the annual surplus participation coefficient α exceeds 100%, which implies that reserves fall below the target buffer ratio and may even fall below 0 in extreme cases. Although such a distribution rule may not be very realistic, it is necessary for companies without initial reserves and asset volatility of 10% to offer low guaranteed interest rates at a fair premium.

We also find that for a fixed $\tilde{g}$, an increase in the volatility σ leads to a sharp decrease in α, due to a greater possibility of a larger annual excess bonus. Thus, the annual bonus option element becomes more valuable and, consequently, the participation coefficient α needs to be lowered to keep the contract fair. Certainly, the initial reserve situation significantly influences the size of α. For zero initial reserves, the company must build up reserves before providing annual surplus participation and therefore fair contracts require a higher participation rate α.

Since the guaranteed rate $\tilde{g}$ is compounded discrete, even for $\tilde{g} = 4\%$ some surplus distribution is necessary to fulfil the equilibrium condition (2).

Shortfall

As in the PTP model, a shortfall occurs if:

$$A(T) < P(T) = P_0 \prod_{t=1}^{T} \left(1 + \max \left\{ \tilde{g}, \alpha \left(\frac{B(t-1)}{P(t-1)} - \gamma \right) \right\} \right).$$

Since the customer payoff $P(T)$ depends on the guaranteed interest rate $\tilde{g}$, the target buffer ratio γ, and the annual participation coefficient α, the considered lower partial moments are functions of these parameters.

Isoquants

Figure 3 contains parameter combinations $(\tilde{g}, \alpha)$ of contracts leading to the same market value under $\mathbb{Q}$ (see Table 3), as well as for contracts resulting in an identical shortfall probability (LPM_0) of 3% and 15% under $\mathbb{P}$.

The two parameters, $\tilde{g}$ and α, have a similar effect on risk since the shortfall probability is an increasing function of $\tilde{g}$ and α. Thus, in order to obtain a constant shortfall probability, the surplus participation coefficient α decreases with increasing $\tilde{g}$. If the guaranteed interest rate $\tilde{g}$ is greater than 0.54%, no $\alpha > 0$ can be found that leads to a shortfall probability of 3%. Only very conservative combinations of $\tilde{g}$ and α lead to such a shortfall probability, e.g., $\tilde{g} = 0.5\%$ and $\alpha = 4.1\%$. It can be seen that the 15% iso-shortfall probability curve is close to the curve representing fair contracts. For low guaranteed interest rates, the iso-shortfall probability curve is above, for low values of $\tilde{g}$ it is below, the curve of fair contracts. Any point above the 15% shortfall probability curve represents a parameter combination $(\tilde{g}, \alpha)$ with a higher shortfall risk; any point below represents a parameter combination with lower shortfall risk. This implies that

