PRIIP-KID: Providing Retail Investors with Inappropriate Product Information?

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Abstract

Providers of so-called packaged-retail and insurance-based investment products ("PRIIPs") have to draw up a standardized key-information document ("KID") since 1st of January 2018 when they offer such products in the European Union. In addition to some standard information on the product and its provider, this key information document discloses the product's riskiness by means of a summary risk indicator, its performance potential by means of so-called performance scenarios and its included costs by means of a summary cost indicator. The European Commission has issued regulatory technical standards stating how the risk indicator, the performance scenarios and the cost indicator shall be calculated. This paper analyzes these "calculation recipes", focusing on the risk indicator and the performance scenarios. Since, the European Commission issues these formulae without providing the assumed methodologies, our analyses on the one side shed light on the (presumed) underlying ideas and on the other side detect methodical and technical errors.

We show that the risk indicator's formula can be derived in a Black-Scholes setting considering a single premium investment. Since insurance companies are generally required to produce key information documents for regular premium payments as well, we show that an application of this formula to regular premium payments overestimates the products' "true" risk. Therefore, we propose amended formulae for the risk indicator for regular premium payments which perform much better than the current specification.

Further, we identify methodical and technical errors prevailing in the requirement regarding the (presumed) performance scenarios' calculation.

Taking into account the revision of the PRIIPs-directive at the end of 2018, this paper provides a good starting point for fixing current methodical and technical issues when risk indicator and performance scenarios are assessed.

Keywords: PRIIP; regulation; insurance-based investment products; financial modelling

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1 Introduction

Since 1st of January 2018, providers of packaged retail and insurance-based investment products (so-called „PRIIPS“) have to produce a key-information document (so-called „KID“) following regulation EU 1286/2014 issued by the European Commission (cf. European Commission (2014)). This key information document has to be provided to the customer in "good time" before the actual purchase of the considered product and contains among others an indication of products' potential

- risk by means of a "summary risk indicator"
- return by means of so-called performance scenarios
- costs by means of a "summary cost indicator"

For deriving the required figures on risk, return and costs, the European Commission issued additional regulatory technical standards (so-called "RTS") in European Commission (2017).[1] The key information document has to be produced assuming a retail investor either to invest a single premium of 10.000 EUR or in addition – when insurance-based investment products are considered – a regular premium payment of 1.000 EUR annually instead of the single premium investment. Further, some "recommended holding period" / maturity of T years has to be specified for the calculations by the product provider. European Commission (2017) then assigns each product subject to the PRIIP-regulation to one of four different "product categories" which are briefly summarized as follows:

- *Category 1* comprises
  - products where retail investors may lose more than their invested premiums,
  - derivative-like products such as futures, options, swaps, etc.
  - and products whose prices are only determined on a less than monthly basis.

[1] Note, originally the PRIIP-regulation should have entered into force already on December 31st, 2016, based on underlying regulatory technical standards issued on 30th of June 2016. However, the European Parliament objected to these regulatory technical standards and hence the PRIIP-regulation was postponed to the beginning of 2018 and the underlying regulatory technical standards had to be revised in the meantime.

Hence, due to the types of products considered here, products of category 1 typical insurance-based investment products will not qualify for category 1.

- *Category 2* covers products that provide a "linear" non-structured exposure to their underlying investments. Generally most of (non-structured) investment funds, such as equity, fixed income or balanced funds will therefore qualify as products of category 2.
- *Category 3* in contrast covers products that offer "non-linear" structured exposure to their underlying investments. E.g. guarantee funds managed according to some portfolio insurance technique and hence typically providing path-dependent (non-linear) exposure to their underlying investments qualify for category 3.
- Finally, *category 4* covers all products whose "*values depend in part on factors not observed in the market*" (cf. European Commission (2017)) and especially includes insurance-based investment products that are equipped with some profit participation which is generally not directly observed in the market.

For each of these product categories European Commission (2017) provides quantitative "recipes" and methodological advice to perform the required calculations, however without reasoning these formulae. Hence, this paper is on the one side concerned with deriving the (potential) ideas behind the proposed approaches and on the other side critically assesses if these provided recipes are accurate. We focus on the quantitative assessment of the risk and the return measures in our analyses and do not undertake an assessment of the proposed methodology on the disclosure of costs. In particular we show:

(1) The formula for calculating the risk indicator is (presumably) based on a single premium investment. Therefore, we show that its application to regular premium payments yields results that generally overestimate the products' "true" risk. After highlighting these issues, we propose amended formulae to address the risk indicator for regular premium payments.

(2) If our understanding of the idea behind the calculation methodology is correct, the proposed formulae for deriving the performance scenarios of products of category 2 are technically wrong and yield to inappropriate results as well.

Since European Commission (2014) already scheduled a review of the PRIIP-regulation itself during 2018, the results of this paper may provide a good starting point to solve the above issues in a potentially amended version of the corresponding regulatory technical standards.

The remainder of this paper is organized as follows: Section 2 introduces the financial model used throughout our analyses which is (at least to our understanding implicitly) assumed by European Commission (2017) for deriving the risk indicator. Assuming this financial model, Section 3 then deals with the proposed calculation methodology required for the risk indicator, sheds light on the idea underlying these calculations and shows that this approach in general yields wrong results if applied to regular premium payments. Then, we propose and analyze different modified calculation recipes that allow for an appropriate treatment of regular premium payments as well. Section 4 analyzes the derivation of the performance scenarios for so-called products of category 2 and detects technical errors in these formulae. Finally, Section 5 concludes.

2 Financial model

Throughout the paper, we apply a Black-Scholes model (cf. Black and Scholes (1973)) equipped with parameters μ and σ and denote this model as $BS(\mu, \sigma)$. Hence, for $\mu \in \mathbb{R}, \sigma \geq 0$ and $S_0 = 1$, we consider the stochastic differential equation

$$dS_t = S_t(\mu dt + \sigma dW_t)$$

where $W(t)$ is a Wiener Process under the considered probability measure. This stochastic differential equation can then be solved as

$$S_T = \exp\big((\mu - 0.5\sigma^2)T + \sigma W_T\big).$$

Hence, S_T follows a log-normal distribution for any $T \in \mathbb{R}^+$.

Sections 3 and 4 require the calculation of different percentiles of the underlying model. Therefore, we indicate how different percentiles of a $BS(\mu, \sigma)$ can be derived. For $\alpha \in (0,1)$ and the corresponding percentile x_α, we obtain

$$\mathbb{P}(S_T \leq x_\alpha) = \alpha$$

$$\Leftrightarrow$$

$$\mathbb{P}\left((\mu - 0.5\sigma^2)T + \sigma W_T \leq \ln(x_\alpha)\right) = \alpha$$

$$\Leftrightarrow$$

$$\mathbb{P}\left(\frac{W_T}{\sqrt{T}} \leq \frac{(\ln(x_\alpha) - (\mu - 0.5\sigma^2)T)}{\sigma\sqrt{T}}\right) = \alpha$$

Since $\frac{W_T}{\sqrt{T}}$ follows a standard normal distribution, we finally get

$$\frac{(\ln(x_\alpha) - (\mu - 0.5\sigma^2)T)}{\sigma\sqrt{T}} = z_\alpha \Leftrightarrow x_\alpha = \exp\left((\mu - 0.5\sigma^2)T + z_\alpha\sigma\sqrt{T}\right)$$

with z_α denoting the α^{th}-percentile of the standard normal distribution, i.e. $z_\alpha = \Phi^{-1}(\alpha)$ where $\Phi(.)$ is the cumulative distribution function of a standard normal random variable.

3 Risk: Derivation of the summary risk indicator

The summary risk indicator consists of an assessment of a product's market risk by a so-called market risk measure ("MRM") and the product provider's creditworthiness by means of a so-called credit risk measure ("CRM"). Both measures MRM and CRM are then combined to result in a summary risk indicator ("SRI") which is a number between 1 and 7. Following European Commission (2017), Annex II, point 52 Table 1 shows the summary risk indicator as a function of MRM and CRM.

	MRM 1	MRM 2	MRM 3	MRM 4	MRM 5	MRM 6	MRM 7
CRM 1	1	2	3	4	5	6	7
CRM 2	1	2	3	4	5	6	7
CRM 3	3	3	3	4	5	6	7
CRM 4	5	5	5	5	5	6	7
CRM 5	5	5	5	5	5	6	7
CRM 6	6	6	6	6	6	6	7

Table 1 Summary risk indicator (SRI) as combination of market risk measure (MRM) and credit risk measure (CRM)

The credit risk measure builds on the product provider's rating whereas the market risk measure is derived from the product's exposure to capital market risk and may hence differ when different products issued by the same provider are considered. The following analyses focus on the market risk measure, i.e. the product's "riskiness", only. We are therefore not concerned with the product provider's creditworthiness.[2]

In what follows, Section 3.1 first shows how to derive the product's riskiness as required by European Commission (2017) and then indicates the potential idea behind this formula. Section 3.2 then briefly deals with the single premium case whereas Section 3.3 extends the calculation approach to a consideration of regular premium payments.

3.1 Calculation recipe by European Commission (2017) and potential idea behind the formula

The regulatory technical standards issued by the European Commission (cf. European Commission (2017), Annex II, point 1) state that the product's Value-at-Risk at a confidence level of 97.5% should be used as a market risk measure. Hence, the European Commission requires deriving the 2.5th-percentile of the product's probability distribution of benefit payments at the end of the recommended holding period (maturity). This percentile is then mapped to a so-called "Value-at-Risk equivalent volatility" (VEV) which then transforms to the market risk measure in a numerical scale of 1 to 7 following Table 2 (cf. European Commission (2017), Annex II, point 2).

