PRIIP-KID: Appearances are deceiving or why to expect the unexpected in a generic KID for multiple option products

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Abstract

Providers of so-called packaged-retail and insurance-based investment products ("PRIIPs") in the European Union have to draw up a standardized key information document ("KID") since 1st of January 2018. In addition to some standard information on the product and its provider, this key information document discloses the product's riskiness by means of a summary risk indicator, its performance potential by means of performance scenarios and its costs by means of a detailed disclosure of charges including a summary cost indicator. For products with multiple investment options ("MOPs"), the regulation requires product providers to disclose a range for the summary cost indicator for the available investment options in the product. For feasibility, in practice often only synthetic investment options instead of the whole available investment portfolio are considered to derive an estimate for the required range. Based on a standard approach used in the German and Austrian insurance market, we analyze those investment options which actually yield the lower and especially upper bound of the required range. By considering different unit-linked products with investment guarantees our results show that those synthetic investment options typically used in practice may provide a false estimation of the range and, in particular, significantly underestimate the real upper bound. Further, we provide guidance how the "worst-case" combination of the investment option's volatility and its charges can be found such that the summary cost indicator is maximized for the different products considered here.

Keywords: PRIIP-KID; insurance-based investment product; multiple option product; option-based-portfolio insurance (OBPI); constant-proportion-portfolio insurance (CPPI)

1 Introduction

Since 1st of January 2018, providers of packaged retail and insurance-based investment products (so-called "PRIIPs") have to provide a key information document (so-called "KID") following regulation EU 1286/2014 issued by the European Commission (cf. European Commission, 2014). PRIIPs contain pretty much any (packaged) investment product sold by insurance companies, banks and asset managers within the European Union and hence this regulation practically affects the whole Pan-European financial market. Only old age provision products offered within occupational pensions or state subsidized pension products are exempt from this regulation.

The PRIIP regulation was initiated in the aftermath of the financial crisis in 2008 where retail investors suffered tremendous losses. Citing the recitals of European Commission (2014) highlights the motivation of the European legislator to protect the retail investor from similar (unforeseen) losses in the future by introducing common pre-contractual disclosure requirements on the considered products, cf.:

(1) Retail investors are increasingly offered a wide variety of packaged retail and insurance-based investment products (PRIIPs) when they consider making an investment. Some of these products provide specific investment solutions tailored to the needs of retail investors, are frequently combined with insurance coverage or can be complex and difficult to understand. Existing disclosures to retail investors for such PRIIPs are uncoordinated and often do not help retail investors to compare different products, or understand their features. Consequently, retail investors have often made investments without understanding the associated risks and costs and have, on occasion, suffered unforeseen losses.

(2) *Improving the transparency of PRIIPs offered to retail investors is an important investor protection measure and a precondition for rebuilding the confidence of retail investors in the financial market, in particular in the aftermath of the financial crisis.* […]

Therefore, a key information document has to be provided to the customer in "good time" before the actual purchase of the considered product and contains, among others, an indication of the products'

- risk by means of a description of the products' market risk and the provider's default risk, including the calculation of a "summary risk indicator",
- return by means of "performance scenarios",

- costs by means of a detailed disclosure of charges including a "summary cost indicator".

For deriving the required figures on risk, return and costs, the European Commission issued additional regulatory technical standards (so-called "RTS") by European Commission (2017). The key information document has to be produced assuming a retail investor either to invest a single premium or in addition – when insurance-based investment products are considered – a regular premium payment instead of the single premium investment. Further, some "recommended holding period" / maturity of T years has to be specified for the calculations by the product provider. European Commission (2017) then assigns each product subject to the PRIIP-regulation to one of four different "product categories" and specifies different relevant methodologies to perform the required calculations (cf. Graf, 2019 for further details).

The four product categories can be briefly summarized as follows:

- *Category 1* comprises products where retail investors may lose more than their invested premiums, derivative-like products such as futures, options, swaps, etc. and products whose prices are only determined on a less than monthly basis.
- *Category 2* covers products that provide a "linear" non-structured exposure to their underlying investments. Generally most of (non-structured) investment funds, such as equity, fixed income or balanced funds will therefore qualify as products of category 2.
- *Category 3* in contrast covers products that offer "non-linear" structured exposure to their underlying investments. E.g. guarantee funds managed according to some portfolio insurance technique and hence typically providing path-dependent (non-linear) exposure to their underlying investments qualify for category 3.
- Finally, *category 4* covers all products whose "*values depend in part on factors not observed in the market*" (cf. European Commission, 2017) and especially includes insurance-based investment products that are equipped with some profit participation which is generally not directly observed in the market.

The summary risk indicator is given by a number between 1 (low risk) and 7 (high risk). In order to determine the summary risk indicator, the product's market risk (by a Value-at-Risk approach) as well as the provider's credit risk (taking into account external credit ratings) are assessed. The performance scenarios require disclosure of the product's potential benefits provided the so-called "stress, unfavorable, moderate and favorable" scenario. These scenarios are derived from different percentiles of the product's probability distribution of maturity benefits. Graf (2019) critically treats the current requirements for the derivation of the summary risk indicator for regular premium payments and the specifications of the performance scenarios for products of category 2.

This paper addresses the disclosure of costs, especially taking products with multiple investment options (and hence different charges and potentially different underlying asset returns) into account and focuses on products of category 4.