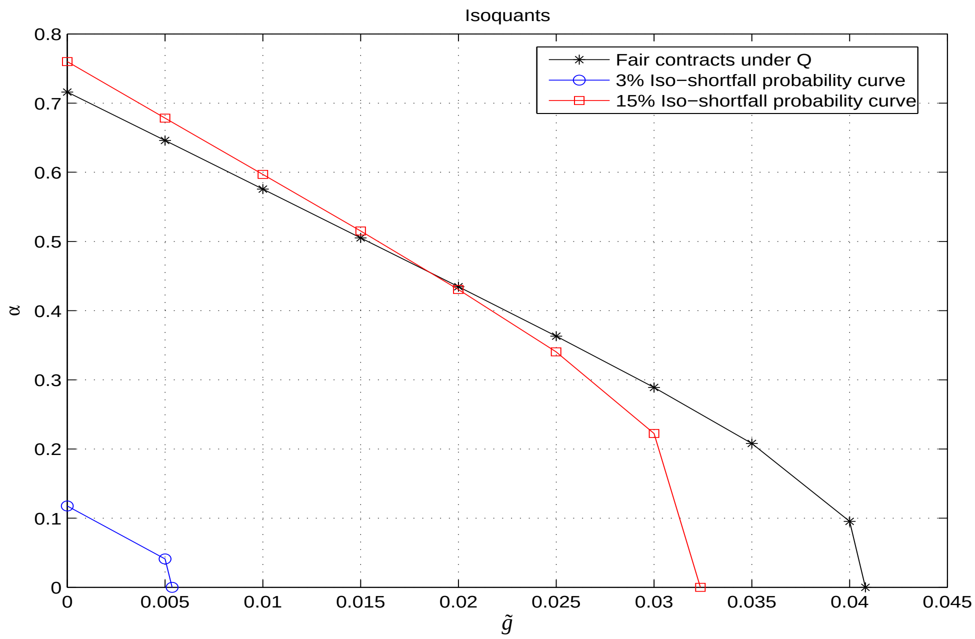

Figure 3: CS Model. Isoquants for $\sigma = 10\%$, $A_0 = 100$, $B_0 = 10$, $\gamma = 10\%$, $T = 10$.

in the above example any parameter combination of fair contracts with $\tilde{g} > 2\%$ leads to a shortfall probability greater than 15%, whereas any parameter combination of fair contracts with $\tilde{g} < 2\%$ leads to a shortfall probability lower than 15%. The guaranteed interest rate $\tilde{g}$ appears to be the key risk driver for the shortfall probability in this example.

Figure 3 shows that a same market price under $\mathbb{Q}$ does not imply the same shortfall probability under $\mathbb{P}$. Thus, we need to analyze the risk of fair contracts in more detail, which we do in the next section.

Risk of fair contracts

Next, we study the risk associated with the fair contracts in Table 3, starting with contracts where $B_0 = 10$. The lower partial moments in terms of $\tilde{g}$ are displayed in Figure 4 for $\sigma = 10\%$ and $\sigma = 15\%$.

Recall that every point on the curves represents a fair contract with the same value under the risk-neutral measure $\mathbb{Q}$. Every $\tilde{g} \in \{0, \ldots, 0.04\}$ is associated with a unique value of α that can be found in Table 3. As in the PTP model, the figures confirm that the same market value does not imply equal risk. In Figure 4, all three risk measures increase in $\tilde{g}$ despite the decrease in α. Hence, for fair contracts with $B_0 = 10$, $\tilde{g}$ has a dominating effect on risk compared to α. This implies that companies with adequate initial reserves could significantly reduce the risk of new and existing contracts, if doing so is legally permitted, by reducing the guaranteed interest rate

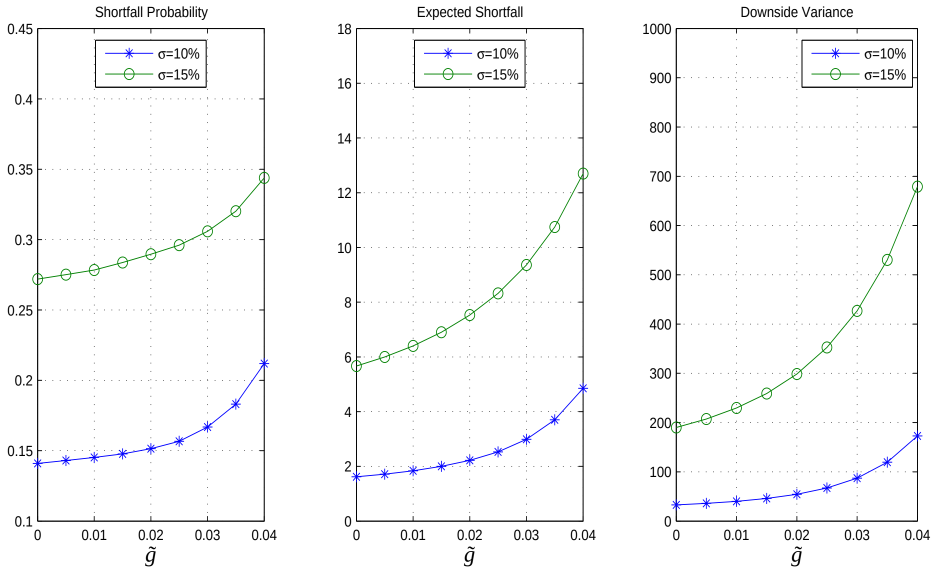

Figure 4: CS Model. Risk of fair contracts in Table 3 as a function of $\tilde{g}$ for $B_0 = 10$ with $\gamma = 10\%$.

and at the same time increasing ongoing surplus distribution such that the market value of the contract is unchanged.