VEV	< 0.5%	0.5% - 5%	5% - 12%	12% - 20%	20% - 30%	30% - 80%	>80%
MRM	1	2	3	4	5	6	7

Table 2 Value-at-Risk-equivalent volatility (VEV) and resulting market risk measure (MRM)

[2] European insurance companies which are subject to the Solvency II-regulation may typically (at least) qualify for a CRM of 2.

Further, European Commission (2017) states in Annex II, point 13 how to derive the VEV from a given 97.5% Value-at-Risk by[3]

$$VEV = \frac{\sqrt{3.842 - 2 \cdot VaR_{ReturnSpace}} - 1.96}{\sqrt{T}}$$

where $VaR_{ReturnSpace}$ is the log-return corresponding to the 2.5th-percentile of the product's probability distribution of maturity benefits when a recommended holding period of T years is considered. As already indicated in Section 1 European Commission (2017) provides different methodologies on how to derive $VaR_{RaturnSpace}$ for different product categories considered.[4] However, these different requirements for calculating $VaR_{ReturnSpace}$ are not in the scope of this paper. Instead, we focus on the derivation of the VEV from a given $VaR_{ReturnSpace}$. We particularly give a (possible) reasoning for the above formula and highlight some issues with this approach, especially when regular premium payments are considered.

We will now show how the above formula for VEV was derived (at least to our understanding): Consider a $BS(\mu, VEV)$-model (cf. Section 2). VEV's intuition is to align some (externally calculated) percentile – i.e. $VaR_{ReturnSpace}$ – with the (synthetic) volatility VEV of a corresponding Black-Scholes-model.

To see this, compute the α^{th}- percentile of above Black-Scholes-model as

$$\frac{(\ln(x_\alpha) - (\mu - 0.5VEV^2) \cdot T)}{VEV \cdot \sqrt{T}} = z_\alpha$$

with z_α denoting the α^{th}-percentile of a standard normal random variable and further setting $\ln(x_\alpha) = VaR_{ReturnSpace}$. Hence, we consider one equation with two unknown parameters μ and VEV. If we assume $\mu = 0$, we obtain

$$\frac{(\ln(x_\alpha) + 0.5VEV^2 \cdot T)}{VEV \cdot \sqrt{T}} = z_\alpha$$

[3] Here, we explicitly follow the notation of European Commission (2017).

[4] For products of category 2 an analytical formula following Cornish and Fisher (1938) is proposed whereas products of category 3 and 4 typically require some simulation approach to derive $VaR_{ReturnSpace}$. Finally, the derivation of MRM of category 1 products does not require any calculation, but is directly set to either 6 or 7 by European Commission (2017).

$$\Leftrightarrow$$

$$0.5 \cdot T \cdot VEV^2 - z_\alpha \cdot \sqrt{T} \cdot VEV + \ln(x_\alpha) = 0$$

which is a quadratic equation in the parameter VEV and can therefore be solved as

$$VEV = \frac{z_\alpha \cdot \sqrt{T} + \sqrt{z_\alpha^2 \cdot T - 4 \cdot 0.5 \cdot T \cdot \ln(x_\alpha)}}{2 \cdot 0.5 \cdot T}$$

$$\Leftrightarrow$$

$$VEV = \frac{z_\alpha + \sqrt{z_\alpha^2 - 2 \cdot \ln(x_\alpha)}}{\sqrt{T}}$$

Setting $\alpha = 2.5\%$ and $\ln(x_\alpha) = VaR_{ReturnSpace}$ yields $z_\alpha = -1.96$ and

$$VEV = \frac{-1.96 + \sqrt{3.842 - 2 \cdot VaR_{ReturnSpace}}}{\sqrt{T}}$$

which equals the above definition of the Value-at-Risk equivalent volatility stated by European Commission (2017). We therefore strongly believe that this is how the formula for VEV was derived by the European Commission. Note that according to this derivation, European Commission (2017) essentially assumes a drift of $\mu = 0$ when they compute the VEV. Hence, the underlying $VaR_{ReturnSpace}$ also has to be derived assuming a "0-drift world" for reasons of consistency. This is ensured by the further methodological requirements specified by European Commission (2017) regarding the calculation of $VaR_{ReturnSpace}$ for the different product categories.[5]

The following box summarizes how European Commission (2017) proposes to derive the market risk measure (MRM) from the Value-at-Risk equivalent volatility (VEV), respectively from the product's 97.5%-Value-at-Risk $VaR_{ReturnSpace}$.

[5] E.g. the formula for products of category 2 to derive the $VaR_{ReturnSpace}$ directly assumes a drift of 0, whereas the requirements for products of category 3 first propose a risk-neutral simulation and then again discount the simulated 2.5th-percentile with the risk-free rate for obtaining $VaR_{ReturnSpace}$ which then also resembles a "0-drift world" (cf. European Commission (2017), Annex II, point 12 and points 16-22).

Summary of the European Commission's approach on deriving the market risk measure

- Derive $VaR_{ReturnSpace}$ as the log-return of the product's 2.5th -percentile assuming a "0-drift" world.
- In a $BS(0, VEV)$-model, map this $VaR_{ReturnSpace}$ to the Value-at-Risk equivalent volatility (VEV) applying $VEV = \frac{\sqrt{3.842-2\cdot VaR_{ReturnSpace}}-1.96}{\sqrt{T}}$
- Derive the market risk measure (MRM) from the VEV following Table 2.

Potential issues

Note that the above derivation of VEV presumably assumes a single premium investment in a $BS(0, VEV)$-model and is therefore potentially not valid (not even defined) when regular premium payments are considered instead. This issue, its potential consequences and possible amendments to the VEV calculation for regular premium payments are analyzed in Section 3.3.[6] Further, the assumption of a "0 drift-world" may result in inappropriate estimates of the products' 2.5th-percentile, especially when path-dependent products such as CPPI-type products are considered. This additional issue is however not further addressed in the remainder of this paper.

The following sections 3.2 and 3.3 carry out some numerical analyses for single and regular premium payments. These analyses are based on Monte-Carlo simulation performing 10^6 simulated trajectories of (different versions of) the underlying financial model as introduced in Section 2.[7] Throughout our numerical study we investigate different volatilities σ ranging from 0%, 0.5%, 1%, 1.5%, … to 30% and different recommended holding periods / maturities T ranging from 1, 2, … to 40 years.

[6] Since the VEV calculation recipe was apparently only provided for single premium products by European Commission (2017), the industry had to come up with some derivations for regular premium payments as well. The German insurance market for example followed an approach similar to the ideas analyzed in Section 3.3.1.

[7] Although the Black-Scholes-model is analytically very tractable, closed form solutions for the probability distribution of regularly investing in a Black-Scholes-model do (to the best of our knowledge) not exist. Hence, we rely on Monte-Carlo simulation instead.

Note, as shown above, the derivation of the VEV is analytically correct for the single premium case. Hence, Section 3.2 “only” indicates the accurateness of our simulations carried out. In section 3.2, we estimate the 2.5th-percentile of a single premium investment in the $BS(0,\sigma)$-model over a time horizon of T years and then re-engineer the volatility σ from the product’s 2.5th-percentile applying above formula.

Section 3.3 then expands the calculations to the regular premium case. Again we estimate the 2.5th-percentile of a regular premium investment in the $BS(0,\sigma)$-model over a time horizon of T years and then investigate different methodologies to re-engineer the volatility σ from the product’s 2.5th-percentile.

3.2 The single premium case

This section deals with the VEV’s formula as provided by European Commission (2017) considering a single premium investment in a $BS(0,\sigma)$-model for $\sigma = 0, 0.5\%, 1\%, \dots, 30\%$ over a time horizon of $T = 1,2, \dots, 40$ years.

We simulate the corresponding Black-Scholes models $BS(0,\sigma)$, estimate the products’ log-return $VaR_{ReturnSpace}$ by deriving the 2.5th-percentile and re-engineer the original volatility by $\frac{\sqrt{3.842-2\cdot VaR_{ReturnSpace}}-1.96}{\sqrt{T}}$.

Fig 1 summarizes the results, where we show the difference between the “re-engineered” volatility – i.e. the VEV – and the original volatility – i.e. σ – which originally fed into the Black-Scholes model. For example, Fig 1 shows that setting $\sigma = 30\%$ and $T = 40$, the re-engineered volatility is 2bp larger than the original volatility of 30%, i.e. it is 30.02%.