European Commission (2017) describes how the key information document for products equipped with multiple investment options (so-called MOPs – multiple option products) shall be produced. The regulator requires to either draw up a separate key information document for each possible investment option (cf. §10a of European Commission (2017), in what follows called "10a-approach") or to produce a generic key information document describing the features of the considered product and additionally providing separate quantitative information on the available investment options (cf. §10b of European Commission (2017), in what follows called "10b-approach"). Following the 10b-approach, the generic key information document has to provide a quantitative assessment of the range of risk indicators and charges for the considered investment options, but does not require quantitatively disclosing related performance scenarios. Hence, for the generic KID it would in general be sufficient to analyze the "least and most risky" and the "least and most expensive" investment option within the product. The question how to identify these peak value funds is not at all trivial as will be shown in this paper.

Whereas European Commission (2017) provides concrete methodological requirements for the calculations when products of category 2 and 3 are considered, a "*robust and well recognized industry and regulatory standard*" shall be applied for insurance-based investment products of category 4 instead. Based on such an industry standard developed in the German and Austrian insurance market, we will show that the actual definition of the least and most expensive investment option in terms of the required disclosure of charges is not straight forward and hence typical rule-of-thumb approximations may yield to an inappropriate disclosure in the generic KID.[1] We will further analyze if and how these investment options may differ when different products are considered.

The remainder of this paper is organized as follows: Section 2 describes the content of the key information document in more detail and especially shows how costs shall be disclosed therein. Section 3 introduces the different products considered in our analyses.

[1] Note, in an ideal world, product providers would compute the required numbers for each available investment option and then derive the corresponding ranges from that. However, in practice often (due to the required simulations for the products considered) these derivations are only performed for some limited number of "synthetic" funds assumed to yield the "least and most expensive" investment option.

We focus on unit-linked products with and without investment guarantee and especially investigate the impact of different portfolio insurance strategies such as option-based-portfolio insurance (OBPI) and constant-proportion-portfolio insurance (CPPI). Section 4 describes the modelling approach that is applied in the German and Austrian market to cover PRIIPs of category 4 – including the financial model used in the analyses. Section 5 summarizes the parameters applied in our numerical analyses whereas Sections 6 and 7 state the main results of our paper by deriving analytical solutions for the considered products, in particular showing that seemingly obvious most expensive investment options can be far from the real peak values. Finally, Section 8 concludes.

2 Content of the KID

This section introduces the requirements for drawing up the generic key information document for a product with multiple investment options when the KID is produced in line with the already mentioned 10b-approach. In addition to narrative explanations on the product provider and the product's specifics, European Commission (2017) requires drawing up a range of the summary risk indicator and the summary cost indicator which cover the available investment options if they were considered in the respective product. In contrast to the 10a-approach, the quantitative disclosure of performance scenarios is not required within the generic KID in the 10b-approach. Here, only some narrative explanation on how the performance may differ for different underlying investment options is mandatory, but no calculations have to be performed.

The summary risk indicator is based on the Value-at-Risk of the underlying product and the credit rating of the product provider and then yields a number between 1 and 7.[2] For products with an investment guarantee, European Commission (2017) allows deriving the summary risk indicator based on the issued guarantee and hence no calculation of the actual Value-at-Risk is then necessary. For products without any investment guarantee, the summary risk indicators of the product are likely to coincide with those of the underlying investment options and hence no additional calculation for the generic key information document will be necessary.

Therefore, regarding the generic key information document, calculations of the product for the available investment options are often only necessary for the disclosure of charges by the summary cost indicator. This summary cost indicator is defined as the so-called reduction in yield (RIY) of all charges in the product assuming that the product is held

[2] Graf (2019) treats the derivation of the summary risk indicator, especially for regular premium products, in more detail.

until the end of the recommended holding period (cf. European Commission (2017), Annex VI, point 61). The reduction in yield is the difference of two yields, one of a so-called cost free scenario and the gross return of the product after considering all charges. Technical details and a formal definition of the reduction in yield follow in Section 4.3. In addition to the reduction in yield, a disclosure of the total amount of charges (in EUR) is also mandatory (cf. European Commission (2017), Annex VI, point 62).[3] Following European Commission (2017) the calculations of the reduction in yield and the total amount of charges have to be performed given the moderate performance scenario which corresponds to the product's 50^{th}-percentile of potential benefits. Assuming this scenario, the reduction in yield gives the impact of charges on the product's yield as a per annum figure whereas the total amount of charges depicts the cumulated sum of charges in EUR that occurred during the life of the product in this scenario (cf. Section 4.3 for more details on the actual derivations performed within the considered industry standard).

Therefore, the actual reduction in yield and the total amount of charges not only depend on the investment option's charges but also on the product's 50^{th}-percentile. This is obviously different for different investment options. Hence, it is ad hoc not clear which investment option actually yields to the lowest and especially highest reduction in yield or total amount of charges. In an ideal world, reduction in yield and total amount of charges would be calculated for any available investment option and according to these results the range of least and most expensive investment option would just be recognized accordingly. However, this approach may be unfeasible e.g. when a large number of investment options is available and when Monte-Carlo-simulations have to be performed to derive the required moderate performance scenarios. Therefore, in practice often rule-of-thumb approximations are considered by setting up synthetic investment options (e.g. with lowest/highest volatility and lowest/highest charges of the available investment options), and then deriving reduction in yield and total amount of charges for these funds and the resulting ranges accordingly.

Our analyses in Sections 6 and 7 will however show that these synthetic investment options will in general yield a wrong assessment of the possible range of costs and may especially underestimate the most expensive investment option tremendously. Moreover, we will show that for different products such as OBPI or CPPI (cf. Sections 3, 6 and 7), different combinations of volatility and charges will actually yield the most expensive investment option within the product.

[3] The reduction in yield and the amount of charges shall be disclosed assuming an investment horizon equal to the recommended holding period. In addition, the cost figures shall also be disclosed after a period of one year and at halftime of the recommended holding period.

