The Impact of Surplus Distribution on the Risk Exposure of With Profit Life Insurance Policies Including Interest Rate Guarantees

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Abstract

This paper analyzes the numerical impact of different surplus distribution mechanisms on the risk exposure of a life insurance company selling with profit life insurance policies with a cliquet-style interest rate guarantee. Three representative companies are considered, each using a different type of surplus distribution: A mechanism, where the guaranteed interest rate also applies to surplus that has been credited in the past, a slightly less restrictive type in which a guaranteed rate of interest of 0% applies to past surplus, and a third mechanism that allows for the company to use former surplus in order to compensate for underperformance in “bad” years. Although at outset all contracts offer the same guaranteed benefit at maturity, a distribution mechanism of the third type yields preferable results with respect to the considered risk measure. In particular, throughout the analysis, our representative company 3 faces ceteris paribus a significantly lower shortfall risk than the other two companies. Offering “strong” guarantees puts companies at a significant competitive disadvantage relative to insurers providing only the third type of surplus distribution mechanism.

1. Introduction

Many with profit life insurance policies contain an interest rate guarantee. Often, this guarantee is given on a point-to-point basis, i.e. the guarantee is only relevant at maturity of the contract. Other products (which are predominant, e.g., in the German market), however, offer a so-called cliquet-style (or year-by-year) guarantee. This means that the policy holders have an account to which every year a certain rate of return has to be credited. Typically, life insurance companies try to provide the guaranteed rate of interest plus some surplus on the policy holders' accounts.

There are different mechanisms defining how the annual surplus can be distributed to the insured. These mechanisms vary from country to country and sometimes from insurance company to insurance company. They can be divided in three different categories and combinations thereof:

a) Surplus may be credited to the policy reserves. In this case, it is guaranteed that this surplus will earn the guaranteed rate of interest in future years.

b) Surplus may be credited to a surplus account that is owned by the policy holder and may therefore not be reduced anymore. Thus, there is a guaranteed interest rate of 0% on money that is in this surplus account.

c) Surplus may be credited to a terminal bonus account. Money that has been credited to this account will only be distributed to the insured at maturity of their contracts but not (or only partially) if they cancel the contract. Furthermore, money may be taken from this account in order to pay interest rate guarantees (on the policy reserves) if in some year the return on assets is not sufficient to pay for these guarantees.

It is obvious that c.p., insurance companies using different surplus distribution mechanisms may have a significantly different risk profile. In the past, this may have been of minor importance since there was a comfortable margin between market interest rates and the guaranteed rates that were typically offered within life insurance policies. Recently, however, these margins have been significantly reduced, in particular for contracts that have been sold years ago with rather high guaranteed rates. This development illustrates that analyzing and

managing an insurance company's financial risks should not only be restricted to management of the assets but also be concerned with reducing risks that result from the product design.

A number of papers have recently addressed interest rate guarantees, in particular Briys and de Varenne (1997), Grosen and Jorgensen (2000), Jensen et al. (2001), Hansen and Miltersen (2002), Grosen and Jorgensen (2002), Bacinello (2003), Miltersen and Persson (2003), Tanskanen and Lukkarinen (2003), Bauer et al. (2006), and Kling, Richter and Russ (2006).

Briys and de Varenne (1997) compute closed-form solutions for market values of liabilities and equities in a point-to-point guarantee framework. In their model the policy holder receives a guaranteed interest and is also credited some bonus, determined as a certain fraction of net financial gains (when positive). The paper also looks at the impact of interest rate guarantees on the company's risk exposure by analyzing interest rate elasticity and duration of insurance liabilities.

Contrasting the just-mentioned approach, Grosen and Jorgensen (2000) consider cliquet-style guarantees and introduce a model that takes into account an insurer's use of the average interest principle. In addition to a policy reserve (the customer's account) they introduce a "bonus reserve", a buffer that can be used to smoothen future bonus distributions. They analyze a mechanism that credits bonus to the customer's reserve based upon the current ratio of bonus reserve over policy reserve. A bonus is paid only if this ratio exceeds a given threshold. Thus, the actual distribution of surplus indirectly reflects current investment results but primarily focuses on the company's ability to level out insufficient results in the future. The authors decompose the contract into a risk free bond, a bonus and a surrender option. They compute contract values by means of Monte Carlo simulation, and also calculate contract default probabilities for different parameter combinations.[1] However, they calculate default probabilities under the risk neutral probability measure Q. Therefore, the numerical results are only of limited explanatory value.

Grosen and Jorgensen (2002) discuss a model based upon the framework used by Briys and de Varenne (1997). They incorporate a regulatory constraint for the insurer's assets according to which the company is closed down and liquidated if the market value of assets drops below a

[1] Jensen et al. (2001) extend the findings of Grosen and Jorgensen (2000). As one extension, among others, they introduce mortality risk. Another paper that incorporates mortality risk as well as the surrender option is Bacinello (2003).

threshold at any point in time during the life of the policy. Their results suggest that the introduction of the regulatory constraint significantly reduces the value of the shareholders' default put option and thereby an insurer's incentive to change its assets' risk characteristics to the policy holders' disadvantage.

Miltersen and Persson (2003) also use a cliquet-style framework and allow for a portion of excess interest to be credited not directly to the customer's account but to a bonus account. In their model, the interest that exceeds the guaranteed rate is – if positive – divided into three portions that are credited to the insured's account, the insurer's account, and to a bonus account. In case of investment returns below the guaranteed rate, funds are moved from the bonus account into the policy owner's account. Thus, the bonus account is available for smoothing returns over time. Unlike in the Grosen and Jorgensen (2000) model, however, the buffer consists of funds that have already been designated to the particular customer: Any positive balance on the bonus account is credited to the policy owner when the contract expires. This is used to model so-called "terminal bonuses". In this setting, Miltersen and Persson (2003) derive numerical results on the influence of various parameters on the contract value.[2]

Bauer et al. (2006) investigate the valuation of participating contracts under the German regulatory framework. They present a framework, in which the different kinds of guarantees or options incorporated in participating contracts with interest rate guarantees can be analyzed separately. The practical implementation of two different numerical approaches to price the embedded options is discussed. The authors find that life insurers currently offer interest rate guarantees below their risk-neutral value. Furthermore, the financial strength of an insurance company considerably affects the value of a contract.

While the primary focus in the literature is on the fair (i.e. risk-neutral) valuation of the life insurance contract, Kling, Richter and Russ (2006) concentrate on the risk a contract imposes on the insurer, measured by means of shortfall probabilities under the so-called "real-world probability measure P". Assuming cliquet-style guarantees, they study the impact interest rate guarantees have on the insurer's shortfall probability and how default risks depend on charac-

[2] Contrasting the mechanism discussed in Miltersen and Persson (2003), life insurance contracts often employ a distribution policy that does not accumulate undistributed surplus on an individual basis, but for a greater pool of customers. A model that allows for this technique can be found in Hansen and Miltersen (2002).

teristics of the contract, on the insurer's reserve situation and asset allocation, on management decisions, as well as on regulatory parameters.