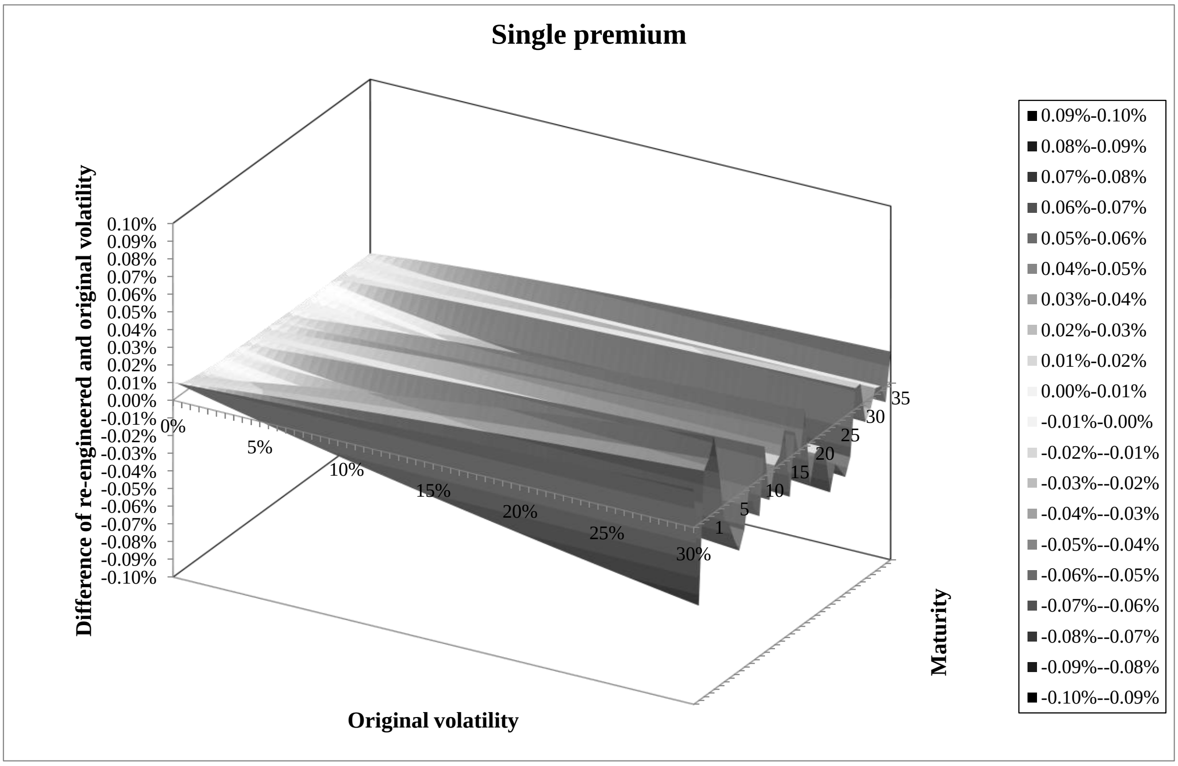

Fig 1 Single premium: Difference of re-engineered and original volatility

Fig 1 shows that all differences are extremely small indicating that the applied Monte-Carlo approach yields accurate estimates for the considered 2.5th-percentiles.

For being able to compare these results for the single premium case with the regular premium case more easily, Fig 2 shows the same numbers however applying a larger scaling on the y-axis from -10% to 10% instead of -0.1% to 0.1%.

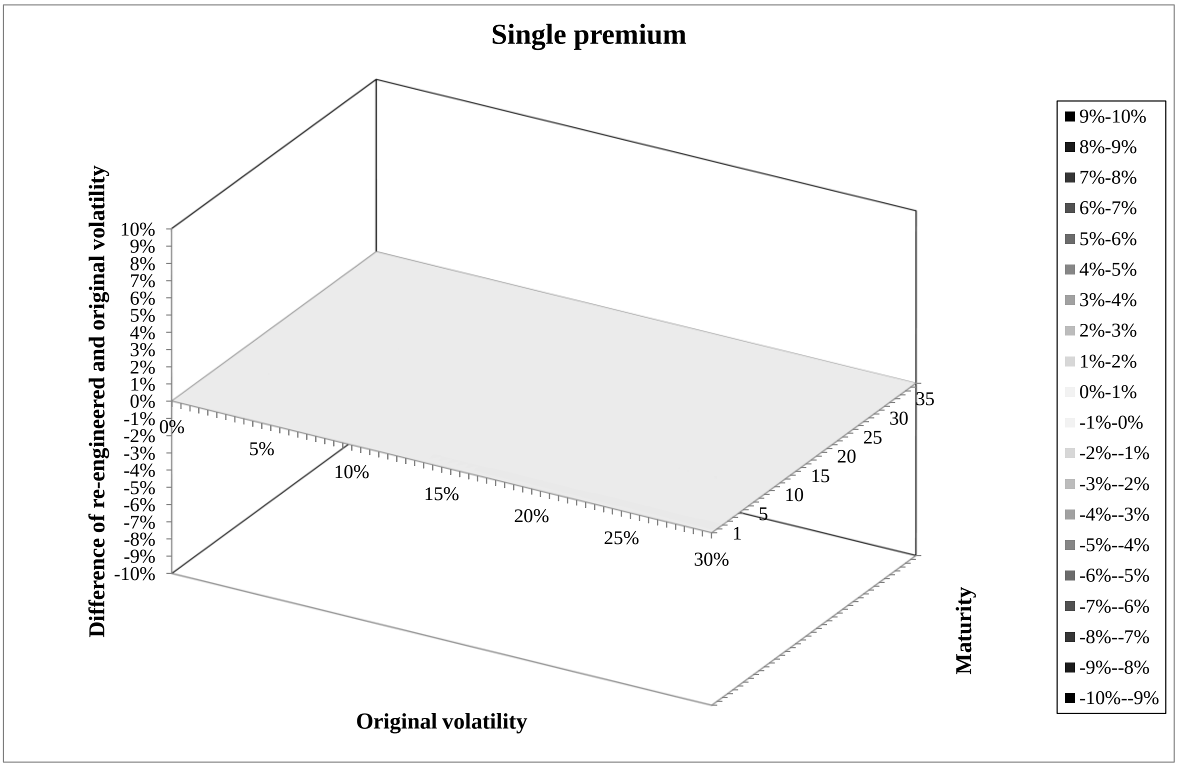

Fig 2 Single premium: Difference of re-engineered and original volatility (different scaling)

3.3 The regular premium case

In this section we extend our analysis to regular premium payments. European Commission (2017) requires to draw up key information documents for insurance-based investment products generally assuming both, a single premium of 10.000 EUR as well as annual premium payments of 1.000 EUR, each over the recommended holding period T (cf. European Commission (2017), Annex VI, point 90).

Since European Commission (2017) only provides a formula for deriving the VEV which is presumably based on a single premium investment (cf. Section 3.2), we (and all product providers in practice) first have to address the qualitative meaning of VEV (as indicated by the formula for a single premium payment) and try to consistently extend this approach to regular premium payments. We assume that the European Commission’s intended qualitative requirement on calculating the VEV for regular premium payments shall coincide with the reasoning for the single premium case. Hence, following our derivations, the qualitative requirement for deriving the VEV when regular premiums are considered would then read as follows:

Qualitative requirement on the VEV for regular premium payments

For a (pre-calculated) 97.5%-Value-at-Risk of a product equipped with regular premium payments find the VEV such that an investment of the same regular premium payments assuming a Black-Scholes-model $BS(0, VEV)$ yields the same 97.5%-Value-at-Risk.

This section will now – given this qualitative requirement – treat different calculation methodologies for computing the VEV based on the 97.5%-Value-at-Risk.

Therefore, consider the random variable $W_\sigma(T) := \sum_{t=0}^{T-1} 1.000 \cdot \frac{S(T)}{S(t)}$ which gives the maturity benefit of an annual investment of 1.000 EUR where $S(t)$ follows a $BS(0, \sigma)$-model over T years. We estimate the corresponding 97.5%-Value-at-Risk of $W_\sigma(T)$ by means of Monte-Carlo simulation and then try to re-engineer the volatility σ (i.e. the "true" VEV) that originally fed into the $BS(0, \sigma)$-model.

The different methodologies for re-engineering the volatility based on the investment's 97.5%-Value-at-Risk will be given in the following Sections. Section 3.3.1 analyzes an approach which builds on "heuristically" extending the formula provided by European Commission (2017) to take regular premium payments into account. This approach is currently e.g. applied in the German insurance market and also proposed by the European Fund and Asset Management Association (EFAMA).[8] However, it turns out that the re-engineered volatility VEV obtained by this methodology tends to significantly overestimate the original volatility σ that fed into the considered Black-Scholes model. Therefore, Section 3.3.2 introduces another approach primarily based on a lognormal approximation of the resulting probability distribution obtained by regularly investing in a Black-Scholes-model. We show that this approach in contrast tends to generally underestimate the original volatility. Hence, Section 3.3.3 introduces a combination of both

[8] Note, this formula is not defined by European Commission (2017) but was developed by the industry (cf. e.g. EFAMA (2017a) and EFAMA (2017b)) to come up with an approach that is both (very) close to the formula proposed by the European Commission and in addition at least applicable to regular premium payments as well.

methodologies which delivers astonishingly appropriate estimates of the true volatility based on the product's 97.5%-Value-at-Risk. Finally, Section 3.3.4 briefly sketches an approach to derive the MRM by just simulating the Value-at-Risk only for "critical" volatilities and for different recommended holding periods once in advance and then aligning the product's 97.5%-Value-at-Risk with these tabulated values accordingly.

3.3.1 Approach 1: (Heuristic) Extension of the formula by European Commission (2017)

Section 3.1 introduces the VEV calculation as

$$VEV = \frac{\sqrt{3.842 - 2 \cdot VaR_{ReturnSpace}} - 1.96}{\sqrt{T}}$$

where $VaR_{ReturnSpace}$ corresponds to the log-return of the 2.5th-percentile of a single premium investment into the considered product. Hence, equivalently $VaR_{ReturnSpace} = \ln\left(\frac{S_{T,2.5\%}}{S_0}\right)$ where $S_{T,2.5\%}$ equals the 2.5th-percentile of the considered $BS(0, VEV)$-model. Let r denote the annualized log-return corresponding to this 2.5th-percentile, i.e. $r = \frac{VaR_{ReturnSpace}}{T}$, then above formula can be rewritten as $VEV = \frac{\sqrt{3.842-2 \cdot r \cdot T}-1.96}{\sqrt{T}}$.