Figure 4 also illustrates that variance of the reference portfolio has a great influence on the risk. An increment of 5% in σ doubles the probability of shortfall for fair contracts. The effect on expected shortfall and downside variance is even stronger.

In contrast to the uniform observations for $B_0 = 10$, *LPM* curves for $B_0 = 0$ in Figure 5 are more complex. The effect of the interaction between $\tilde{g}$ and α on the risk of fair contracts depends on the asset volatility and on the risk measure, i.e., on the weight n assigned to the extent of the shortfall. Moreover, as expected, the risk level is generally higher than for $B_0 = 10$.

For an asset volatility $\sigma = 10\%$, the lower partial moments in Figure 5 decrease for low values of the guaranteed interest rate $\tilde{g}$ and increase for values of $\tilde{g}$ close to the risk-free rate $r = 4\%$. This means that for each lower partial moment of degree 0, 1, and 2, there is an inflection point with least risk. With increasing degree n, the inflection point gets smaller, implying that, depending on the risk measure, there exists a guaranteed interest rate $\tilde{g}$ with least risk for fair contracts. In our example, $\tilde{g} = 3\%$ seems to be close to a risk-minimizing choice for the shortfall probability, and $\tilde{g} = 2\%$ results in risk-minimization for expected shortfall and downside variance.

If one considers the expected shortfall as the relevant risk measure, the parameter combinations $\tilde{g} = 0.5\%, \alpha = 182.7\%$ and $\tilde{g} = 3.3\%, \alpha = 42.8\%$ lead to the same expected shortfall under $\mathbb{P}$ and to the same market value under $\mathbb{Q}$. However, by choosing a guaranteed interest rate in between these values, e.g., $\tilde{g} = 2\%$ and $\alpha = 107.1\%$, expected shortfall can be reduced without changing the market value of the contract. One could even go one step further: by choosing any guaranteed interest rate $\tilde{g}$ between 0.5% and 3.3%, the company can find a surplus distribution coefficient α that leads to a higher market value under $\mathbb{Q}$ and a lower expected shortfall under $\mathbb{P}$ at the same time. For example, make $\tilde{g} = 2\%$ and $\alpha = 120\%$.

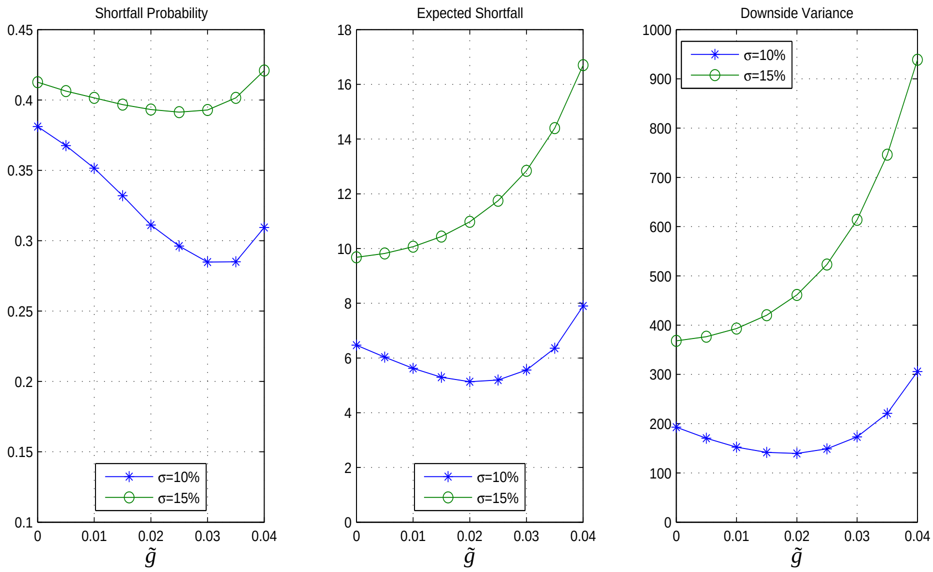

Figure 5: CS Model. Risk of fair contracts in Table 3 as a function of $\tilde{g}$ for $B_0 = 0$ with $\gamma = 10\%$.