The present paper analyzes the numerical impact of different surplus distribution mechanisms on the risk exposure of a life insurance company selling with profit life insurance policies with an interest rate guarantee. We employ the model framework introduced in Kling, Richter and Russ (2006), but extend the model such that the different surplus mechanisms described above and any combinations can be compared. The model also allows for the comparison of the two major types of interest rate guarantees: cliquet-style guarantees and point-to-point guarantees, as described above. The focus of our numerical analysis, however, is on the different surplus distribution mechanisms.

The paper is organized as follows. In Section 2, we introduce our model for the insurance company, in particular a simple asset model and a representation of the insurer's liabilities in a steady state. Furthermore, we give a detailed description of the surplus distribution mechanisms described above and introduce shortfall probabilities as the relevant risk measure for all our analyses. Section 3 contains a variety of results analyzing the different risk levels of insurers using different surplus mechanisms as well as the impact of several parameters on these risk levels. Section 4 concludes and provides some outlook on possible future research.

2. The model framework

2.1 The insurer's financial situation

In our model, we use a simplified illustration of the insurer's balance sheet. We expand the model from Kling, Richter and Russ (2006) so that different surplus distribution mechanisms can be included:

Assets	Liabilities	
	E_t	
	L_t	policy holders' accounts
A_t	S_t	policy holders' accounts
	B_t	policy holders' accounts
	R_t	reserves

A_t	A_t

Figure 1: Model of the insurer's financial situation

Here, A_t, denotes the market value of the insurer's assets at time t. The liability side comprises five entries:

- E_t is the time t book value of the company's equity which is assumed to be constant over time.
- L_t is the time t book value of the policy reserves. The insurer guarantees the policy holder an annual interest rate g on this account. Note that any surplus that is credited to this account will also have to earn at least the guaranteed rate in the future.
- S_t is the time t book value of the policy holders' individual surplus accounts. Consistent with German legislation, we assume that the guaranteed rate need not be credited on this account but once surplus is distributed to this account it may not be reduced at any time in the future, i.e., the guaranteed rate of interest on this account is 0% p.a.
- B_t is the time t book value of the bonus account for terminal bonuses. It is owned by the policy holders but not on an individual basis. Parts of this account are paid out to maturing contracts as a terminal bonus. It may however also be used to provide guarantees for other accounts in the future.
- R_t is the reserve account which is given by $R_t = A_t - L_t - S_t - B_t - E_t$. It consists mainly of asset valuation reserves.

Even though there is an international trend towards fair value accounting, book value accounting will still be important in some countries for several reasons. In Germany, e.g., certain minimum surplus distribution rules imposed by the legislator/regulator will continue to be based on book value earnings according to the German Commercial Code. Thus, in order to realistically model these minimum requirements, book values of the assets and liabilities will be relevant even after the introduction of international accounting standards.

Our model allows for dividend payments. Whenever dividends D_t are paid out to equity holders, A_t is reduced by the corresponding amount. To simplify notation, we assume that such payments occur annually, at times $t = 1,2,\ldots,T$, where T denotes some finite time horizon.

2.2 The asset model

Similar to the approach in Kling, Richter and Russ (2006), we use a very simple model for the assets: We assume a complete, frictionless and continuous market. Between dividend payments, we let A_t follow a geometric Brownian motion

$$\frac{dA_t}{A_t} = \mu dt + \sigma dW_t , \tag{1}$$

where W_t denotes a Wiener process on some probability space $(\Omega,\mathcal{F},P)$ with a filtration F, to which W is adapted. Both, μ and σ are deterministic and constant over time.[3]

Including dividend payments D_t, we get for $t = 1,2,\ldots,T$

$$A_t^- = A_{t-1}^+ \cdot e^{\int_{t-1}^{t} \mu - \frac{\sigma^2}{2} du + \int_{t-1}^{t} \sigma dW_u} = A_{t-1}^+ \cdot e^{\mu - \frac{\sigma^2}{2} + \sigma \int_{t-1}^{t} dW_u} \qquad \text{and} \qquad A_t^+ = A_t^- - D_t ,$$

where A_t^- and A_t^+ denote the asset value at time t just before and immediately after the dividend payment. Analogously, L_t^- and L_t^+, B_t^- and B_t^+, S_t^- and S_t^+, and R_t^- and R_t^+ denote the corresponding values immediately before and immediately after the dividend payment. The numerical analysis in Section 3 assumes A_t to consist of stocks and bonds with s denoting the stock portion of the (continuously rebalancing) portfolio.

2.3 Insurance Benefits and Guarantees on the Insurance Liabilities

For the sake of simplicity, our model considers an insurance company in a "steady state": We assume that contracts corresponding to some constant fraction ξ of the insurer's liabilities terminate each year due to maturity, surrender or death. The company pays out the corresponding

[3] The model also allows for time dependent choices of μ and σ.

values of the policy holders' accounts, i.e. $\xi \cdot \left(L^{+}_{t-1} + S^{+}_{t-1} + B^{+}_{t-1}\right)$.[4] We assume that the sum of premiums P_{t-1} collected at the beginning of year t (resulting from new business as well as regular (annual) premium from "old" contracts) also equals $P_{t-1} = \xi \cdot \left(L^{+}_{t-1} + S^{+}_{t-1} + B^{+}_{t-1}\right)$.[5]

Since the premiums collected are added to the policy reserves, the value of the policy holders' accounts before guarantee provision and surplus distribution are given by

$$L^{-}_{t} = (1-\xi)L^{+}_{t-1} + P_{t-1},\ S^{-}_{t} = (1-\xi)S^{+}_{t-1} \text{ and } B^{-}_{t} = (1-\xi)B^{+}_{t-1}.$$

The values L_t^+, S_t^+ and B_t^+ then depend on amount and type of surplus distribution. By definition of the different accounts (see Section 2.1), we get the following lower bounds:

$$L^{+}_{t} \geq L^{-}_{t} \cdot (1+g),\ S^{+}_{t} \geq S^{-}_{t} \text{ and } B^{+}_{t} \geq 0.$$

The amount of distribution to the different policy holders' accounts and to equity holders each year depends on the earnings on book value as well as decisions made by the company's management. Following German legislation, we assume that there is a "*minimum participation rate*" requiring that at least a certain portion δ of the earnings on book value has to be credited to the policy holders' accounts.

Earnings on book value are subject to accounting rules giving insurance companies certain freedom to create and dissolve asset valuation reserves. Following the approach introduced in Kling, Richter and Russ (2006), we assume that at least a portion y of the increase in market value has to be identified as earnings in book values in the balance sheet.[6] The parameter y therefore represents the degree of "*restriction in asset valuation*" immanent in the relevant

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[4] We assume that there are neither gains nor losses due to mortality and thus ignore death benefits that might exceed the value of the policy holder's account. This means that the cost of insurance, i.e. the part of the premium that is charged for the death benefit, is calculated with best estimate mortality rates and exactly covers any death benefits that exceed the policy holders' accounts.