3 Considered products

This section introduces the considered products on a high-level basis to motivate the financial modelling and to introduce some nomenclature applied in the industry standard described in Section 4. Details and further results of the considered products are then provided in Sections 6 and 7, respectively. In our analyses, we consider two basic versions of guarantee products with different investment options (here: equity funds with different volatility) assuming a single premium payment P and a recommended holding period / maturity of T years. We distinguish products based on option-based-portfolio insurance (OBPI) and constant-proportion-portfolio insurance (CPPI) for some specified investment guarantee G_T at maturity. In addition, we analyze a pure unit-linked product without any guarantee. This can be understood as a special case of the option-based insurance product with a guarantee of zero.

We consider premium proportional charges βP which reduce the amount invested to $(1-\beta)P$. In addition, account proportional charges c – quoted as an annual fee – are deducted on a continuous basis from the client's account. Further additional charges c_A occur for the part of the client's account which is actually invested in the considered equity fund. Hence, for this part the account proportional charges increase to $(c+c_A)$.

For some equity fund with spot-value $A(t)$ at time t, charges β, c and c_A the different product mechanisms (CPPI and OBPI) on a high-level basis then read as follows:

Constant-proportion portfolio insurance (CPPI)

The CPPI product consists of an investment in two assets, a riskless asset and a risky financial instrument which is given by the equity fund $A(t)$. The insurance company's general assets deliver a (technical) guaranteed rate of return r_g and therefore serve as the riskless asset within the product. Hence, in order to ensure the guarantee G_T, the product's investment is being rebalanced on a continuous basis between the riskless and risky asset taking charges β, c and c_A into account.

Option-based portfolio insurance (OBPI)

For this product, the contract consists of two components, an account value and an additional put option (which is not part of the account value). Hence, the development of the client's account value – i.e. the development of the fund investment after deduction of charges without guarantee – is given by

$$(1-\beta)P \cdot \frac{A(t)}{A(0)} \cdot \exp(-(c+c_A)t), 0 < t < T.$$

At maturity $t = T$, an investment guarantee G_T on the investment is provided by the additional put option that covers for losses of the account value at maturity. Thus, the payoff of the contract at maturity is given by

$$\max\left(G_T, (1-\beta)P \cdot \frac{A(T)}{A(0)} \cdot \exp(-(c + c_A)T)\right).$$

Note, in this paper we are neither concerned with the appropriate price (e.g. its "fair" value) of the option, nor the underlying hedge portfolio from a product provider's point of view. We just assume that for given charges c and c_A as well as an investment in the underlying fund $A(t)$, the product provider at least provides the guarantee G_T at maturity.

4 An industry standard for products of category 4

This section describes the industry standard applied in the German and Austrian German insurance market for products of category 4 as e.g. introduced by DAV (2018) or AVÖ (2018).[4] This industry standard was proposed by the German Actuarial Association (DAV) and builds on an already previously existing methodology for calculating so-called risk-return classes for state-subsidized old age provision products in Germany (cf. Produktinformationstelle Altersvorsorge, 2017) performing Monte-Carlo-simulations of the considered products.[5] It therefore constitutes a "*robust and well recognized industry and regulatory standard*" for a treatment of category 4 products in respect with the requirements from European Commission (2017).

Next, Section 4.1 introduces the underlying stochastic capital market model for equities and interest rates applied in this industry standard. Based on this stochastic model, Section 4.2 depicts the modelling of equity funds. Note, this industry standard further develops the modelling of additional asset classes – such as the insurance company's general assets – which are however not in the scope of this paper and therefore not addressed here. Section 4.3 shows how – given this modelling framework – the cost disclosure by means of the reduction in yield and the total amount of charges for different products shall be derived.

[4] The industry standard proposed by DAV (2018) was also presented by Niemeyer and Rieck (2018) at the International Conference of Actuaries (ICA) in Berlin, 2018.

[5] In Germany, for a product to qualify as a state-subsidized product it has to be certified in advance. Part of this certification process is the derivation of a risk-return class. This derivation is done by an organization called "Produktinformationsstelle Altersvorsorge".

4.1 Financial model

Equity returns are modelled applying a generalized Geometric Brownian Motion and interest rates are assumed to follow an Additive Two-Factor Gaussian model ("G2++") or Two-Factor-Hull-White model, respectively. We start with an introduction of the interest rate model and first assume the existence of a risk-neutral probability measure $\mathbb{Q}$. Since our analyses will however need the specification of the objective ("real-world") measure $\mathbb{P}$, a change of measure will be performed by introducing corresponding risk premiums. Finally, we introduce the applied equity model.[6]

The G2++ model first assumes the short rate $r_{\mathbb{Q}}(t)$ to be driven by two potentially correlated stochastic factors $x_{\mathbb{Q}}(t), y_{\mathbb{Q}}(t)$. By no-arbitrage arguments the price $P(t,T)$ of a zero-coupon bond at time t with time-to-maturity $(T-t)$ can be derived as a function of the underlying factors.[7]

The factors' dynamics are given by

$$dx_{\mathbb{Q}}(t) = -ax_{\mathbb{Q}}(t)dt + \sigma dW_t^{x_{\mathbb{Q}}},\ \ x_{\mathbb{Q}}(0) = 0$$

$$dy_{\mathbb{Q}}(t) = -by_{\mathbb{Q}}(t)dt + \eta dW_t^{y_{\mathbb{Q}}},\ \ y_{\mathbb{Q}}(0) = 0$$

with $(W_t)^{x_{\mathbb{Q}}}, (W_t)^{y_{\mathbb{Q}}}$ being correlated Wiener processes under $\mathbb{Q}$ with instantaneous correlation ρ, i. e. $dW_t^{x_{\mathbb{Q}}} dW_t^{y_{\mathbb{Q}}} = \rho dt$.