For a volatility of $\sigma = 15\%$, only the shortfall probability shows the same effect in the considered interval while expected shortfall and downside variance are increasing in $\tilde{g}$.

In our analysis of risk created by cliquet guarantees, we focused on the relation between the guaranteed interest rate $\tilde{g}$ and the participation parameter α. For a contract with positive initial reserves $B_0 = 10$ we found $\tilde{g}$ to be the dominating parameter responsible for risk. In this case, the insurer could keep the market value the same and reduce risk by increasing the annual bonus participation and lowering the guaranteed interest rate if doing so would not violate any legal requirements.

In contrast, an insurance contract with zero initial reserves $B_0 = 0$ leads to a more complex picture as the *LPM* curve characteristics show different dynamics. The impact of α and $\tilde{g}$ on risk varies depending on asset volatility and *LPM* degree n. As a result, the influence of the annual participation rate α should not be underestimated since it can have a significant impact on the risk of fair contracts and even dominate the effect of the guaranteed interest rate $\tilde{g}$. This concern makes it worthwhile to analyze models of practical relevance in different countries and identify key risk drivers for other cliquet-style models.

5 A DANISH CLIQUET-STYLE MODEL

We now turn to a model of participating life insurance contracts that has practical relevance in Denmark. In this section we study a model suggested by Hansen and Miltersen (2002) that is a hybrid of the CS model discussed above, and the model of Miltersen and Persson (2003).

Dynamics of the liabilities and customer payoff

Hansen and Miltersen's (2002) model has a smoothing surplus distribution mechanism similar to that of the CS model. In addition, a positive terminal bonus reserve is transferred to the policyholder and adds to the maturity payment, whereas a negative bonus reserve must be covered by the insurance company. The insurance company issues a series of options on annual returns, which are covered by the bonus reserve. The policyholder pays an annual percentage fee ξ, which is collected in the company's account C.

The percentage fee ξ is directly subtracted from the policy interest rate and transferred to C. To be consistent with Hansen and Miltersen (2002), the policy reserve is compounded continuously:

$$P(t) = P(t-1)e^{\left\{\max\left[g,\ln\left(1+\alpha\left(\frac{B(t-1)}{(P(t-1)+C(t-1))}-\gamma\right)\right)\right]-\xi\right\}}.$$

Note that the fee ξ represents a fundamental control mechanism for the maturity payment because it directly reduces the policy interest rate.

To calculate the buffer ratio $B(t)/(P(t)+C(t))$, $P+C$ is modeled as follows:

$$(P(t)+C(t)) = (P(t-1)+C(t-1))e^{\left\{\max\left[g,\ln\left(1+\alpha\left(\frac{B(t-1)}{(P(t-1)+C(t-1))}-\gamma\right)\right)\right]\right\}}.$$

The difference between P and $P+C$ is the annual payment fee transferred to company's account C, i.e.:

$$C(t) = (P(t)+C(t)) - P(t),$$

and the bonus account B is residually determined as:

$$B(t) = B(t-1) + A(t) - A(t-1) - (P(t)+C(t)) + (P(t-1)+C(t-1)).$$

Summarizing, the customer payoff adds up to:

$$\begin{aligned} L_T &= P(T) + S(T) = P(T) + B(T)^+ \\ &= P_0 \prod_{i=1}^{T} e^{\left\{\max\left[g,\ln\left(1+\alpha\left(\frac{B(i-1)}{(P(i-1)+C(i-1))}-\gamma\right)\right)\right]\right\}} e^{-T\xi} + B(T)^+. \end{aligned} \quad (4)$$

Fair contracts

As in the related CS model, there are no analytical expressions for expectations and thus Monte Carlo simulation is used for evaluations. Following Hansen and Miltersen, we assume that the contract is not backed by initial reserves, i.e., $B_0 = 0$. Instead, we focus on the guaranteed interest rate and the newly introduced payment fee ξ. In our analysis, $r = 4\%$, $T = 10$, $\gamma = 10\%$, $P_0 = 100$, and $B_0 = 0$.