[5] Since we ignore death benefits that exceed the policy holders' accounts, of course P_t does not include the cost of insurance.

[6] This means that the sum of the increase in book value $\left[\left(A^{+}_{t} - R^{+}_{t}\right) - \left(A^{+}_{t-1} - R^{+}_{t-1}\right)\right]$ and the dividend payments D_t has to exceed $y \cdot \left(A^{-}_{t} - A^{+}_{t-1}\right)$.

accounting rules. Furthermore, the insurer can reduce reserves in order to increase the book value of assets without any restrictions by selling assets whose market value exceeds the book value.

2.4 Surplus distribution and dividend payments

This section deals with the **amount** of surplus that is credited to the policy holders in any given year, whilst the next section introduces different surplus distribution **mechanisms**. Surplus that is distributed to the policy holders' accounts and dividends that are paid to the shareholders are determined by the insurance company's management every year. Our general model allows for any management decision rule at time t that is F_t-measurable. In the numerical analysis, however, we will focus on one decision rule that seems to prevail in Germany: In the past, insurance companies used to keep surplus distribution to policy holders and dividends to shareholders rather constant over years, building and dissolving reserves in order to smoothen returns. Only when the reserves reached a rather low level, they started reducing surplus. Therefore, we apply a decision rule that considers this: As long as reserves are in a "comfortable" range, some constant target policy return rate is credited to policy holders' accounts. If crediting this target rate would lead to an "uncomfortably low" reserve level, surplus is reduced. If reducing surplus is not sufficient, first reserves are further dissolved and then the bonus account is reduced. On the other hand, if crediting the target policy return rate would yield to a very high level of reserves, surplus is increased above the target policy return rate. The technical details of this simple idea are explained in the remainder of this section:

As long as the reserve quota $x_t = \dfrac{R_t}{L_t^+ + S_t^+ + B_t^+ + E_t}$ stays within a given range $[a;b]$, a target policy return rate $z > g$ is credited to the sum of the policy holders' accounts. Furthermore, equity holders receive a target dividend rate α of company's equity. Thus, we get

$L_t^+ + S_t^+ + B_t^+ = (1+z)(L_t^- + S_t^- + B_t^-)$, and $A_t^+ = A_t^- - \alpha E_{t-1}$ i.e. the surplus Su_t^{PH} provided to policy holders and the dividend payments are given by

$$Su_t^{PH} = z(L_t^- + S_t^- + B_t^-) - gL_t^- \quad \text{and} \quad D_t = \alpha E_{t-1}. \tag{2}$$

Note that at this point we do not specify, how the surplus is distributed to the accounts L, S and B. This depends on the particular distribution mechanisms, that will be introduced in the next section.

As long as the reserve quota remains in $[a;b]$, this policy is followed. If however crediting z and α to policy holders and shareholders, respectively, would lead to a reserve quota below a or above b, surplus and dividends from (2) are reduced or increased by multiplying both with a constant factor $c \geq 0$ that leads to a reserve quota $x_t = a$ or $x_t = b$.

If no such factor $c \geq 0$ exists, this means that even paying no surplus and no dividends would lead to a reserve quota below a. In this case, only the guaranteed rate of interest is provided to the policy reserves while the surplus and the bonus account remain unchanged and no dividends are paid, i.e.,

$$L_t^+ = (1+g)L_t^-, S_t^+ = S_t^-, \; B_t^+ = B_t^- \text{ and } A_t^+ = A_t^-.$$

This is only possible if it results in a reserve quota between 0 and a. Otherwise, the bonus account is reduced by the amount needed to keep reserves at 0, i.e. we let $L_t^+ = (1+g)L_t^-$, $S_t^+ = S_t^-$, $A_t^+ = A_t^-$, and $B_t^+ = A_t^+ - L_t^+ - S_t^+ - E_t$ and thus $R_t = 0$. Of course, this is only possible if $A_t^+ \geq (1+g)L_t^- + S_t^+ + E_t$, since otherwise, this would result in $B_t^+ < 0$.

If the bonus account is not sufficient to provide the guaranteed rate of interest, i.e. if $A_t^- < (1+g)L_t^- + S_t^- + E_{t-1}$ then $L_t^+ = (1+g)L_t^-$, $S_t^+ = S_t^-$, $A_t^+ = A_t^-$, and $B_t^+ = 0$ which leads to negative reserves. Our model allows for negative reserves as long as there is enough equity to back the liabilities. We speak of a shortfall if there is not enough equity, see below.

Finally, we have to check in each of the cases above, whether these rules comply with the restriction in asset valuation and the minimum participation rate. The restriction in asset valuation (see footnote 6), is violated if $X_{BV} := y\left(A_t^- - A_{t-1}^+\right) - \left(\left(A_t^+ - R_t^+\right) - \left(A_{t-1}^+ - R_{t-1}^+\right) + D_t\right) > 0$. In this case, we distribute the exceeding book value X_{BV} increasing the surplus provided to the policy holders by $\delta \cdot X_{BV}$ and the dividends by $(1-\delta) \cdot X_{BV}$.

If $\delta\left[\left(A_t^+ - R_t^+\right) - \left(A_{t-1}^+ - R_{t-1}^+\right) + D_t\right] - \left(\left(L_t^+ - L_{t-1}^+\right) + \left(S_t^+ - S_{t-1}^+\right) + \left(B_t^+ - B_{t-1}^+\right)\right) > 0$, the surplus provided to the policy holders is increased by this amount in order to fulfill the minimum participation rule. This is achieved by reducing the dividends given to the shareholders accordingly.

2.5 Surplus distribution mechanisms

Once the amount of surplus has been determined according to the management decision rule given in Section 2.4, the surplus distribution mechanism has to be specified. We will analyze the impact of different surplus distribution mechanisms by considering three different model companies. We assume that all companies start out with the same balance sheet. In particular, we assume that the values of E_0, L_0, S_0 and B_0 are the same for each company which means that in the past the companies provided surplus in the same manner and only apply the different mechanisms described below for future surplus. The mechanisms chosen for the future are:

Company 1: All surplus as determined in Section 2.4 above is credited to the policy reserves L_t. In this case, the guaranteed rate of return also applies to past surplus. Company 1 therefore promises cliquet-style guarantees. Note that this is the type of surplus distribution that leads to the highest future liabilities and to the least amount of flexibility for the company.

Company 2: All surplus as determined in Section 2.4 above is credited to the surplus account S_t. The policy reserves L_t are increased only by the guaranteed rate of interest. This type of surplus distribution provides more flexibility for the insurer, since for the account S, the guaranteed interest is only 0%.

Company 3: Surplus as determined in Section 2.4 above is credited to the bonus account B_t. The policy reserves L_t are increased only by the guaranteed rate of interest. This type of surplus distribution provides the highest degree of flexibility for the company, since the bonus account will only be distributed to the individual policy holders at maturity of their contracts. In the meantime, money may be taken from this account to pay for interest rate guarantees if in some year the return on assets is not sufficient.