Building on the annualized log-return, a natural (heuristic) extension to derive the VEV for a product when regular premium payments are considered is therefore given by the following approach:

Approach 1: (Heuristic) Extension of the formula by European Commission (2017)

- Let r denote the product's internal rate of return given regular premium payments of 1.000 EUR on an annual basis and a (pre-calculated) 97.5%-Value-at-Risk VaR, i.e. let r be the solution of $VaR = \sum_{t=0}^{T-1} 1000 \cdot e^{r \cdot (T-t)}$

- Applying this internal rate of return r, set the VEV as

$$VEV = \frac{\sqrt{3.842 - 2 \cdot r \cdot T} - 1.96}{\sqrt{T}}$$

Fig 3 shows the results when approach 1 is applied for deriving the VEV when regular premium payments are considered. We depict the difference between the re-engineered volatility VEV and the volatility σ that was originally applied in the $BS(0,\sigma)$-model. From Fig 3, we conclude that the VEV derived by approach 1 generally overestimates the original volatility σ. The estimation error increases with the considered recommended holding period T and with the original volatility σ. Whereas an original volatility of $\sigma = 5\%$ is "only" overestimated by 0.71% for a recommended holding period of $T = 40$ years – and hence VEV=5.71% is obtained – an application of approach 1 delivers VEV = 37.47% for an original $\sigma = 30\%$ and a maturity $T = 40$ years. For both, short recommended holding periods (e.g. no longer than 5 years) and low volatilities (e.g. lower than 5% p.a.), the estimation error's impact is rather low with a maximum observed overestimation of the true volatility of 0.53% p.a. for $\sigma = 5\%$ and $T = 5$.

However, since recommended holding periods – especially with regards to insurance-based investment products – are typically much longer than 5 years, this approach generally (significantly) overestimates a product's VEV when regular premium payments are considered.[9] Therefore, the application of approach 1 for regular premium payments should be thoroughly revisited.

[9] Recommended holding periods for insurance-based investment products typically assumed e.g. in the German or Austrian market reach up to 40 years, whereas in contrast e.g. the French market for some products assumes a recommended holding period of one year (cf. Institut des Actuaires (2017)).

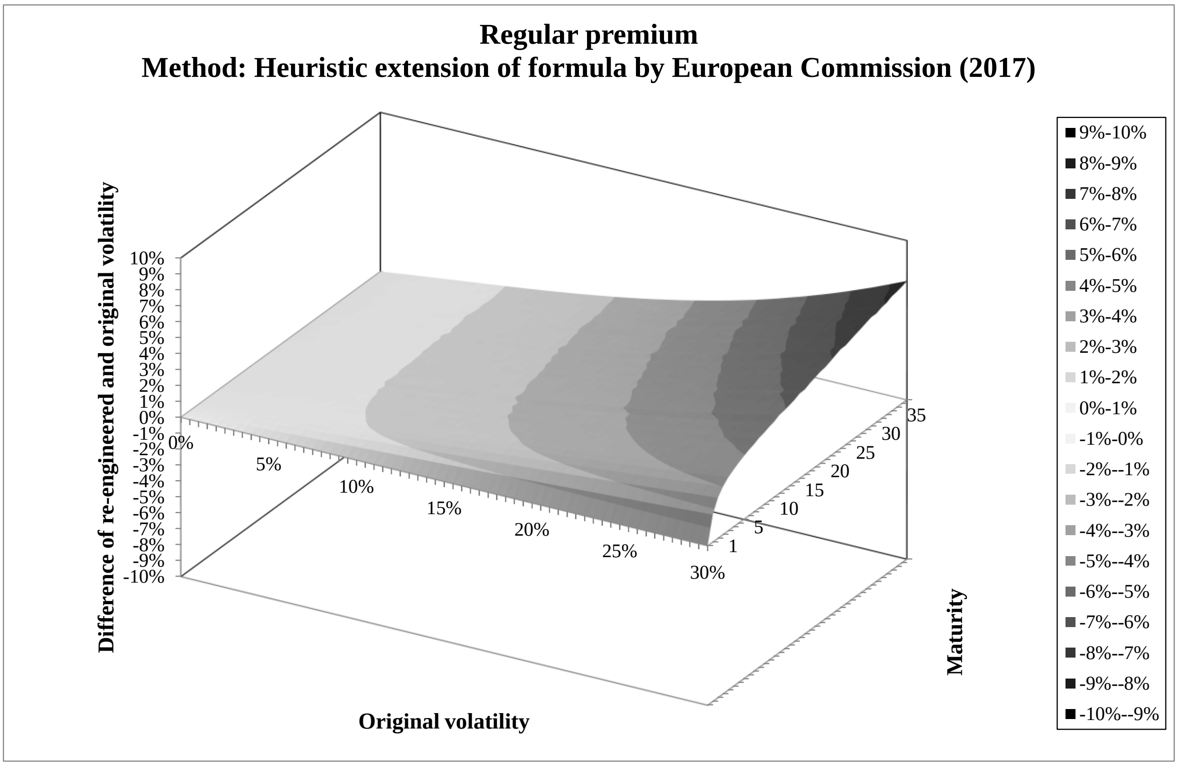

Fig 3 Regular premium: Difference of re-engineered and original volatility, Method: Heuristic extension of RTS formula

3.3.2 Approach 2: Approximation by a log-normal distribution

Section 3.3.1 showed that a heuristic extension of the formula provided by European Commission (2017) for regular premium payments always (and often significantly) overestimates the product's "real" VEV. Hence, this section proposes a different approach to compute the VEV which builds on a log-normal approximation of the resulting probability distribution of maturity benefits considering an annual investment into a $BS(0,\sigma)$-model.

For a given $BS(0,\sigma)$-model we obtain

$$W_\sigma(T) = \sum_{t=0}^{T-1} 1.000 \cdot \frac{S(T)}{S(t)} = 1.000 \cdot \frac{S(T)}{S(T-1)} + \frac{S(T)}{S(T-1)} \cdot \sum_{t=0}^{T-2} 1.000 \cdot \frac{S(T-1)}{S(t)}$$
$$= \frac{S(T)}{S(T-1)} \cdot \big(1.000 + W_\sigma(T-1)\big)$$

Since $\frac{S(T)}{S(T-1)}$ is independent of $W_\sigma(T-1)$, we compute the expectation of $W_\sigma(T)$ recursively as

$$\mathbb{E}[W_\sigma(T)] = \mathbb{E}\left[\frac{S(T)}{S(T-1)}\right] \cdot (1.000 + \mathbb{E}[W_\sigma(T-1)])$$

further using $\mathbb{E}[W_\sigma(1)] = \mathbb{E}\left[\frac{S(1)}{S(0)} \cdot 1.000\right]$.

Following the same arguments, the second moment of $W_\sigma(T)$ can be derived as

$$\mathbb{E}[W_\sigma^2(T)] = \mathbb{E}\left[\left(\frac{S(T)}{S(T-1)}\right)^2\right] \cdot (1.000^2 + 2 \cdot 1.000 \cdot \mathbb{E}[W_\sigma(T-1)] + \mathbb{E}[W_\sigma^2(T-1)])$$

applying $\mathbb{E}[W_\sigma^2(1)] = 1.000^2 \cdot \mathbb{E}\left[\left(\frac{S(1)}{S(0)}\right)^2\right]$.

Further, note that $\frac{S(t)}{S(t-1)}, \forall t = 1, \dots, T$ are independent identically distributed copies of a log-normal random variable with moments

$$\mathbb{E}\left[\left(\frac{S(t)}{S(t-1)}\right)^k\right] = \exp(k \cdot (-0.5\sigma^2) + 0.5 \cdot k^2 \cdot \sigma^2)$$

Hence, for given $BS(0, \sigma)$-model, one is able to derive the first two moments of $W_\sigma(T)$ analytically applying above recursive formulae.

We now approximate $W_\sigma(T)$ with a log-normal random variable $Z_\sigma \sim LN\left(\mu_{W_\sigma(T)}, \sigma_{W_\sigma(T)}\right)$ by matching the first two moments of Z_σ with the first two moments of $W_\sigma(T)$, i.e. by solving the equation set

$$\exp\left(\mu_{W_\sigma(T)} + 0.5\sigma^2_{W_\sigma(T)}\right) = \mathbb{E}[W_\sigma(T)]$$
$$\exp\left(2\mu_{W_\sigma(T)} + 2\sigma^2_{W_\sigma(T)}\right) = \mathbb{E}[W_\sigma^2(T)]$$

which can be solved as

$$\mu_{W_\sigma(T)} = \ln(\mathbb{E}[W_\sigma^2(T)]) - \ln(\mathbb{E}[W_\sigma(T)]) - 1.5\sigma^2_{W_\sigma(T)}$$
$$\sigma^2_{W_\sigma(T)} = \ln(\mathbb{E}[W_\sigma^2(T)]) - 2 \cdot \ln(\mathbb{E}[W_\sigma(T)])$$

After specification of $\mu_{W_\sigma(T)}$ and $\sigma^2_{W_\sigma(T)}$, the 97.5%-Value-at-Risk VaR_{Z_σ} (i.e. the 2.5$^{\text{th}}$-percentile) of Z_σ is readily derived as

$$2.5\% = \mathbb{P}\left(Z_\sigma \leq VaR_{Z_\sigma}\right) = \mathbb{P}\left(\ln(Z_\sigma) \leq \ln\left(VaR_{Z_\sigma}\right)\right)$$
$$= \mathbb{P}\left(\frac{\ln(Z_\sigma) - \mu_{W_\sigma(T)}}{\sigma_{W_\sigma(T)}} \leq \frac{\ln\left(VaR_{Z_\sigma}\right) - \mu_{W_\sigma(T)}}{\sigma_{W_\sigma(T)}}\right)$$

Hence, with z_α denoting the 2.5^{th}-percentile of a standard-normal random variable, we get $VaR_{Z_\sigma} = \exp\left(\mu_{W_\sigma(T)} + z_\alpha \cdot \sigma_{W_\sigma(T)}\right)$. These derivations yield our second approach on a possible calculation of the product's VEV for regular premium payments:

Approach 2: Approximation by a log-normal distribution

Consider a product's (pre-calculated) 97.5%-Value-at-Risk VaR given regular premium payments of 1.000 EUR on an annual basis.