The short rate $r_{\mathbb{Q}}(t)$ is then specified as

$$r_{\mathbb{Q}}(t) = x_{\mathbb{Q}}(t) + y_{\mathbb{Q}}(t) + \psi(t)$$

with a deterministic function $\psi(t)$ defined as

$$\psi(t) = f^M(0,t) + \frac{\sigma^2}{2a^2}(1-e^{-at})^2 + \frac{\eta^2}{2b^2}(1-e^{-bt})^2 + \rho\frac{\sigma\eta}{ab}(1-e^{-at})(1-e^{-bt})$$

where $f^M(0,t) = -\frac{\partial \ln\left(P^M(0,t)\right)}{\partial t}$ equals the instantaneous forward rate extracted from the initial yield curve implied by the capital market's zero-bond prices $P^M(0,t), \forall t$. This deterministic function ensures that obtained model prices $P(0,t)$ coincide with the yield curve $P^M(0,t)$ observed in the market at $t=0$ for all considered maturities t. In this setting, the initial yield curve $P^M(0,t)$ is (up to some maturity $\hat{t}$) specified by applying the

[6] Cf. Korn and Wagner (2018) for a general setup of this model in the considered $\mathbb{P}/\mathbb{Q}$-setting and Graf and Korn (2020) for an application in line with this paper and the same choice of risk premiums under $\mathbb{P}$.

[7] The following (risk-neutral) specifications closely follow Brigo and Mercurio (2006).

Nelson-Siegel-Svensson-approach (cf. Svensson, 1994) for modelling the corresponding spot rates $z(t)$. For maturities longer than $\hat{t}$ a flat interest rate curve is assumed by then setting the spot rate to $\hat{z}$.

Thus, the spot rate $z(0,t)$ is given by [8]

$$z(0,t) = \begin{cases} \frac{1}{100} \cdot \left(\beta_0 + \beta_1 \left(1 - e^{-\frac{t}{\tau_1}}\right) \frac{\tau_1}{t} + \beta_2 \left(\left(1 - e^{-\frac{t}{\tau_1}}\right) \frac{\tau_1}{t} - e^{-\frac{t}{\tau_1}} \right) + \beta_3 \left(\left(1 - e^{-\frac{t}{\tau_2}}\right) \frac{\tau_2}{t} - e^{-\frac{t}{\tau_2}} \right) \right), & t \le \hat{t} \\ \hat{z}, & t > \hat{t} \end{cases}$$

with parameters $\beta_0,\ \beta_1,\ \beta_2,\ \beta_3, \tau_1$ and τ_2 . The German Federal Reserve Bank also applies the above Nelson-Siegel-Svensson-approach and accordingly publishes calibrated parameters[9] (cf. Schich, 1997). Given these parameters, we further set $P^M(0,t) = \left(1 + z(0,t)\right)^{-t}$.[10]

In order to model the short rate $r(t)$ under the objective probability measure $\mathbb{P}$, additionally risk premiums d_x and d_y on the underlying stochastic factors are considered. We do so by amending the mean reversion levels of $x_{\mathbb{P}}(t)$ and $y_{\mathbb{P}}(t)$ under $\mathbb{P}$ and obtain

$$dx_{\mathbb{P}}(t) = a\left(d_x - x_{\mathbb{P}}(t)\right)dt + \sigma dW_t^{x_{\mathbb{P}}},\ \ x_{\mathbb{P}}(0) = 0$$

$$dy_{\mathbb{P}}(t) = b\left(d_y - y_{\mathbb{P}}(t)\right)dt + \eta dW_t^{y_{\mathbb{P}}},\ \ y_{\mathbb{P}}(0) = 0$$

This yields to

$$\begin{aligned} x_{\mathbb{P}}(t) &\coloneqq x_{\mathbb{Q}}(t) + d_x(1 - e^{-at}) \\ y_{\mathbb{P}}(t) &\coloneqq y_{\mathbb{Q}}(t) + d_y(1 - e^{-bt}) \\ r(t) &\coloneqq x_{\mathbb{P}}(t) + y_{\mathbb{P}}(t) + \psi(t) \end{aligned}$$

[8] The parameters used in our analysis (cf. Section 5, Tab 2) yield the corresponding spot rate in percentage points. Therefore, an additional adjustment with $\frac{1}{100}$ has to be performed.

[9] Calibrated parameters by the German Federal Reserve bank may be found via https://www.bundesbank.de/dynamic/action/de/statistiken/zeitreihen-datenbanken/zeitreihen-datenbank/759778/759778?statisticType=BBK_ITS&listId=www_skms_it03c&treeAnchor=GELD (last checked: June 2020).

[10] Note, Schich (1997) in contrast to Svensson (1994) sets $P^M(0,t) = \left(1 + z(0,t)\right)^{-t}$ as compared to $P^M(0,t) = \exp(-z(0,t)t)$. Hence, for the parameters to be consistent we follow Schich (1997) for the definition of $P^M(0,t)$.