Note that although these three mechanisms lead to a different degree of flexibility and thus a different risk for the insurer, the guaranteed maturity value that is shown to the policy holder at outset, is the same in all cases.

The differences between the different surplus mechanisms are illustrated by the following example of a two year contract with a guaranteed rate of interest of 5% p.a.:

We assume that the company has neither equity nor positive reserves, the accounts S_t and B_t are 0 at $t = 0$, the value of the assets A_t and the value of the liabilities L_t are both 100. In the first

year, assets increase by 15. The insurance company credits the guarantee to the policy reserves, a surplus of 5 to the policy holders and hidden reserves are increased by 5. In the second year, assets remain unchanged. The final payoff for the policy holder and the insurance company's final solvency situation is therefore given by:

Company 1: In the first year, policy reserves are increased to 110. Thus, in the second year, a guaranteed increase of 5.5 has to be credited to the policy reserves. The final guarantee for the policy holder is 115.5. The company is unable to pay its liabilities at t = 2 since the asset value is only 115.

Company 2: In the first year, policy reserves are increased by 5 and the surplus account S_t is increased by 5. Thus, in the second year, a guaranteed increase of 5.25 has to be credited to the policy reserves. The final guarantee for the policy holder consists of the guaranteed policy reserves and the value of the surplus account and is thus given by 115.25. The value of the company's assets at t = 2 (115) is also below the value of the liabilities (115.25), however by a slightly smaller margin.

Company 3: In the first year, policy reserves are increased by 5 and the bonus account B_t is increased by 5. In the second year, a guaranteed increase of 5.25 has to be credited to the policy reserves. The final guarantee for the policy holder consists of the guaranteed policy reserves only and is thus given by 110.25. This can be provided by reducing the bonus account B_2. The rest of the bonus account is paid out as terminal bonus. Thus, the value of the assets matches the payout to the insured.

2.6 Shortfall

We considered a fixed time horizon of T years. If at any balance sheet date t=1,2,…,T, the market value of the assets is lower than the book value of the policy holders' accounts, i.e. if $A_t^+ < L_t^+ + S_t^+ + B_t^+$, this constitutes a shortfall.

We let the stopping time τ be the first balance sheet date with a shortfall or $\tau = T+1$ if no shortfall occurs. Our numerical analyses in the next section will use the shortfall probability $P(\tau \le T)$ as a risk measure. In our model, there are many parameters that have an influence on this shortfall probability, in particular parameters describing the regulatory framework (the guaranteed rate of interest g, the minimum participation rate δ, the restriction in asset valuation

y), parameters describing the insurance company's financial situation and management decisions (the initial reserve situation x:=x_0, the portion of stocks in the asset portfolio s, target dividend rate α, target policy return rate z, target range for the reserve quota $[a;b]$), capital market parameters, (drift μ and volatility σ of the asset portfolio), the considered time horizon T, the percentage ξ of the liabilities maturing every year, and the surplus distribution mechanism (model company 1, 2 or 3).

3. Analysis

In what follows, we will study the model companies introduced above in order to analyze the effect of different surplus distribution mechanisms on an insurer's shortfall probability. As mentioned in Section 2.2, we assume A_t to be a well diversified portfolio consisting of stocks and bonds with s denoting the stock portion of the (continuously rebalanced) portfolio. We assume the portfolio to follow the process (1). Furthermore, assuming an expected return of 8% and a volatility of 20% for the stock portion of the portfolio, an expected return of 5% and a volatility of 3.5% for the bond portion of the portfolio, as well as a slightly negative correlation (ρ = -0.1) between stock and bond returns,[7] the parameters of the process (1) are uniquely determined for any given stock portion s.

Since no analytical solutions for the shortfall probability exist, we use Monte Carlo simulation methods. We generated the normally distributed random sample required to project the Geometric Brownian Motion using a Box-Muller transformation, cf. e.g. Fishman (1996). The required uniformly distributed random sample was created by the random number generator URN03 described in Karian and Dudewicz (1991). For each combination of parameters, 100,000 simulations of A_t were performed. In each sample path, the development of the insurer's balance sheet over time was calculated, where the development of the accounts L, S and B was derived using the surplus mechanisms and surplus amounts described above. The Monte Carlo estimate for the shortfall probability is the relative portion of sample paths in which a shortfall occurs.

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[7] As in Kling, Richter and Russ (2006), we used data of a German stock index (DAX) and a German bond index (REXP) of the years 1988 to 2003 to get estimates for drift, volatility and correlation of stocks and bonds. Since historical bond returns seem to be too high compared to current low interest rates, we reduced the drift for the bond portion to 5%.

If not stated otherwise we keep the following parameters fixed in this section:

We assume that the restriction in asset valuation is y = 50% and use a minimum participation rate of δ = 90% as required by German regulation. At t = 0, we assume the balance sheet to consist of 2% equity, 91.5% policy reserves and 6.5% bonus account which represents a typical balance sheet of a German life insurance company.

Furthermore we assume that the insurer aims to provide a target policy return rate of z = 5% to the policy holders accounts and a target dividend rate of α = 10% as long as the reserve quota stays within a range of [a; b] = [5%; 30%].

We assume that contracts corresponding to ξ = 10% of the liabilities leave the company every year and set the time horizon of our analysis at T = 10 years.

As a starting point, we look at shortfall probabilities as a function of the initial reserve quota and compare results for two different values of the guaranteed interest rate, g = 2.75% and g = 4%.[8] Additionally, we consider two different asset allocations by assuming a stock ratio in the portfolio of s = 10% and s = 30%, respectively. The results are displayed in Figure 2. Interestingly, results indicate that for a given set of parameters companies 1 and 2 behave almost identically. In other words, the question of whether the guaranteed rate of return or just a guaranteed rate of 0% is promised on past surplus does not make a major difference under these conditions.[9] For company 3, however, outcomes differ significantly. Generally, all other things equal, company 3 faces a much lower risk of shortfall, as it has the greatest flexibility in using former surplus as “emergency funds” to provide interest guarantees in bad years.

The guaranteed rate of interest and the stock ratio have a considerable impact on the likelihood of shortfall, in particular when initial reserves are low. The different diagrams in Figure 2 show, that for low reserve quota levels, increasing the guaranteed rate from 2.75% to 4% causes an increase in the shortfall probability of about 15% and increasing the stock ratio from

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[8] The current maximum guaranteed rate for new business in the German market is 2.75%. There are still many older contracts in force that have been sold with higher rates up to 4%.

[9] This also remains true for most of the sets of results described in the following. However, the difference becomes larger if the time horizon is increased.

10% to 30% causes an increase in the shortfall probability of more than 20%. Both effects diminish for higher initial reserve quotas.

Obviously, the shortfall probability decreases as initial reserves increase. However, the marginal effect of the initial reserve quota is greater for a higher interest guarantee.