For calculating the VEV,

- find the $BS(0, VEV)$-model such that $VaR_{Z_{VEV}} = VaR$
- with $VaR_{Z_{VEV}} = \exp\left(\mu_{W_{VEV}(T)} + z_\alpha \cdot \sigma_{W_{VEV}(T)}\right)$

where $\mu_{W_{VEV}(T)}$ and $\sigma_{W_{VEV}(T)}$ are derived as

$$\mu_{W_{VEV}(T)} = \ln(\mathbb{E}[\mathrm{W}_{\mathrm{VEV}}^2(T)]) - \ln(\mathbb{E}[W_{VEV}(T)]) - 1.5\sigma_{W_{VEV}(T)}^2$$
$$\sigma_{W_{VEV}(T)}^2 = \ln(\mathbb{E}[\mathrm{W}_{\mathrm{VEV}}^2(T)]) - 2 \cdot \ln(\mathbb{E}[W_{VEV}(T)])$$

Note, typically for deriving the VEV following approach 2, a numerical procedure, e.g. based on a bisection algorithm is necessary to find the required volatility.

Fig 4 depicts the results when approach 2 is applied. These results show that approach 2 generally underestimates the original volatility. Similar to the results based on approach 1 (cf. Fig 3) the estimation error increases with the considered recommended holding period T and with the original volatility σ, but performs slightly better than approach 1, especially for (true) volatilities larger than 5% and less than 15%. Whereas e.g. an original volatility of $\sigma = 10\%$ is underestimated by 0.75% for a recommended holding period of $T = 40$ years – hence VEV = 9.25% is obtained – an application of approach 2 delivers a VEV = 21.85% for an original $\sigma = 30\%$ and $T = 40$ years.

Summarizing, approach 2 delivers at least somehow appropriate results for short recommended holding periods (e.g. no longer than 10 years) and further assuming rather low volatilities. However, similar with the results in Section 3.3.1, an application of approach 2 to (rather long-term) insurance-based investment products may only deliver reasonable results when low volatilities are considered.

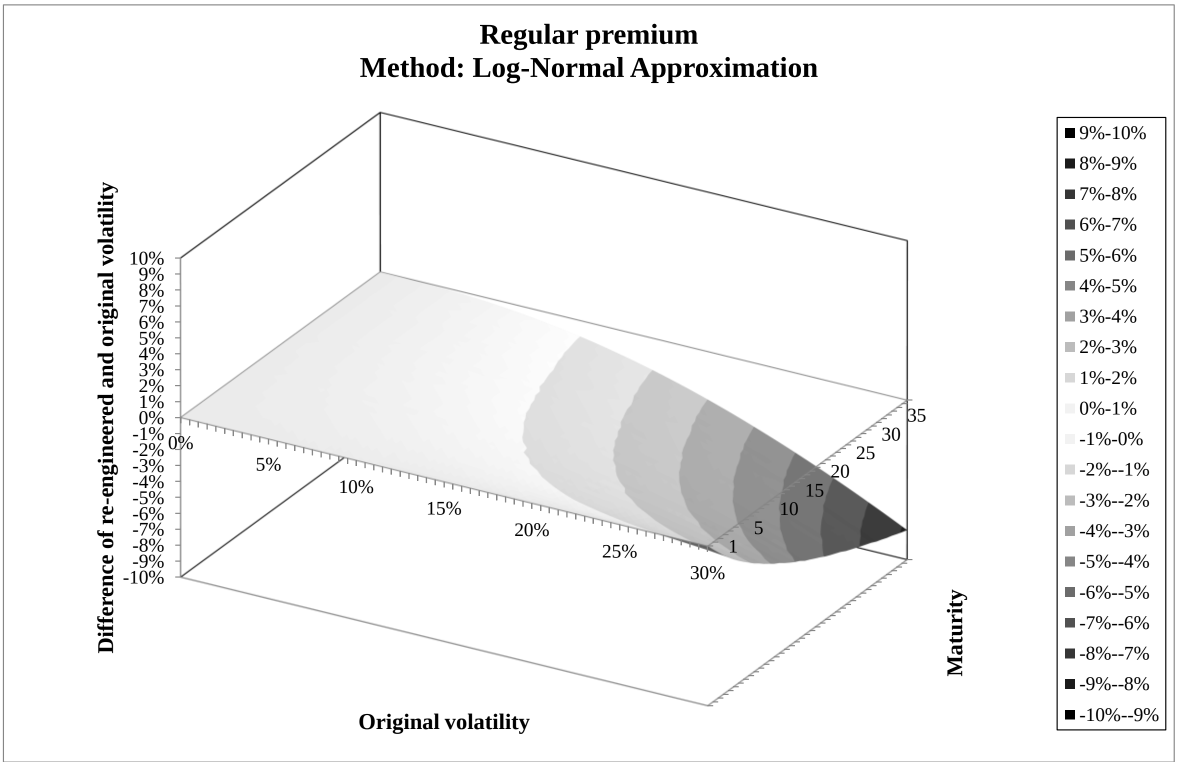

Fig 4 Regular premium: Difference of re-engineered and original volatility, Method: Log-Normal Approximation

3.3.3 Approach 3: Combination of approach 1 and approach 2

Sections 3.3.1 and 3.3.2 showed that the VEV derived by approach 1 and approach 2 generally over- respectively underestimates the volatility of the $BS(0, \sigma)$-model from which the 97.5%-Value-at-Risk was derived.

Hence, a natural (practitioner's) idea to obtain better estimates for the VEV is to combine approach 1 and 2 by simply averaging their results:

Approach 3: Combination of approach 1 and approach 2

Consider a product's (pre-calculated) 97.5%-Value-at-Risk VaR given regular premium payments of 1.000 EUR on an annual basis.

For deriving the VEV,

- compute VEV_1 by applying approach 1,
- compute VEV_2 by applying approach 2,

set $VEV = 0.5(VEV_1 + VEV_2)$.

Fig 5 depicts the results of applying approach 3. These results are (astonishingly) accurate, since in our calculation the maximum observed underestimation (resp. overestimation) of the original volatility was -0.35% (0.88%), observed for an original volatility $\sigma = 30\%$ and a recommended holding period of $T = 40$ and $T = 4$ respectively. In general, the applied methodology delivers very good results when VEV is derived for regular premium payments, especially compared to the methodologies analyzed so far (cf. Sections 3.3.1 and 3.3.2).

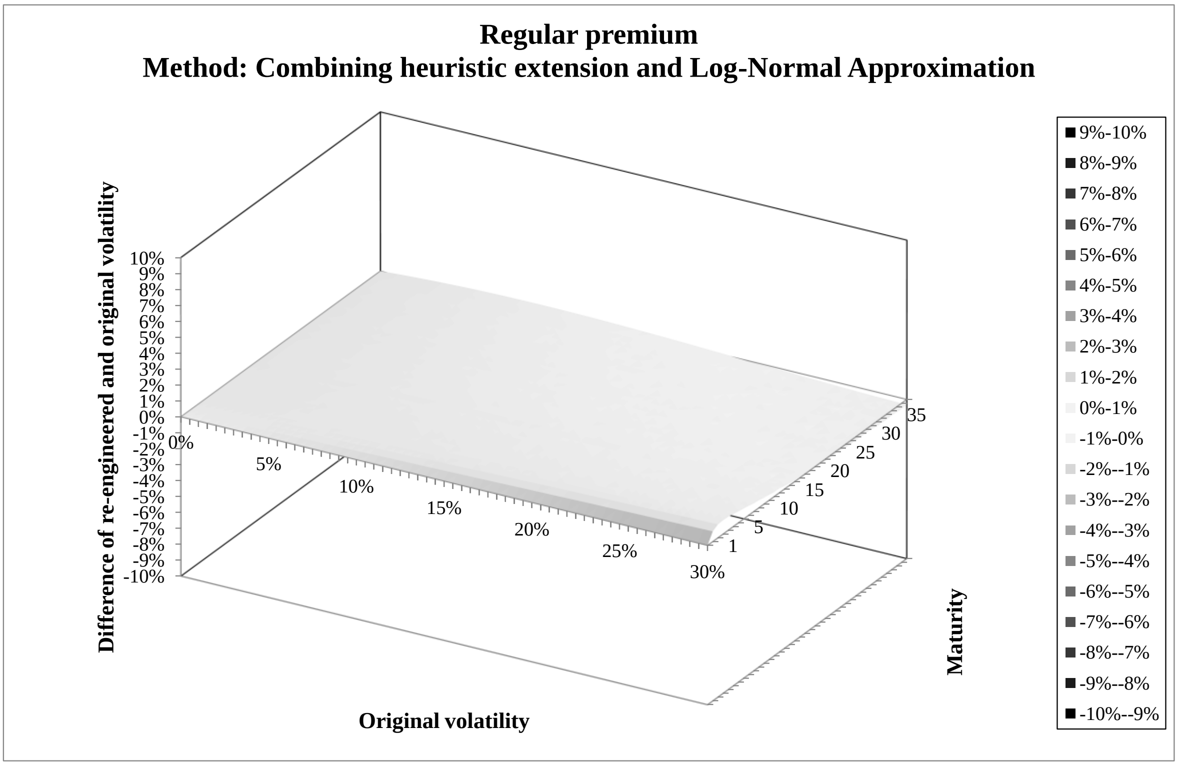

Fig 5 Regular premium: Difference of re-engineered and original volatility, Method: Combined approach

Fig 6 shows the same results when a different scaling of the y-axis is applied. Fig 6 shows that approach 3 is of course not perfect, however yields results that may be acceptable from a practitioner's point of view and that are superior to those currently derived by the industry (cf. Section 3.3.1 and approach 1).

Hence, in our view this approach yields accurate estimates of the product's riskiness (in terms of Value-at-Risk equivalent volatility) and could therefore be taken into account when the European Commission revises the PRIIP's directive and its regulatory technical standards at the end of 2018.