With this specification, the (no-arbitrage) price of a zero-coupon-bond with time-to-maturity $(T-t)$ at time t is given as

$$P(t,T)=\frac{P^M(0,T)}{P^M(0,t)}\cdot \exp(A(t,T))$$

with

$$A(t,T)=\frac{1}{2}\big(V(t,T)-V(0,T)+V(0,t)\big)-\frac{1-\exp\big(-a(T-t)\big)}{a}x_{\mathbb{P}}(t) -\frac{1-\exp\big(-b(T-t)\big)}{b}y_{\mathbb{P}}(t)$$

and

$$V(t,T)=\frac{\sigma^2}{a^2}\left((T-t)+\frac{2}{a}\mathrm{e}^{-a(T-t)}-\frac{1}{2a}\mathrm{e}^{-2a(T-t)}-\frac{3}{2a}\right)+ \frac{\eta^2}{b^2}\left((T-t)+\frac{2}{b}\mathrm{e}^{-b(T-t)}-\frac{1}{2b}\mathrm{e}^{-2b(T-t)}-\frac{3}{2b}\right)+ 2\rho\frac{\sigma\eta}{ab}\left((T-t)+\frac{1}{a}\left(\mathrm{e}^{-a(T-t)}-1\right)+\frac{1}{b}\left(\mathrm{e}^{-b(T-t)}-1\right) -\frac{1}{a+b}\left(\mathrm{e}^{-(a+b)(T-t)}-1\right)\right).$$

Note, we want to stress the interplay between $\mathbb{P}$ and $\mathbb{Q}$ here: Realizations of the considered state variables $x_{\mathbb{P}}$ and $y_{\mathbb{P}}$ under the real-world measure $\mathbb{P}$ at time t feed into (risk-neutral) pricing formulae, e.g. derived for the pricing of zero-coupon-bonds $P(t,T)$ under $\mathbb{Q}$.

Now, we introduce the modelling of equity returns by applying a generalized Geometric Brownian motion. The spot price of the so-called “base” equity process $S(t)$ has the following dynamics

$$dS(t)=S(t)\Big((r(t)+\lambda)dt+\sigma_S dW_t^S\Big), S(0)=1$$

where (W_t^S) is a Wiener process uncorrelated to $\left(W_t^{x_{\mathbb{Q}}}\right)$ and $\left(W_t^{Y_{\mathbb{Q}}}\right)$, σ_S is the volatility and λ specifies the risk premium earned over the short rate $r(t)$. Note, the specified dynamics yield solutions of $r(t)$ and $S(t)$ as

$$r(t)=\sigma\int_0^t e^{-a(t-s)}dW^{x_{\mathbb{Q}}}(s)+d_x(1-e^{-at})+\eta\int_0^t e^{-b(t-s)}dW^{y_{\mathbb{Q}}}(s)+\ d_y(1-e^{-bt}) +\ \psi(t)$$

$$S(t) = S(0) \cdot \exp\left(\int_0^t r(s)ds + (\lambda - 0.5\sigma_S^2)t + \sigma_S W^S(t)\right)$$

We therefore conclude that $r(t)$ and $\int_0^t r(s)ds$ follow a normal distribution and hence $S(t)$ follows a log-normal distribution in this setting.

4.2 Considered investment options: equity funds

This section introduces the modelling of the considered investment options by means of equity funds based on the stochastic model as specified above. In this modelling approach different equity funds may generally differ by their volatility. In practice, they obviously may also differ by their charges. For the sake of notation, these potentially different charges are however accounted for in the considered products which invest into these equity funds as introduced in Section 3. Therefore, we can refrain from including charges into the modelling of equity funds in this section.

Let $A(t)$ denote the spot price of an equity fund equipped with volatility σ_A. The considered industry standard assumes that per additional "unit of risk" (in terms of volatility) compared to the base equity process $S(t)$ some additional risk premium should be earned. Hence, the risk premium λ_A is set as $\lambda_A := \lambda \frac{\sigma_A}{\sigma_S}$ where λ is the risk premium of the already specified "base" equity process. The fund's dynamics then read as

$$dA(t) = A(t)\Big((r(t) + \lambda_A)dt + \sigma_A dW_t^S\Big)$$

which yields

$$\mathrm{A}(t) = A(0) \cdot \exp\left(\int_0^t r(s)ds + \left(\lambda \frac{\sigma_A}{\sigma_S} - 0.5\sigma_A^2\right)t + \sigma_A W^S(t)\right), A(0) = 1.$$

4.3 Disclosure of charges

This section introduces how the summary cost indicator (i.e. the reduction in yield and the total amount of charges) given the modelling assumptions so far can be derived. The reduction in yield RIY of a product is defined as the difference of two yields, one yield of a so-called cost free scenario (i.e. some assumed asset return before taking any charges of the product into account) and one yield after considering all charges of the product. The difference of these two yields is then labelled as the effect of all charges on the product's yield ("reduction in yield").

European Commission (2017, Annex VI, p. 71) requires that for the derivation of the cost indicator "*[...] the methodology and the underlying hypothesis used for the estimation of the moderate scenario from the performance scenarios section of the key information document [shall be applied]*". Hence, based on the product's moderate performance scenario – which is defined by the 50th percentile of the product's benefits at the recommended holding period – the summary cost indicator shall be assessed.[11] We denote the yield of the cost free scenario – i.e. before all charges in the moderate scenario – by z^{mod} and the yield after all charges in the moderate scenario by r^{mod} and thus obtain

$$RIY := z^{mod} - r^{mod}.$$

Since the moderate performance scenario is specified by a single number (the product's median benefit payment), but does not specify any actual capital market path which led to this outcome, we first need to specify z^{mod} and r^{mod} for the calculation of the RIY as well as the "whole" moderate scenario – in terms of an artificial capital market path over the whole life of the contract – which yields the product's 50th percentile for the additionally required calculation of total amount of charges. Obviously, there is no unique solution to both problems, since there are e.g. an infinite number of potential paths leading to the product's median benefit. We therefore use and describe the approach proposed in the German insurance market (resp. industry standard) in what follows. We want to stress here that different methodologies to derive the reduction in yield "given the moderate scenario" are possible and could therefore be subject of further research. However, in this paper we focus on studying the methodology proposed in the considered industry standard.