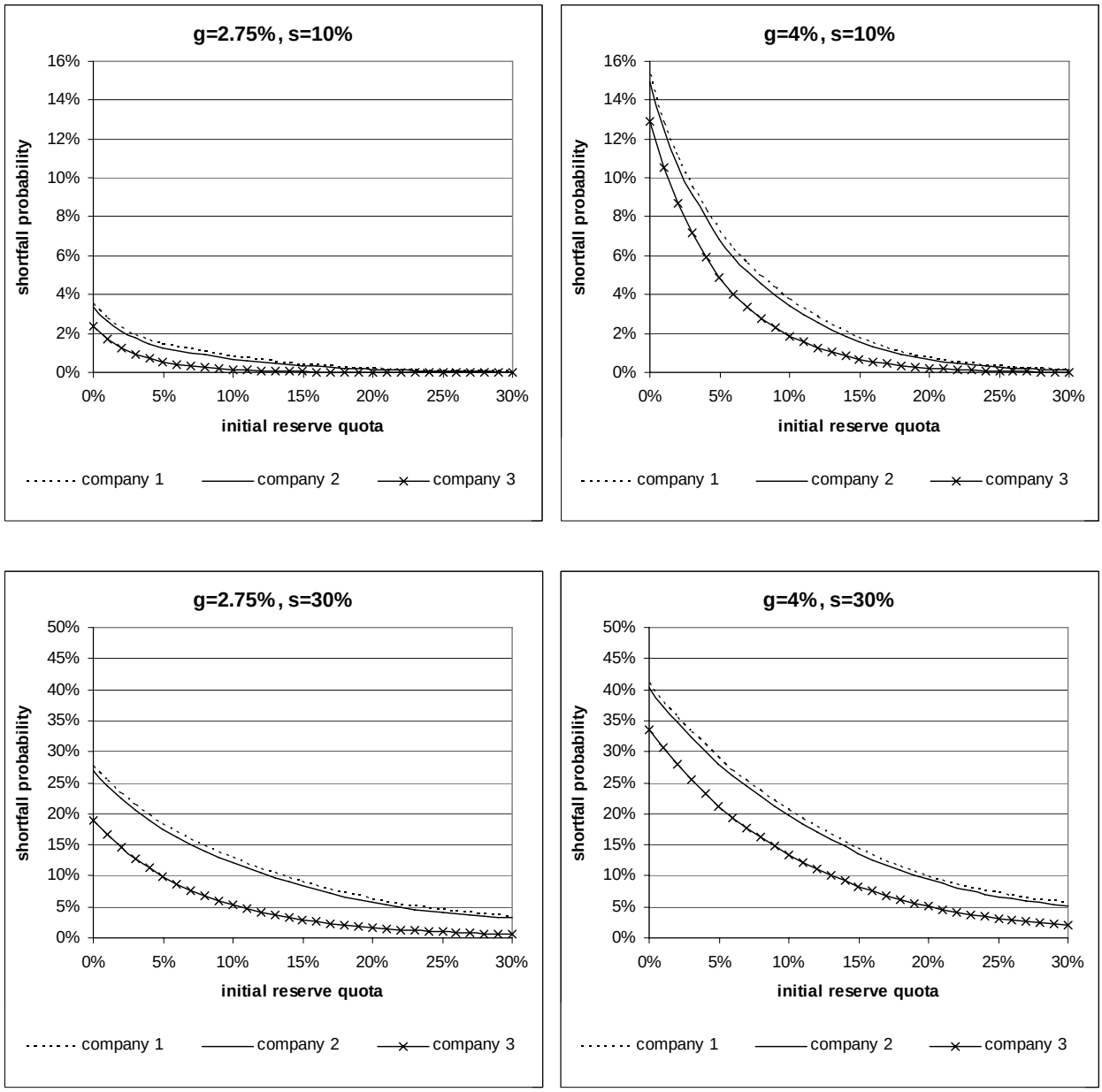

Figure 2: Shortfall probability as a function of the initial reserves for different values of the guaranteed rate of interest and for different asset allocations

Figure 3 shows the shortfall probability as a function of the guaranteed rate of interest for different values of the initial reserve quota x and the stock portion s. Our calculations again

confirm the straightforward proposition that crediting an interest rate guarantee inflicts a much higher risk on a company with a poor initial reserve level. For instance, given a reserve level of 20% and a stock portion of 30%, company 1 can offer a guaranteed rate of 2.75% at a shortfall probability of roughly 8%, whereas, all other things equal, a reduction in the reserve quota to 5% would bring up the shortfall risk to about 20%. At the lower reserve quota, company 3, however, would be able to provide the same guaranteed interest rate with a shortfall probability of only 10%. Again, ceteris paribus company 3 is characterized by a considerably lower shortfall risk.

Additionally, we find that not only the shortfall probability, but also the marginal impact of increasing the guaranteed interest rate is generally greater where reserves are low.

It should also be noted that for low levels of the guaranteed interest rate, the probability of shortfall tends to zero in the case of a 10% stock portion. As the guarantee is decreased, the bond portion of the insurer's asset portfolio becomes more and more likely to be sufficient to generate the minimum interest, while at the same time it limits the shortfall exposure because of the lower volatility. Whereas with a greater portion of stocks the shortfall risk remains significantly positive for companies 1 and 2, insurer 3 would still be able to basically avoid shortfall risk for low guaranteed rates at least in the case with higher initial reserves.

Furthermore, it must be highlighted that given a maximum tolerable shortfall probability, company 3 would at the same reserve level always be able to offer a higher guaranteed rate of interest, i.e. create a higher guaranteed maturity value at inception of the contract. For instance, consider the parameters $x = 20\%$ and $s = 30\%$. In this situation, at a shortfall probability of 5% company 1 would only be able to promise a guaranteed rate of 2%, while company 3 could offer 4%. This, of course, puts companies 1 and 2 at a significant competitive disadvantage, as for a given asset allocation company 3 can offer contracts with a higher initial guarantee return without exposing itself to greater risk.