Parameter combinations (g, ξ) of fair contracts satisfying Equation (2) are shown in Table 4 for $\sigma = 10\%$ and $\sigma = 15\%$ and several values of α. Our results are consistent with the parameter combinations (g, α, ξ) found by Hansen and Miltersen.[11]

		g				
	α	0%	1%	2%	3%	4%
	20%	0.18%	0.32%	0.54%	0.87%	1.32%
$\sigma = 10\%$	50%	0.23%	0.37%	0.59%	0.90%	1.33%
	90%	0.31%	0.46%	0.68%	0.99%	1.41%
	20%	0.64%	0.86%	1.16%	1.54%	2.00%
$\sigma = 15\%$	50%	0.77%	1.00%	1.28%	1.64%	2.08%
	90%	0.96%	1.19%	1.48%	1.84%	2.27%

Table 4: Danish Model. Values of ξ for fair contracts.

With increasing guaranteed interest rate g and fixed α, there is a greater possibility of a higher maturity payment and, therefore, the fee ξ must be raised to keep the contract fair. As in the other models, an increase of the asset volatility σ to 15% makes the bonus option element more valuable and therefore requires an increase in ξ for fixed α to counterbalance this effect.

Shortfall

As described in our basic model, a shortfall occurs if:

$$A(T) < P(T) = P_0 \prod_{i=1}^{T} e^{\left\{\max\left[g, \ln\left(1+\alpha\left(\frac{B(i-1)}{(P(i-1)+C(i-1))}-\gamma\right)\right)\right]\right\}} e^{-T\xi}.$$

Since the customer payoff $P(T)$ and the company's account $C(T)$ depend on the guaranteed interest rate g, the target buffer ratio γ, the annual participation coefficient α, and the payment fee ξ, the considered lower partial moments are again functions of these parameters. Once more, path dependency makes it impossible to derive closed-form solutions for these lower partial moments and so Monte Carlo simulation is employed to get numerical results.

[11] Hansen and Miltersen used the Newton algorithm on g for ξ given.

Isoquants

Figure 6 displays parameter combinations (g, ξ) of contracts with the same market price under $\mathbb{Q}$ (cf. Table 4) as well as combinations with a shortfall probability of 3% and 10% under $\mathbb{P}$. Obviously, g and ξ have opposite effects on risk as the shortfall probability is increasing in g and decreasing in ξ. This leads to the result that for increasing g, ξ also must increase so as to keep the shortfall probability constant.