In line with Section 3, we assume a single premium payment of $P = 1$. With V_T denoting the client's payoff at maturity, the moderate performance scenario builds on the median of V_T, say V_T^{mod}, i.e. $V_T^{mod} = \inf\{x \in \mathbb{R}: F_{V_T}(x) \geq 0.5\}$ where F_{V_T} denotes the cumulative distribution function of V_T. This benefit payment immediately yields the gross return after charges r^{mod} from the retail investor's point of view as $r^{mod} = \frac{\ln(V_T^{mod})}{T}$.

The specification of z^{mod} – i.e. the "yield in the cost free scenario before any charges" – is less straight forward. In line with the German industry standard (cf. DAV, 2018), we assume that all underlying assets of the product before considering any charges perform with the same constant yield. Under this yield, we invest into different assets according to the product's algorithm / asset allocation and deduct all charges of the product accordingly.

[11] Note, European Commission (2017) also requires disclosure of the reduction in yield and the total amount of charges for interim investment periods after 1 year and after the contract's halftime. We will focus on the recommended holding period only, but our results also transform to earlier time points.

z^{mod} is then specified by the constant yield which actually yields a benefit payment equal to V_T^{mod}. In other words z^{mod} is the deterministic gross return before taking any charges into account which yields the same maturity benefit as the stochastically derived V_T^{mod} and hence specifies one possible path underlying the moderate performance scenario. Given z^{mod}, we calculate the reduction in yield RIY as $RIY := z^{mod} - r^{mod}$.

In addition to the derivation of the reduction in yield, z^{mod} is also applied to compute the total amount of charges C (in EUR) as an additional part of the required cost disclosure. In practice, charges would typically be deducted on a monthly basis, so $C = \sum_{i=0}^{12T} C_i$ where C_i equals the charges deducted at time $i = 0, \frac{1}{12}, \dots, T$ again assuming a deterministic projection of the product applying the gross return z^{mod}. The products considered in this paper (cf. Section 3) generally allocate and deduct their charges on a continuous basis, so the total amount of charges C will be – based on this methodology – given by $C = \lim_{\Delta t \to 0} \sum_{i=0}^{k-1} C_i$ for a partition of the recommended holding period into k equidistant time steps of length Δt, i.e. $k = \frac{T}{\Delta t}$. In Sections 6 and 7 we will derive analytical solutions for the reduction in yield RIY and the total amount of charges C for the different products, respectively. We will then further conclude that different funds (differing by their volatility and their charges) may actually maximize the reduction in yield and the total amount of charges and hence caution has to be taken when synthetic funds to compute the summary cost indicator's range in the generic key information document are specified.

5 Parameter set for numerical analyses

In the following sections we analyze the introduced sample products (cf. Section 3) and derive closed-form solutions for the reduction in yield RIY and the corresponding total amount of charges C. These solutions can then be used to determine lower and upper bounds for RIY and C and to identify those funds which actually lead to these minimum/maximum values. Consequently, these results can then serve as an "educated guess for the critical funds" when more complex products or different premium payment flows are considered and presumably some Monte-Carlo-simulation instead of an analytical solution has to be performed.

With the following results we will show that in general not always those funds equipped with the highest charge c_A will actually yield to the highest reduction in yield or the highest total amount of charges. In addition, the fund which leads to the highest reduction in yield may in general not yield to the highest amount of charges vice versa.

In what follows, Section 6 treats CPPI-products whereas Section 7 analyzes OBPI-products introduced (cf. Section 3). Our numerical analyses are performed for a single premium

$P = 1$ with the parameter set as summarized by Tab 1 and Tab 2 obtained by Produktinformationsstelle Altersvorsorge (2017).

Parameter	a	b	σ	η	ρ	d_x	d_y	λ	σ_S
Value	0.389	0.097	0.0182	0.019	-0.924	0.016	-0.00295	0.04	0.2

Tab 1 Capital market parameters

Parameter	β_0	β_1	β_2	β_3	τ_1	τ_2	$\hat{t}$	$\hat{z}$
Value	0.556	-1.37525	26.25197	-25.3854	5.62709	5.03144	20	0.00814

Tab 2 Nelson-Siegel-Svenson parameters

In Produktinformationsstelle Altersvorsorge (2017), the calibration of the "$\mathbb{Q}$ parameters" – i.e. a, b, σ, η and ρ – is performed such that prices of interest rate derivatives at a certain reference date are reproduced reasonably well by the model. The "$\mathbb{P}$-parameters" – i.e. d_x and d_y – are specified such that real-world forecasts by the OECD are obtained (in expectation) by the model. Finally, the specification of λ and σ_S follows an expert judgement.

6 CPPI-products

Based on the dynamics of the considered CPPI-product derived in Section 6.1, we calculate the reduction in yield and the total amount of charges in Section 6.2. In Section 6.3, we derive parameter combinations – i.e. volatility and charges – which maximize the respective cost figures.

6.1 Portfolio dynamics

The CPPI-products considered in this paper are mainly motivated by products present in many European countries that are often referred to as "dynamic hybrid products". A dynamic hybrid product is a client-individual CPPI managed by the insurance company where as a riskless asset the insurer's general assets are used. In other words, the insurance company's traditional product with a constant guaranteed rate (technical rate) r_g serves as the riskless asset in the CPPI. Note, in practice the investment in the traditional product would also be entitled to an additional surplus participation (above r_g) which is however not considered within this paper.

Therefore, the considered CPPI-product is equipped with some guaranteed maturity benefit $G_T > 0$. At time t this product invests in an equity fund $A(t)$ with volatility σ_A (cf. Section 4.2) and the insurance company's general assets $B(t)$ which provides a constant return given by the technical rate r_g. Hence, the underlying assets' dynamics read as

$$dA(t) = A(t)\Big((r(t) + \lambda_A)dt + \sigma_A dW_t^S\Big)$$
$$dB(t) = r_g B(t)dt$$

Within the considered product, the equity fund and the riskless asset are continuously rebalanced assuming a multiplier m to ensure the guarantee at contract's maturity. In addition, the product provider charges an upfront fee β proportional to the single premium, volume-based fees c p.a. for the whole portfolio and an additional fee c_A p.a. only for those parts of the portfolio that are actually invested in the equity fund.