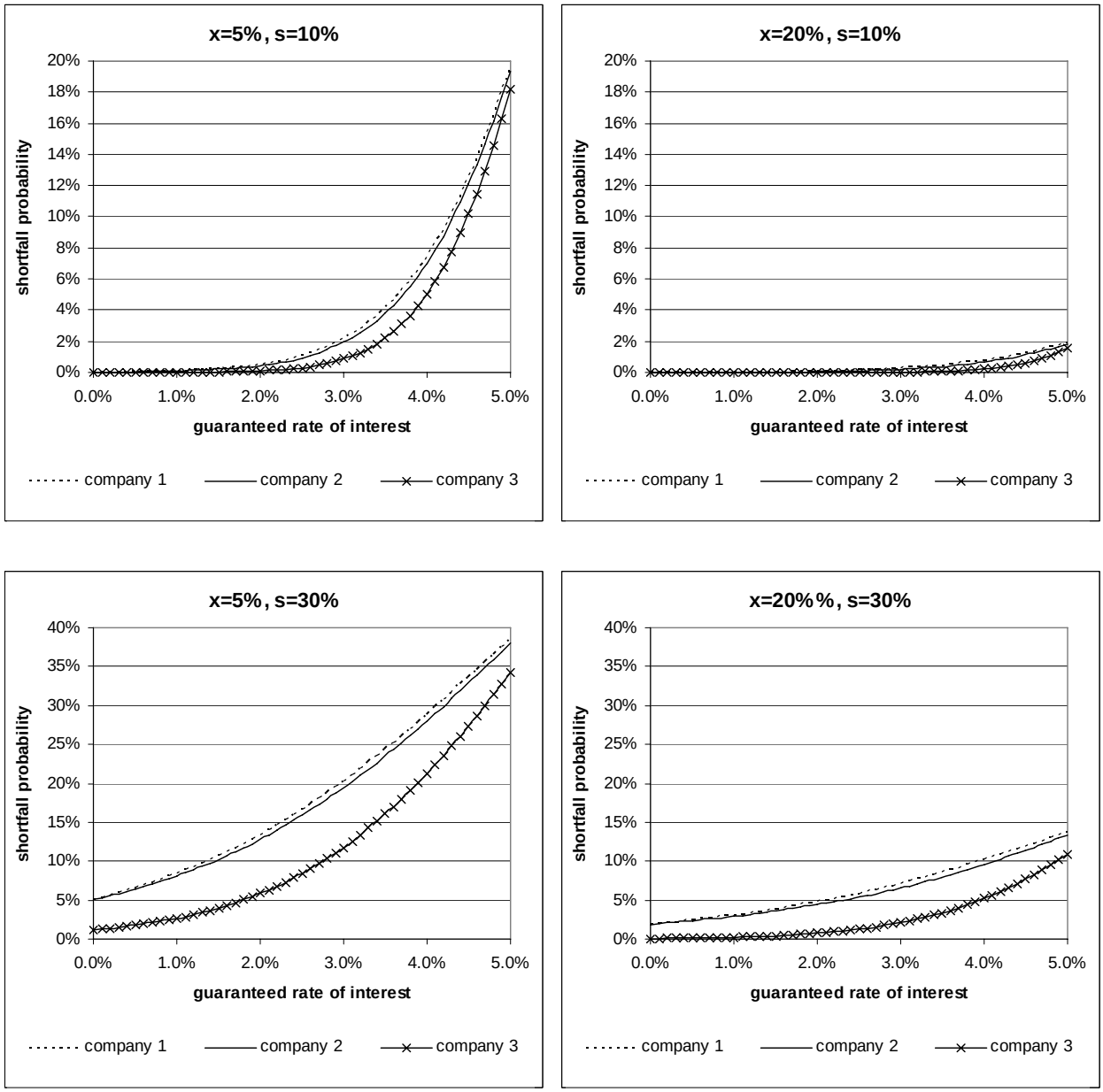

Figure 3: Shortfall probability as a function of the guaranteed rate of interest for different values of the initial reserve quota and for different asset allocations

Considering the shortfall probability as a function of the stock ratio (for g = 2.75%), it can be noted that the shape of this function is not strongly affected by a change in the initial reserve situation (from x = 5% to x = 20%), cf. figure 4. However, a significant difference can be observed for low stock ratios. The function seems to be convex in this area – and concave elsewhere – for all considered parameter sets. For reasons of diversification, of course the probability of shortfall decreases in the stock ratio up to a certain point, (roughly s =10%). This means that very low stock ratios (in particular: s = 0%) are strictly dominated by greater levels of

s which allow a higher expected return at the same risk of shortfall. This effect is particularly pronounced in a situation with low initial reserves.

Given a maximum tolerable shortfall probability, figure 4 shows that company 3 would at the same reserve level and for the same guaranteed rate of interest be able to enter a riskier asset allocation, i.e. create a higher expected return for policy holders. For an initial reserve quota of x = 20%, at a shortfall probability of 5% company 1 would only be able to allow for a stock ratio of 27% while company 3 could afford 38% in stocks. This, of course, puts companies 1 and 2 at a significant competitive disadvantage, as for a given interest guarantee company 3 can offer contracts with a higher expected return without exposing itself to greater risk.

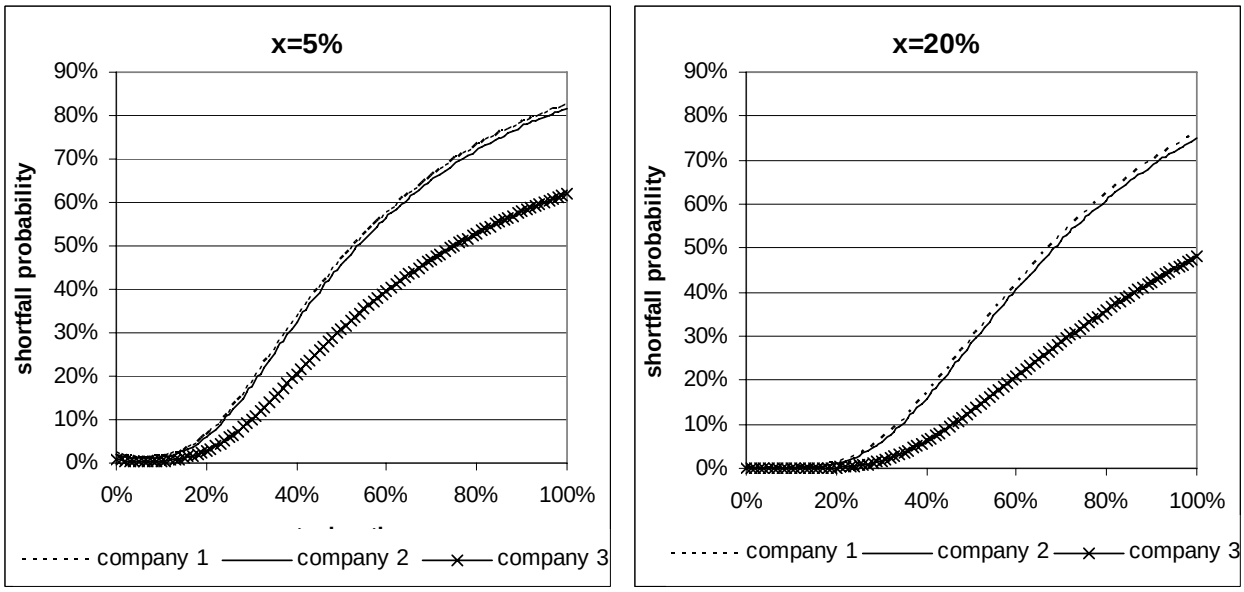

Figure 4: Shortfall probability as a function of the stock ratio s for different values of the initial reserve quota

Figure 5 now shows how results react to variations of the time horizon T. Again, our results indicate that company 3 is considerably more stable than companies 1 and 2. For instance, even in the scenario with low reserves (x=5%), a guaranteed rate of interest of 4% could be provided by company 3 at a shortfall probability of about 15% within 40 years, whereas companies 1 and 2 offering the same guarantee would face the same risk of shortfall within a period of only about 17 years. In order to reduce the 40-year shortfall probability to 15%, companies 1 and 2 would need to have an initial reserve quota of 20%.