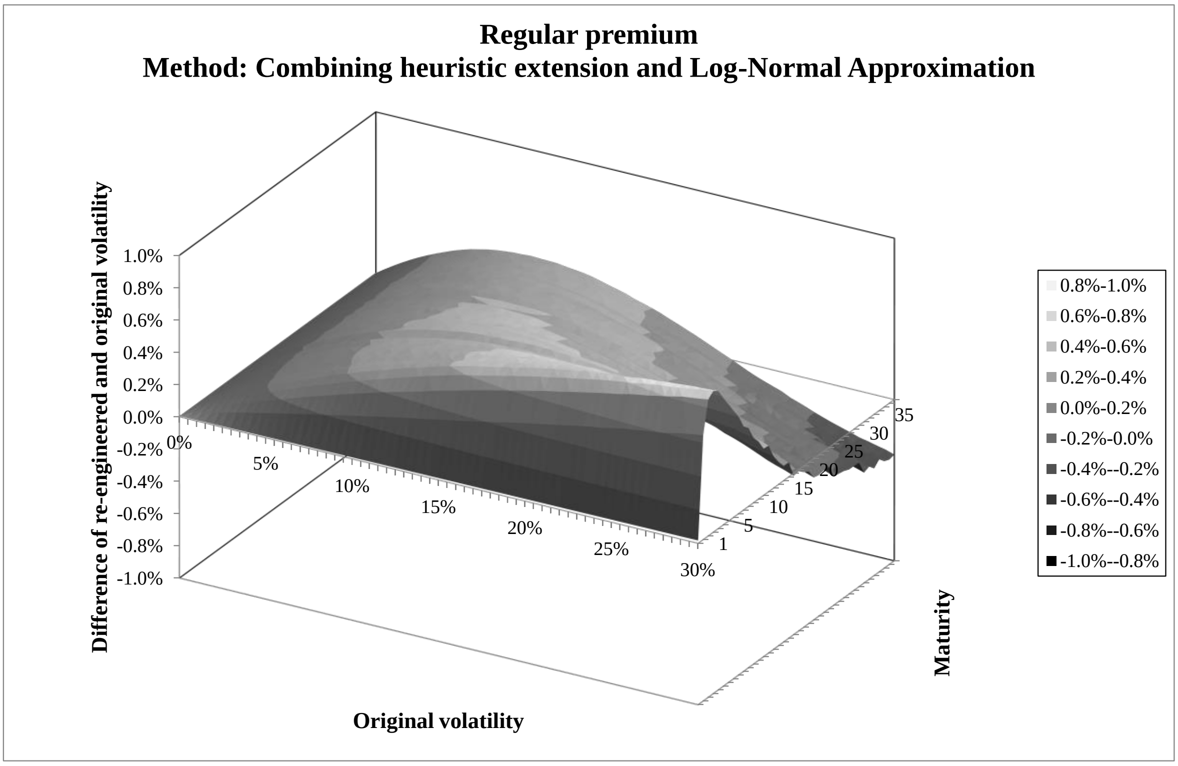

Fig 6 Regular premium: Difference of re-engineered and original volatility, Method: Combination of approach 1 and 2 (different scaling)

3.3.4 Approach 4: Tabulation of Value-at-Risk obtained by Monte-Carlo simulation for "critical" volatilities

Finally, since $W_\sigma(T)$ is not known in closed form, neither of the above approaches delivers the "exact" solution (just an appropriate one from a practitioner's point of view).

Therefore, to obtain the market risk measure (MRM) as a function of the VEV one could also tabulate the 2.5th-percentile of $W_\sigma(T)$ – estimated by means of Monte-Carlo simulation – only for the "critical" volatilities 0.5%, 5%, 12%, 20%, 30% and 80% which separate the different MRM classes $1-7$ (cf. Table 1) for some (long enough) recommended holding periods $T = 1, \ldots$. The product's MRM would then simply be obtained by looking up its pre-computed 2.5th-percentile within the "critical" percentiles. This approach would not need any further numerical algorithm after the critical percentiles had been obtained once. They could even be tabulated by the European Commission itself and then be provided within the revised regulatory technical standards.

4 Return: The performance scenarios

Besides indicating the potential riskiness of the considered products (cf. Section 3), European Commission (2017) also requires the disclosure of four different scenarios on the products' possible returns in terms of potential future benefits. These so-called performance scenarios read as *unfavorable*, *moderate*, *favorable* and an additional so-called *stress scenario*. In this context, the unfavorable scenario corresponds to the products' 10th-percentile, the moderate scenario corresponds to the products' 50th-percentile, the favorable scenario corresponds to the products' 90th-percentile and finally the stress scenario corresponds to the products' 5th-percentile under "stressed" assumptions (cf. European Commission (2017), Annex IV, points 5-8).[10]

The maturity benefit given these different scenarios has to be disclosed at the very end of the recommended holding period and at different intermediate time points as well. We now focus our analysis only on the end of the recommended holding period, but our findings similarly hold when intermediate time points are considered instead. Further, our analyses only treat products of category 2 following the categorizations by European Commission (2017) as indicated in Section 1.

4.1 Calculation recipe by European Commission (2017)

For products of category 2, European Commission (2017), Annex IV, points 9-11 proposes formulae for deriving the required performance scenarios based on a so-called Cornish-Fisher expansion (cf. Cornish and Fisher (1938)) of historically observed log-returns. For ease of notation, let X denote the random variable corresponding to these log-returns on an annual basis.[11] Further, let

- T denote the recommended holding period (in years),
- M_1 denote the log-return's expectation, i.e. $M_1 = \mathbb{E}[X]$

[10] When holding periods equal or less than one year are considered, the 1st-percentile is applied instead of the 5th-percentile.

[11] European Commission (2017) would estimate the following moments based on e.g. daily returns observed from the underlying time series over the last five years. Here, for the sake of simplicity, we assume an annual time scale for our derivations. Our conclusion naturally holds if different time scales are considered as well.

- σ denote the log-return's volatility, i.e. $\sigma = \sqrt{\mathbb{E}[(X - M_1)^2]}$
- μ_1 denote the log-return's skewness, i.e. $\mu_1 = \frac{\mathbb{E}[(X-M_1)^3]}{\sigma^3}$
- μ_2 denote the log-return's excess kurtosis, i.e. $\mu_2 = \frac{\mathbb{E}[(X-M_1)^4]}{\sigma^4} - 3$

Following European Commission (2017), the spot value corresponding to above log-returns considered for T years[12] under these different scenarios is then calculated as follows:

unfavorable scenario

$$e^{M_1 T + \sigma\sqrt{T}\left(-1.28 + 0.107\cdot\frac{\mu_1}{\sqrt{T}} + 0.0724\cdot\frac{\mu_2}{T} - 0{,}0611\cdot\frac{\mu_1^2}{T}\right) - 0.5\sigma^2 T}$$

moderate scenario

$$e^{M_1\cdot T - \frac{\sigma}{6}\cdot\mu_1 - 0.5\sigma^2\cdot T}$$

favorable scenario

$$e^{M_1 T + \sigma\sqrt{T}\left(1.28 + 0.107\cdot\frac{\mu_1}{\sqrt{T}} - 0.0724\cdot\frac{\mu_2}{T} + 0{,}0611\cdot\frac{\mu_1^2}{T}\right) - 0.5\sigma^2 T}$$

stress scenario

When the stress scenario is defined, in addition to considering a different percentile, the expected return is (presumably) further set to 0 and in addition a "stressed" volatility σ_{stress} is applied, Hence, European Commission (2017) proposes

$$e^{\sigma_{stress}\sqrt{T}\left(z_\alpha + \frac{(z_\alpha^2 - 1)}{6}\cdot\frac{\mu_1}{\sqrt{T}} + \frac{(z_\alpha^3 - 3z_\alpha)}{24}\cdot\frac{\mu_2}{T} - \frac{(2z_\alpha^3 - 5z_\alpha)}{36}\cdot\frac{\mu_1^2}{T}\right) - 0.5{\sigma_{stress}}^2 T}$$

where $z_\alpha = \Phi^{-1}(\alpha)$.

4.2 Issues

The following sections highlight some issues with above formulae and the underlying methodology.

[12] i.e. treating the random variable e^Y with $Y = \sum_{i=1}^{T} X_i$ where X_i are independent copies of X.

First, Section 4.2.1 shows that above formulae contain technical errors. These errors generally yield lower returns than those that would be obtained if the (presumed) idea behind the formulae was correctly applied. Further, with increasing volatility the impact of these errors increases.

Second, Section 4.2.2 concludes that the proposed stress-scenario may under certain circumstances produce better results than the unfavorable (or even favorable) performance scenario which may be hard to explain to the retail investor. Further, since the stress scenario does not account for any charges of the underlying assets – since the expected return is set to 0 – there would be absolutely no impact on the stress-scenario's result if a fund manager arbitrarily increased management fees of the underlying investment vehicle. These probably unintended effects might undermine customer's confidence in the information provided by the PRIIP-KID and the document might be perceived inappropriate and not useful.

4.2.1 Technical errors

In our view, the formulae for deriving the different scenarios given by European Commission (2017) are based on the so-called Cornish-Fisher expansion introduced by Cornish and Fisher (1938). They provide an expansion for the α^{th}-percentile $x_\alpha^{0,1}$ of a standardized (i.e. mean of 0 and standard deviation of 1) random variable $X_{0,1}$ by

$$x_\alpha^{0,1} = z_\alpha + \frac{(z_\alpha^2 - 1)}{6} \cdot \mu_1^{0,1} + \frac{(z_\alpha^3 - 3z_\alpha)}{24} \cdot \mu_2^{0,1} - \frac{(2z_\alpha^3 - 5z_\alpha)}{36} \cdot \left(\mu_1^{0,1}\right)^2$$

with $\mu_1^{0,1} = \mathbb{E}\left[X_{0,1}^3\right]$ and $\mu_2^{0,1} = \mathbb{E}\left[X_{0,1}^4\right] - 3$.