Extending the approach provided by Balder et. al (2009) to stochastic interest rates and our particular product design studied, the product's portfolio value V_t at time t therefore consists of an investment of $\alpha_t V_t$ in the equity fund and $(1-\alpha_t)V_t$ in the insurance company's general assets and evolves according to

$$dV_t = V_t\left(\alpha_t\left(-(c+c_A)dt + \frac{dA(t)}{A(t)}\right) + (1-\alpha_t)\left(-cdt + \frac{dB(t)}{B(t)}\right)\right)$$

where α_t is given as $\alpha_t = \frac{m\overline{C_t}}{V_t}$ and $\overline{C_t} = V_t - F_t$ denotes the so-called "cushion" given the guarantee's net present value F_t (its "floor") as $F_t \coloneqq e^{-(r_g-c)\cdot(T-t)}G_T$.

With $F_0 = e^{-(r_g-c)T}G_T$, the development of the floor as $dF_t = (r_g - c)F_t dt$ and the equity fund's dynamics (cf. Section 4.2), the cushion's dynamics can therefore be derived as

$$\begin{aligned}
d\overline{C_t} &= dV_t - dF_t \\
&= V_t\left(\alpha_t\left(-(c+c_A)dt + \frac{dA(t)}{A(t)}\right) + (1-\alpha_t)\left(-cdt + \frac{dB(t)}{B(t)}\right)\right) - (r_g - c)F_t dt \\
&= m\overline{C_t}(-(c+c_A)dt + (r(t)+\lambda_A)dt + \sigma_A dW_t^S) + (\overline{C_t} + F_t - m\overline{C_t})(-cdt + r_g dt) \\
&\qquad - (r_g - c)F_t dt \\
&= \overline{C_t}\left(\left(m\left(r(t) + \lambda_A - c_A - r_g\right) + r_g - c\right)dt + m\sigma_A dW_t^S\right).
\end{aligned}$$

Hence, we obtain

$$\overline{C_t} = \overline{C_0}\exp\left(m\int_0^t r(s)ds + \left(m\left(\lambda\frac{\sigma_A}{\sigma_S} - c_A - r_g\right) - 0.5m^2\sigma_A^2 + r_g - c\right)t + m\sigma_A W^S(t)\right),$$

$$\overline{C_0} = 1 - \beta - F_0$$

and finally

$$V_t = e^{-(r_g - c)(T-t)} G_T + \overline{C}_t$$

Therefore, V_t (and especially V_T) follows a "shifted" log-normal distribution which allows for an analytical treatment of the considered CPPI-product.

6.2 Calculation of reduction in yield and the total amount of charges

The moderate performance scenario, i.e. the 50th-percentile V_T^{mod} of V_T, can be derived from above shifted log-normal random variable. According to the industry standard described in Section 4.3, in order to compute the reduction in yield RIY and the total amount of charges C, a constant yield z^{mod} which leads to a maturity benefit equal to V_T^{mod} is to be determined at first.

More precisely,

- a deterministic projection of the considered product is to be done assuming
  - a constant performance of z^{mod} for the development of all asset classes involved (before any charges are deducted),
  - a deduction of all charges incurred,
  - and allowing for the product's investment algorithm,
- such that this projection delivers a maturity benefit of V_T^{mod}.

Note, for the considered CPPI-product, the technical rate r_g – although assuming that $B(t)$ will perform with z^{mod} – will not be modified for deriving the floor F_t, i.e. $F_t = e^{-(r_g - c)(T-t)} G_T$ and hence $dF_t = F_t(r_g - c)dt$ still holds $\forall t$. This assumption ensures that the actual asset allocation of the product is not artificially modified when z^{mod} is calibrated to match the moderate performance scenario V_T^{mod}.

So given any (deterministic) rate of return z assuming $\frac{dA(t)}{A(t)} = \frac{dB(t)}{B(t)} = zdt$ yields the (deterministic) evolution of the CPPI-product's cushion $\overline{C}_t(z)$ as

$$\begin{aligned} d\overline{C}_t(z) &= dV_t - dF_t \\ &= V(t)\big(\alpha_t(-(c + c_A)dt + zdt) + (1 - \alpha_t)(-cdt + zdt)\big) - \big(r_g - c\big)F_t dt \\ &= m\overline{C}_t(z)(-(c + c_A)dt + zdt) + \big(\overline{C}_t(z) + F_t - m\overline{C}_t(z)\big)(-cdt + zdt) \\ &\qquad - \big(r_g - c\big)F_t dt \end{aligned}$$

$$= \overline{C_t}(z)\big(z-(c+mc_A)\big)dt + F_t\big(z-r_g\big)dt$$

Hence, the cushion $\overline{C_t}(z)$ fulfills the following linear nonhomogeneous differential equation

$$\overline{C_t}'(z) = \big(z-(c+mc_A)\big)\overline{C_t}(z) + \big(z-r_g\big)\mathrm{e}^{-(r_g-c)(T-t)}G_T$$

This differential equation can be solved applying the technique of "the variation of constants"[12] as

$$\overline{C_t}(z) = F_0 e^{(z-(c+mc_A))t}\frac{\big(z-r_g\big)}{\big(r_g+mc_A-z\big)}\Big(e^{(r_g+mc_A-z)t}+c_0\Big)$$

with some constant c_0. By further setting $c_0 := (1-\beta-F_0) - F_0\frac{z-r_g}{r_g+mc_A-z}$, the initial condition on the cushion $\overline{C_0}(z) = 1-\beta-F_0$ is fulfilled.