It also can be seen from figure 5 that the guaranteed rate of interest makes a significant difference if companies are concerned about long-term shortfall probabilities. By reducing the

guaranteed rate of interest from $g = 4\%$ to $g = 2.75\%$, companies are able to reduce the 40-year shortfall probability by about two thirds, independent of other parameters.

The analyses provided earlier showed that the 10-year shortfall probability of companies 1 and 2 are almost identical while the risk of company 3 differs strongly. However, the longer the considered time horizon T, the greater the difference between the three companies: For initial reserves of $x = 5\%$ and a guaranteed rate of $g = 4\%$, the difference in the 10-year shortfall probability between company 1 and company 3 is less than 3%.This difference is increased to 15% for the 40-year shortfall probability.

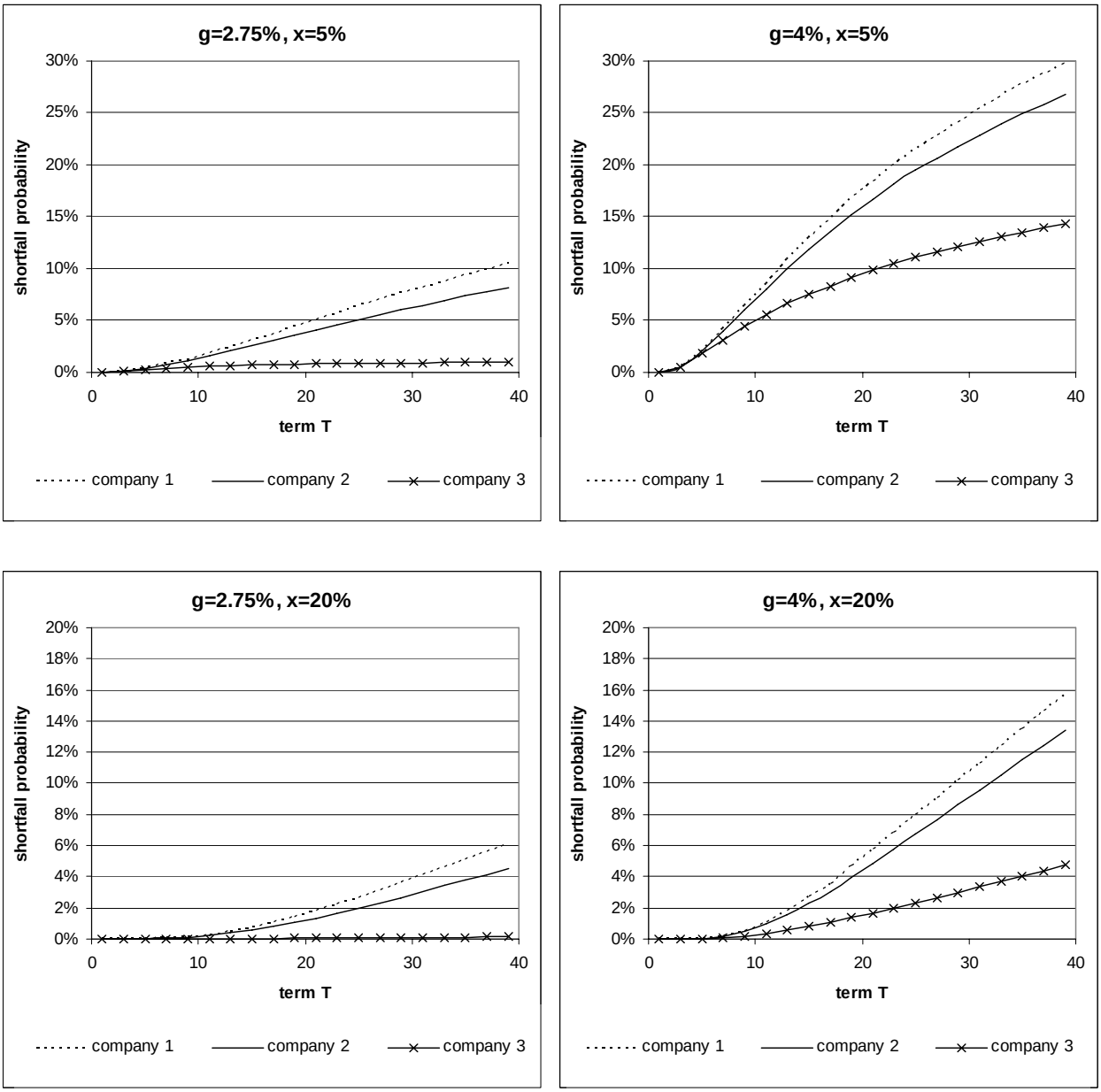

Figure 5: Shortfall probability as a function of the time horizon *T* for different values of the initial reserve quota and the guaranteed rate.

So far, we assumed the same initial balance sheet situation for the three companies considered. This is equivalent to assuming that in the past, the companies behaved exactly the same and will change surplus distribution in the future. We will now analyze whether our results change after the respective different surplus distribution mechanisms have been applied for several years by the different companies. To analyze this effect, in what follows, we focus on so-called forward 10-year shortfall probabilities, defined as contingent probabilities of shortfall in [*t*; *t* +10], given that no shortfall occurred in the first *t* years. One general observation that can be

made from the following analyses is that the differences between the three companies' [t; t +10] forward probabilities tend to be increasing in t. This can be explained by the fact that after t years of applying different surplus mechanisms, the insurers' year t "initial" balance sheets differ.

Figure 6 depicts these forward shortfall probabilities as a function of t for different levels of the guaranteed rate and for different levels of the initial reserve quota. Obviously, an increase in the guaranteed rate always increases the forward shortfall risk, all other things equal. However, it does not seem to have a major impact on the shape of the function.

Interesting observations can be made by comparing results for the two different levels of initial reserves, x = 5% and x = 20%. As shown above, at t = 0, the shortfall risk is significantly lower for larger values of x. But whilst in the low initial reserve situation the forward shortfall probabilities decrease in t, they increase when initial reserves are higher. Although this may seem surprising, it is quite intuitive, if we keep in mind the design of the management rules determining the amount of surplus to be distributed each year: In the lower initial reserves case the insurer will credit lower surplus more frequently as reserves tend toward the lower threshold. Thus, ceteris paribus, it is more likely that the company builds up additional reserves, eventually decreasing its forward shortfall risk. On the other hand, a company with higher initial reserves has a tendency of giving high surplus and thus reducing its reserves. Thus, for large values of t, it is likely that the reserves of the two companies will converge, given that no shortfall occurred before t. This suggests that there is an equilibrium reserve level (and a corresponding default risk). If this level is chosen as the <u>initial</u> reserve level, the graph should be entirely flat.