Now, let $Y := \sum_{i=1}^T X_i$ denote the sum of T independent copies of X, i.e. the probability distribution of the total log-return after investing for T years in an investment vehicle with underlying annual log-returns X.

Then, we get the expectation of Y as $\mathbb{E}[Y] = M_1 T$ and its standard deviation as $\sigma_Y = \sigma\sqrt{T}$. In addition, the skewness of Y is calculated as follows[13]:

[13] Mixed terms such as $\mathbb{E}\left[(X_i - \mathbb{E}[X_i])^2 \cdot \left(X_j - \mathbb{E}[X_j]\right)\right], i \neq j$ vanish due to the independence of X_i and X_j.

$$\mathbb{E}\left[\left(\frac{Y-\mathbb{E}[Y]}{\sigma_Y}\right)^3\right] = \frac{1}{\sigma_Y^3}\sum_{i=1}^{T}\sum_{j=1}^{T}\sum_{k=1}^{T}\mathbb{E}\big[(X_i-\mathbb{E}[X_i])\cdot\big(X_j-\mathbb{E}\big[X_j\big]\big)\cdot(X_k-\mathbb{E}[X_k])\big]$$
$$= \frac{1}{\sigma_Y^3}\sum_{i=1}^{T}\mathbb{E}[(X_i-\mathbb{E}[X_i])^3]$$

and finally

$$\mathbb{E}\left[\left(\frac{Y-\mathbb{E}[Y]}{\sigma_Y}\right)^3\right] = \frac{T}{\sigma_Y^3}\cdot\mu_1\cdot\sigma^3 = \frac{T}{\left(\sigma\cdot\sqrt{T}\right)^3}\cdot\mu_1\cdot\sigma^3 = \frac{\mu_1}{\sqrt{T}}$$

Further, the excess kurtosis of Y is derived as follows:

$$\mathbb{E}\left[\left(\frac{Y-\mathbb{E}[Y]}{\sigma_Y}\right)^4\right]-3$$
$$= \frac{1}{\sigma_Y^4}\sum_{i=1}^{T}\sum_{j=1}^{T}\sum_{k=1}^{T}\sum_{l=1}^{T}\mathbb{E}\big[(X_i-\mathbb{E}[X_i])\cdot\big(X_j-\mathbb{E}\big[X_j\big]\big)\cdot(X_k-\mathbb{E}[X_k])$$
$$\cdot(X_l-\mathbb{E}[X_l])\big]-3$$
$$= \frac{1}{\sigma_Y^4}\left(\sum_{i=1}^{T}\mathbb{E}[(X_i-\mathbb{E}[X_i])^4]\right.$$
$$\left.+6\sum_{i\neq j}\mathbb{E}\left[(X_i-\mathbb{E}[X_i])^2\cdot\big(X_j-\mathbb{E}\big[X_j\big]\big)^2\right]\right)-3$$
$$= \frac{1}{\sigma^4\cdot T^2}\left(T\cdot(\mu_2+3)\sigma^4+6\sum_{i\neq j}\sigma^4\right)-3$$
$$= \frac{\mu_2+3}{T}+\frac{6\frac{T(T-1)}{2}}{\sigma^4\cdot T^2}\sigma^4-3 = \frac{\mu_2}{T}+\frac{3}{T}+3-\frac{3}{T}-3 = \frac{\mu_2}{T}$$

Finally, by applying the Cornish-Fisher-Expansion on the standardized random variable $\frac{Y-\mathbb{E}[Y]}{\sigma_Y}$ we get the α^{th}-percentile of Y as

$$\alpha = \mathbb{P}(Y\leq x_\alpha)$$
$$= \mathbb{P}\left(\frac{Y-\mathbb{E}[Y]}{\sigma_Y}\leq\frac{x_\alpha-\mathbb{E}[Y]}{\sigma_Y}\right)$$

$$\Leftrightarrow \frac{x_\alpha - \mathbb{E}[Y]}{\sigma_Y} = x_\alpha^{0,1}$$

$$\Leftrightarrow x_\alpha = \mathbb{E}[Y] + x_\alpha^{0,1} \cdot \sigma_Y = M_1 T + \ x_\alpha^{0,1} \cdot \sigma\sqrt{T}$$

$x_\alpha^{0,1}$ can be calculated applying the Cornish-Fisher expansion and using the above derivation of skewness and excess kurtosis of Y as

$$x_\alpha^{0,1} = z_\alpha + \frac{(z_\alpha^2 - 1)}{6} \cdot \frac{\mu_1}{\sqrt{T}} + \frac{(z_\alpha^3 - 3z_\alpha)}{24} \cdot \frac{\mu_2}{T} - \frac{(2z_\alpha^3 - 5z_\alpha)}{36} \cdot \frac{\mu_1^2}{T}$$

Transformed to the corresponding spot value, we get the respective α^{th}-percentile as $e^{M_1 T + x_\alpha^{0,1} \cdot \sigma\sqrt{T}}$. This yields the following values for the different percentiles:

unfavorable scenario

Applying $\alpha = 10\%$, instead of $e^{M_1 T + \sigma\sqrt{T}\left(-1.28 + 0.107 \cdot \frac{\mu_1}{\sqrt{T}} + 0.0724 \cdot \frac{\mu_2}{T} - 0{,}0611 \cdot \frac{\mu_1^2}{T}\right) - 0.5\sigma^2 T}$ we get

$$e^{M_1 T + \sigma\sqrt{T}\left(-1.28 + 0.107 \cdot \frac{\mu_1}{\sqrt{T}} + 0.0724 \cdot \frac{\mu_2}{T} - 0{,}0611 \cdot \frac{\mu_1^2}{T}\right)}$$

moderate scenario

Applying $\alpha = 50\%$, instead of $e^{M_1 T \, - \frac{\sigma}{6} \cdot \mu_1 - \, 0.5\sigma^2 T}$ we get

$$e^{M_1 T \, - \frac{\sigma}{6} \cdot \mu_1}$$

favorable scenario

Applying $\alpha = 90\%$, instead of $e^{M_1 T + \sigma\sqrt{T}\left(1.28 + 0.107 \cdot \frac{\mu_1}{\sqrt{T}} - 0.0724 \cdot \frac{\mu_2}{T} + 0{,}0611 \cdot \frac{\mu_1^2}{T}\right) - 0.5\sigma^2 T}$ we get

$$e^{M_1 T + \sigma\sqrt{T}\left(1.28 + 0.107 \cdot \frac{\mu_1}{\sqrt{T}} - 0.0724 \cdot \frac{\mu_2}{T} + 0{,}0611 \cdot \frac{\mu_1^2}{T}\right)}$$

stress scenario

If the underlying idea behind the current specification of the stress scenario actually is to set the expected log-return $M_1 = 0$, then the stress scenario should instead of $e^{\sigma_{stress}\sqrt{T}\left(z_\alpha+\frac{(z_\alpha^2-1)}{6}\frac{\mu_1}{\sqrt{T}}+\frac{(z_\alpha^3-3z_\alpha)}{24}\frac{\mu_2}{T}-\frac{(2z_\alpha^3-5z_\alpha)}{36}\frac{\mu_1^2}{T}\right)-0.5\sigma_{stress}{}^2T}$ be computed as[14]

$$e^{\sigma_{stress}\sqrt{T}\left(z_\alpha+\frac{(z_\alpha^2-1)}{6}\frac{\mu_1}{\sqrt{T}}+\frac{(z_\alpha^3-3z_\alpha)}{24}\frac{\mu_2}{T}-\frac{(2z_\alpha^3-5z_\alpha)}{36}\frac{\mu_1^2}{T}\right)}$$

Consequences

The difference in annualized log-returns of the performance of the correct application of the Cornish-Fisher expansion and the specification by European Commission (2017) is given as $0.5\sigma^2$. The performance scenarios as derived by European Commission (2017) are therefore generally too low. The error increases with increasing volatility such that the performance scenarios as required by European Commission (2017) underestimate the "true" values – i.e. correctly applied Cornish-Fisher expansions – by e.g. 4.5% p.a. when the current volatility was 30% p.a.

Fig 7 depicts this error's impact on the projected performance scenarios as a function of volatility.

[14] The stress scenario's current specification is not motivated by European Commission (2017). Hence, we can only speculate what the actual idea behind the current specification was and correct the potentially similar technical error as present in the other performance scenarios accordingly.