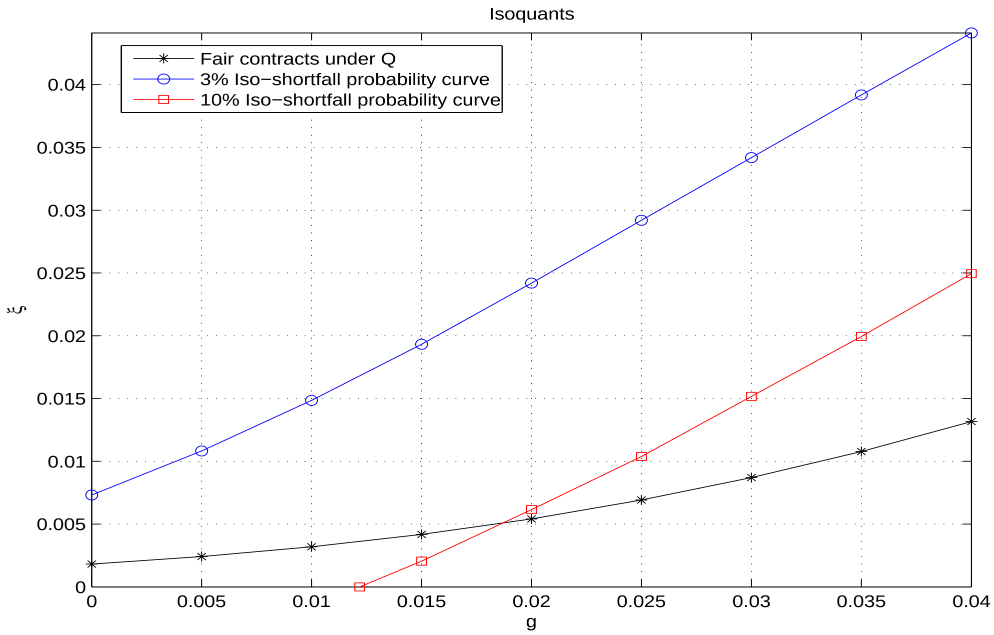

Figure 6: Danish Model. Isoquants for $\sigma = 10\%$, $\alpha = 20\%$, $\gamma = 10\%$.

Combinations of g and ξ that are below an iso-shortfall probability curve therefore represent contracts with higher shortfall probability; parameter combinations above the curve represent contracts with a lower shortfall probability. From this it can be observed that parameter combinations of fair contracts lead to a shortfall probability greater than 3%.

The 10% shortfall probability curve is below the curve representing fair contracts for low values of g ($g \leq 1.5\%$) and above it for high values of g ($g \geq 2\%$), implying that lower values of the guaranteed interest rate result in a lower shortfall probability, even though the contracts are fairly priced. If the guaranteed interest rate is reduced to 1% or below, even for no payment fee ξ, the shortfall probability falls below 10%. Overall, in the example shown, the fee ξ has less influence on the shortfall probability than does the guaranteed interest rate g. The risk of fair contracts is analyzed in more detail in the following section.

Risk of fair contracts

The risk of fair contracts in terms of g is displayed in Figure 7, based on the parameter combinations in Table 4. All three risk measures are increasing in g despite the simultaneously increasing payment fee ξ. For $\alpha = 20\%$ and $\sigma = 10\%$, the shortfall probability dramatically increases from close to 0% when $g = 0\%$ to more than 18% for $g = 4\%$. With increasing α and increasing σ, this effect is weakened, but risk continues to increase with increasing g; thus the effect of g on the risk of fair contracts dominates the effect of ξ.