So, basically, our surplus distribution model implies that companies that have built up high reserves in the past have a tendency to give part of these reserves away to future clients whereas companies with low reserves have a tendency to increase reserves in the future. While this may seem to be a competitive disadvantage for a company with greater initial reserves, we have to keep in mind that this company would be more successful in providing the target interest rate, thus signaling greater stability and ultimately the better product. On the other hand, policy holders should select a company with higher reserves, since this company will provide a higher expected surplus.

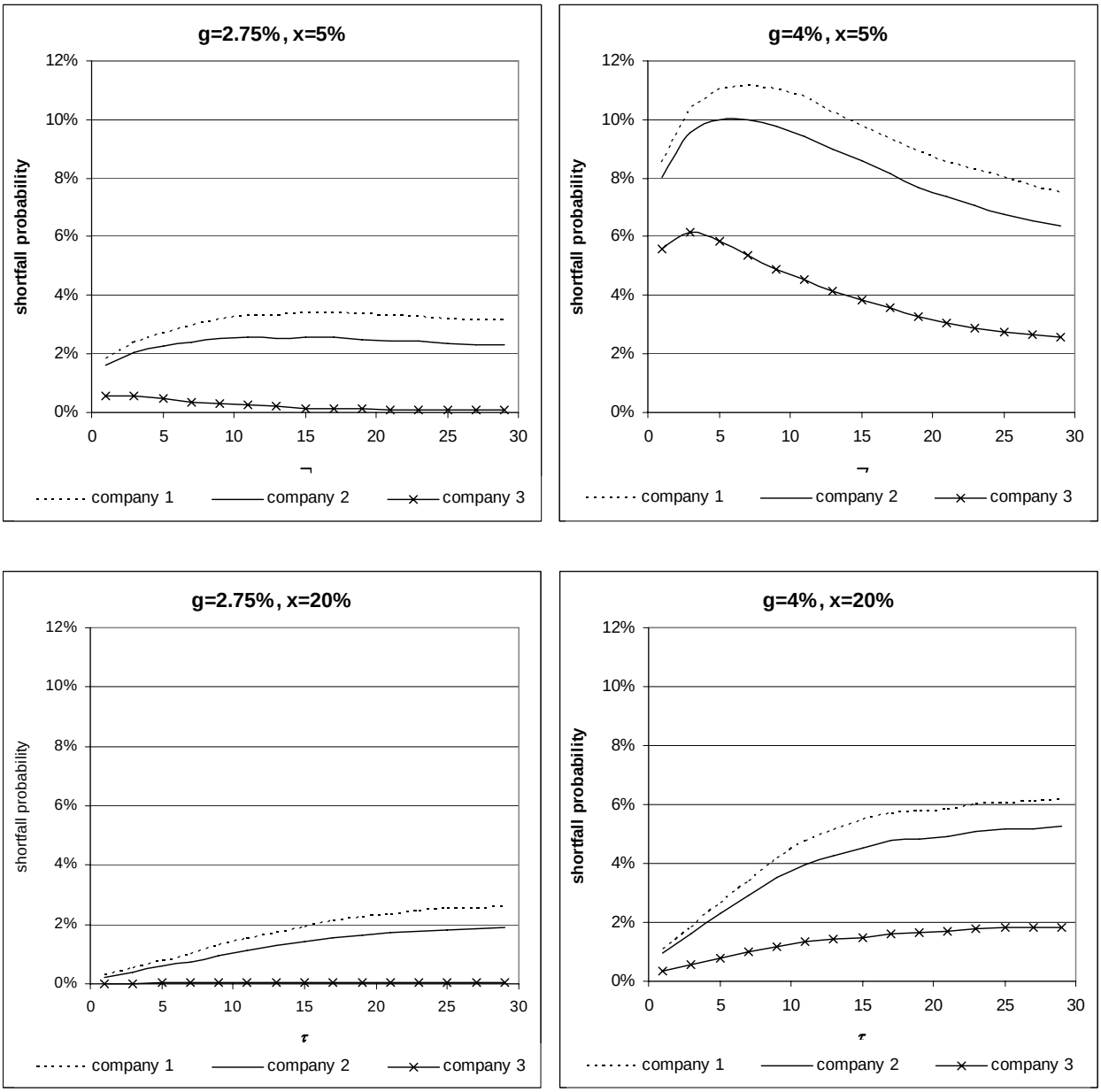

Figure 6: Forward 10-year shortfall probability as a function of the starting time *t* for different values of the initial reserve quota and the guaranteed rate.

4. Conclusions

This paper analyzes the impact of different surplus distribution mechanisms on the risk exposure of a life insurance company selling with profit life insurance policies with a cliquet-style interest rate guarantee. We consider three different types of distribution mechanism: One, where the guaranteed interest rate also applies to surplus that has been credited in the past, a slightly less restrictive mechanism in which a guaranteed rate of interest of 0% applies to past surplus, and a

third mechanism that allows for the company to use former surplus in order to compensate for underperformance in “bad” years.

The results strongly suggest that a distribution mechanism similar to the one introduced for the third type of company is significantly different from the other two distribution mechanisms. Throughout the analysis, our representative company 3 faces ceteris paribus much lower shortfall risk than the other two companies. Thus, a company of our type 3 can afford a much greater portion of stocks in its asset portfolio while maintaining the same shortfall risk, compared to type 1 and 2 companies. This means that, while subject to the same amount of risk, company 3 would be able to invest in a portfolio promising greater expected returns and still offer the same guaranteed maturity benefit. On the other hand, as is straightforward but interesting to note, company 3 would also be able to, all other things equal, provide higher interest rate guarantees than companies 1 and 2 while maintaining the same shortfall risk.

These results should be of particular interest for regulators in the European Union (EU) since on the one hand the surplus mechanism of our type 3 is severely restricted by regulators in some European countries, and on the other hand companies from other EU countries are allowed to sell products with this surplus mechanism across borders (and thus into countries with more restrictive regulation) under the so-called Freedom to provide Services Act. This creates a distortion of competition since companies required to offer “strong” guarantees may be put at a significant competitive disadvantage. This is in particular true for long term contracts.

Thus, our analysis suggests that under certain conditions severe regulation does not only yield suboptimal results, but also seems to be counter-productive with respect to goals that would usually be considered a fundamental purpose of regulation. A major rationale for insurance regulation typically is seen in protecting the customers by keeping insurers solvent. This is particularly important in the area of life insurance where contractual relationships are long-term and the insured become major creditors whose stakes justify strong regulatory intervention.

Naturally, a strong case can also be made for protecting life insurance customers by introducing minimum interest rate guarantees and regulating surplus distribution. One must be aware, however, that inference with the way surplus is distributed, decreases an insurer’s flexibility and, all other things equal, increases shortfall risk.

Overall, shortfall probabilities throughout our analyses are alarmingly high. Partially, this may be attributable to the fact that insurance companies can employ risk management measures not considered in this work, such as adjusting the company's asset allocation when reserves reach a critical level. Although modeling different management decision rules and analyzing their effect on the shortfall risk is possible within our model framework, this is beyond the scope of the present paper. This type of extension, though, might be an interesting topic for future research potentially leading to further valuable insight.

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