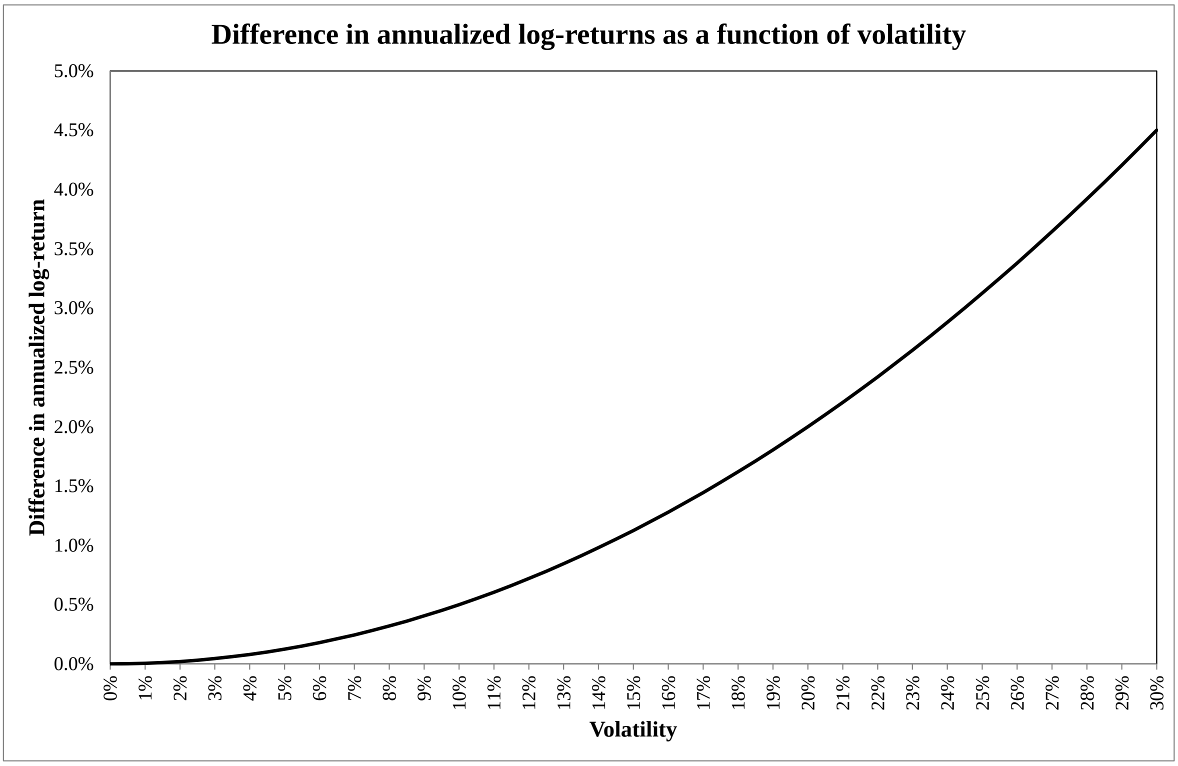

Fig 7 Difference in annualized log-returns of correct calculations and specifications by European Commission (2017) as a function of volatility

4.2.2 Stress scenario can outperform the other performance scenarios

Following European Commission (2017), Annex IV, point 2 the "*stress scenario shall set out significant unfavorable impacts of the product not covered in the unfavorable scenario*". However, this section shows that due to the methodological differences of the stress scenario's and the other performance scenarios' specification, there are situations where the stress scenario can actually yield a higher return than the other performance scenarios.

Consider a $BS(\mu, \sigma)$ −model and (roughly) assume $\sigma_{Stress} = 1.5\sigma$. Following Section 4.2.1, the spot values e.g. in the unfavorable scenario and in the stress scenario are then derived as $e^{(\mu-0.5\sigma^2)T+z_{10\%}\cdot\sigma\sqrt{T}}$ and $e^{z_{5\%}\cdot\sigma_{Stress}\sqrt{T}}$, respectively.[15] Simple algebra then e.g.

[15] The following derivations are performed assuming the performance scenarios' specification stated in Section 4.2.1 instead of the original (potentially flawed) requirements of European Commission (2017). However, the same logic still holds when the current requirement for the stress scenario was considered.

yields that for any $\mu < 0.5\sigma^2 + \frac{\sigma}{\sqrt{T}}(1.5z_{5\%} - z_{10\%})$ the resulting stress scenario outperforms the unfavorable scenario.

Fig 8 shows the necessary μ such that the stress scenario delivers higher returns than the other performance scenarios as a function of volatility σ and assuming a recommended holding period of $T = 40$ years. For example, considering a volatility of 10% p.a., a negative drift of $\mu = -1.37\%$ p.a. is (already) sufficient such that the stress scenario outperforms the unfavorable performance scenario. Assuming the same setting, a drift of $\mu = -3.4\%$ yields a higher return of the stress scenario than the moderate scenario.

In a real-world application – following the requirements by European Commission (2017) – the drift has to be estimated from historical data over a time horizon of the last five years. Hence, an estimate of $\mu = -1.37\%$ is likely to be observed for at least some funds at some point in time. In this setting the moderate scenario will then likely also yield a negative return which might indeed be hard to explain to the retail investor. However, it is not in the scope of this paper to discuss the (albeit very interesting) issue of the currently proposed calibration procedure by European Commission (2017) based on the last five years. Instead we are keen to stress that although the "*stress scenario shall set out significant unfavorable impacts of the product not covered in the unfavorable scenario*", there are situations where it actually delivers a higher return than the unfavorable scenario which clearly seems misleading.

This methodological error should therefore be overcome in a future review of the PRIIP-regulation by adapting the stress scenario's and the other performance scenarios' calculation methodology accordingly.

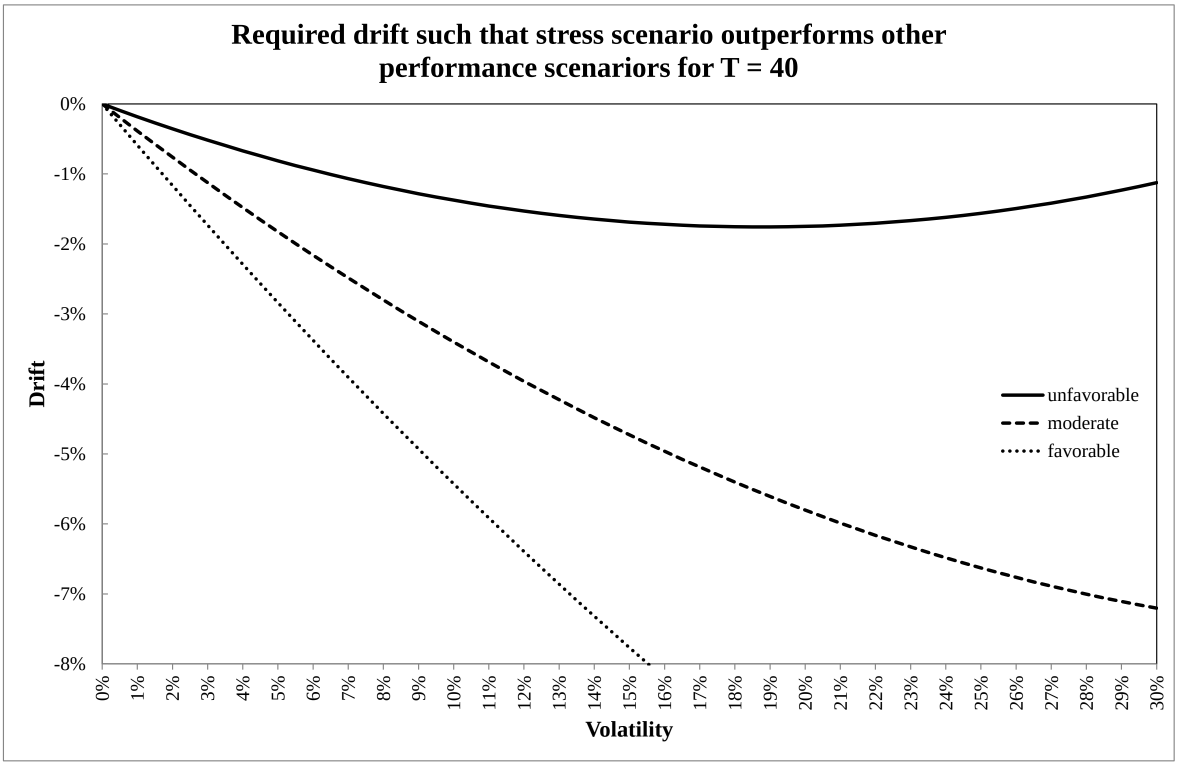

Fig 8 Required drift such that stress scenario outperforms other performance scenarios as a function of volatility and assuming $T = 40$

5 Conclusion

This paper critically assesses the methodology of deriving the market risk measure (MRM) by the so-called Value-at-Risk equivalent volatility (VEV) and the derivation of performance scenarios as required by European Commission (2017) in the context of the PRIIP-regulation (cf. European Commission (2014)).

First, we have derived the theoretical reasoning which appears to underlie the proposed calculation of the VEV. We have shown that this derivation is only valid when products with single premium investments are considered. Since for insurance-based investment products, the product information also has to be disclosed assuming regular premium investments, this formula is not directly applicable in this case. Hence, the industry to our knowledge currently uses an approach for regular premium payments which is closely aligned with the requirements given by the European Commission which are however based on a single premium payment. Based on Monte-Carlo simulations, we have shown that this approach significantly overestimates the "true" VEV when regular premium payments are considered instead. This effect increases when (true) volatilities of the underlying assets and/or the product's maturity/recommended holding period increase. Therefore, we proposed and analyzed approaches which are still numerically tractable and provide a far better approximation for the true VEV when regular premium payments are considered.

Second, we have analyzed the requirements for calculating the so-called performance scenarios for products of so-called category 2 as stated by European Commission (2017). Not further taking into account the possible issues of the required methodology itself – i.e. projecting observed past returns into the future – we have shown that the proposed formulae contain technical errors which in general yield to a systematic underestimation of the products' performance potential. This underestimation increases as the volatility of the considered assets increases.

Summarizing, the current methodology stated by European Commission (2017) yields to a systematic overestimation of risk when regular premium payments are assumed combined with a systematic underestimation of possible performance potential. Both effects are the more pronounced the higher the volatilities of the underlying assets are.

This paper has proposed solutions to both issues considered which may be taken into account during the review of the PRIIP-regulation at the end of 2018.

Further research could additionally focus on a critical assessment of the general methodology of the VEV derivation currently applied and could then analyze if the calculation of a 97.5%-Value-at-Risk assuming a "0-drift", resp. risk-neutral world, is economically meaningful at all. In addition, the currently specified methodology on deriving the performance scenarios is also not valid for a direct application to regular premium payments and could hence be subject to further research as well.

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