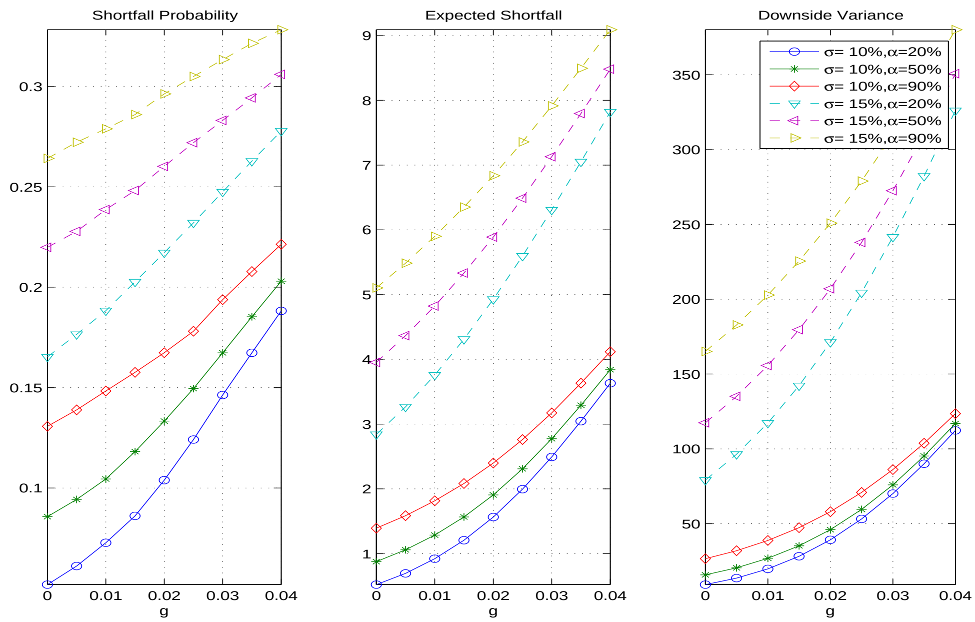

Figure 7: Danish Model. Risk of fair contracts in Table 4 as a function of g with $B_0 = 0$.

We also observe that the annual participation rate α outweighs the impact of ξ on risk. By fixing g, we observe that a larger α (combined with higher ξ) induces a higher shortfall risk. Further calculations revealed that the impact of α on risk of fair contracts outweighs the effect of g as well. This finding further emphasizes that ongoing surplus distribution may be the key risk driver for contracts with cliquet-style guarantees.

Comparing the risk of fair contracts for different asset volatilities σ, we observe that a higher volatility σ combined with a higher fee ξ leads to a higher shortfall probability. The shortfall curves for $\sigma = 15\%$ shown in Figure 7 are clearly above the shortfall curves for $\sigma = 10\%$.

In our study, we assumed $B_0 = 0$ so as to be consistent with Hansen and Miltersen (2002). The absolute level of risk decreases if initial reserves are assumed, but initial reserves appear to have no influence on the basic effects shown in this section. Furthermore, transfer of the terminal bonus to the policyholder changes the market value of the contract without changing the risk.

Thus, terminal bonuses can be used to lower the risk, a finding very similar to the PTP model (see Section 3).

6 SUMMARY

Risk-neutral valuation for insurance contracts is an important scholarly as well as practical issue. Even though this is an appropriate method to handle the valuation of insurance liabilities, the underlying assumption of a perfect hedging strategy cannot be easily implemented by insurance companies. We extend the literature and relate the financial and actuarial approaches by measuring the effects of various contract parameters on actual real-world risk to the insurer for policies with the same value under the risk-neutral measure. We do this by employing several common models containing point-to-point as well as cliquet-style guarantees.

We showed that for all models considered, the risk of fair contracts differs with variations in parameters. This result is significant for future considerations of fair valuation techniques. We further examined one additional model commonly used by life insurance companies in the United Kingdom and suggested by Haberman et al. (2003). For this model, the major risk characteristics of fair contracts are consistent with what we observed in the other models.

In our analysis, we identified key risk drivers. Terminal bonus participation plays a major role in minimizing risk, given that it has no impact on shortfall. Raising the share in the terminal bonus reduces the guaranteed interest rate and the annual surplus participation for fair contracts, thereby lowering risk. This result is common to all models analyzed. For all cliquet-style models, the shortfall probability can be greatly reduced by raising the terminal surplus participation while concurrently lowering the annual participation. The results are even more dramatic for the model with a point-to-point guarantee.

For cliquet-style models, we found that the company's initial bonus reserve has a major influence on the risk imposed by fair contracts when imperfect hedging occurs. Overall, the risk of fair contracts is much lower for positive initial reserves than for zero reserves.

Common to all models considered was that contracts with positive initial reserves demonstrate increasing shortfall probability, expected shortfall, and downside variance as the guaranteed interest rate rises. Hence, in the case of positive reserves, the risk imposed by a fair life insurance contract is mainly driven by the interest rate guarantee, and not by the annual participation coefficient. The contract can remain fair with reduced shortfall risk through a reduction in the guaranteed interest rate.

For contracts with very low positive initial reserves, the findings are much more complex. In this case, the results strongly depend on the underlying model, and vary with risk measures and asset volatility. In particular, the annual surplus participation can dominate the effect of the guaranteed interest rate. Therefore, an insurer may find it preferable to offer higher guarantees with lower annual surplus participation.

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