This finally yields

$$\begin{aligned} V_T(z) &= G_T + \overline{C_T}(z) \\ &= G_T + F_0\mathrm{e}^{(z-(c+mc_A))T}\frac{\big(z-r_g\big)}{\big(r_g+mc_A-z\big)}\Big(e^{(r_g+mc_A-z)T}+c_0\Big). \end{aligned}$$

A numerical root finding algorithm, e.g. via bisection, can then be applied to derive z^{mod} such that $V_T(z^{mod}) = V_T^{mod}$ holds. Based on this constant (deterministic) rate of return z^{mod}, the reduction in yield RIY and the total amount of charges C can then be calculated as follows.

Reduction in yield

The reduction in yield is defined as $RIY = z^{mod} - r^{mod}$ (cf. Section 4.3) with z^{mod} derived from above numerical exercise and $r^{mod} = \frac{\ln(V_T^{mod})}{T}$ corresponding to the retail investor's return on the premium invested in the moderate scenario.

Note, the reduction in yield RIY is typically a function of the fund's underlying volatility σ_A and its charge c_A. In the following section we will analyze candidates for a set of volatility σ_A and charge c_A which yield the highest reduction in yield. We will conclude that the "worst-case charge" will in general not be given by the highest (available) charge

[12] By the variation of constants formula, a solution of the differential equation $f'(t) = g(t)f(t) + h(t)$ is given by $f(t) = e^{\int_0^t g(s)ds}\left(\int_0^t \left(h(s)e^{-\int_0^s g(u)du}\right)ds + c_0\right), c_0 \in \mathbb{R}$

c_A and that the corresponding "worst-case volatility" will further vary with the specified multiplier m.

Total amount of charges

For $z = z^{mod}$ let us now compute the total amount of charges deducted by the product provider and write $\overline{C}_t = \overline{C}_t(z)$ for ease of notation in the remainder of this section.

With $k \in \mathbb{N}$ consider an equidistant partition of $[0, T]$ by k time steps with length $\Delta t = \frac{T}{k}$. For sufficiently small Δt we have $V_{t+\Delta t} \approx F_t e^{(z-c)\Delta t} + \overline{C}_t e^{(z-(c+mc_A))\Delta t}$ and hence the amount of charges C_i deducted at time-step i with $t = i\Delta t$ over the time period Δt can be approximated as[13]

$$\begin{aligned} C_i &= F_t e^{z\Delta t} + \overline{C}_t e^{z\Delta t} - \left(F_t\, \mathrm{e}^{(z-c)\Delta t} + \overline{C}_t e^{(z-(c+mc_A))\Delta t}\right) \\ &= F_0 \exp\left(\left(r_g - c\right)\Delta t\right)^i \mathrm{e}^{z\Delta t}(1 - \mathrm{e}^{-c\Delta t}) + \overline{C}_t \mathrm{e}^{-z\Delta t}\left(1 - \mathrm{e}^{-(c+mc_A)\,\Delta t}\right) \end{aligned}$$

By setting $\kappa \coloneqq F_0 \frac{(z-r_g)}{(r_g+mc_A-z)}$ we further get

$$\begin{aligned} C_i = F_0 e^{z\Delta t}(1 - e^{-c\Delta t}) \exp\left(\left(r_g - c\right)\Delta t\right)^i + \kappa e^{z\Delta t}\left(1 - e^{-(c+mc_A)\,\Delta t}\right) \\ \cdot \left(\exp\left(\left(r_g - c\right)\Delta t\right)^i + c_0 \exp\left(\left(z - (c + mc_A)\right)\Delta t\right)^i\right) \end{aligned}$$

and hence, the total amount of charges is given by

$$\begin{aligned} \beta + \sum_{i=0}^{k-1} C_i = \beta + F_0 e^{z\Delta t}(1 - e^{-c\Delta t}) \sum_{i=0}^{k-1} \exp\left(\left(r_g - c\right)\Delta t\right)^i + \kappa e^{z\Delta t}\left(1 - e^{-(c+mc_A)\,\Delta t}\right) \\ \cdot \sum_{i=0}^{k-1} \left(\exp\left(\left(r_g - c\right)\Delta t\right)^i + c_0 \exp\left(\left(z - (c + mc_A)\right)\Delta t\right)^i\right) \end{aligned}$$

[13] Note, the initial charge β at time $t = 0$ will be accounted for separately and hence is not contained in C_0 here.

If $z \neq (c + mc_A)$ this simplifies to

$$\beta + \sum_{i=0}^{k-1} C_i = \beta + F_0 e^{z\Delta t}(1 - e^{-c\Delta t}) \frac{1 - e^{(r_g - c)T}}{1 - e^{(r_g - c)\Delta t}} + \kappa e^{z\Delta t}\left(1 - e^{-(c+mc_A)\,\Delta t}\right) \cdot \left(\frac{1 - e^{(r_g - c)T}}{1 - e^{(r_g - c)\Delta t}} + c_0 \, \frac{1 - e^{(z-(c+mc_A))T}}{1 - e^{(z-(c+mc_A))\Delta t}} \right)$$

and for $z = (c + mc_A)$ we obtain

$$\beta + \sum_{i=0}^{k-1} C_i = \beta + F_0 e^{z\Delta t}(1 - e^{-c\Delta t}) \frac{1 - e^{(r_g - c)T}}{1 - e^{(r_g - c)\Delta t}} + \kappa e^{z\Delta t}\left(1 - e^{-(c+mc_A)\,\Delta t}\right) \cdot \left(\frac{1 - e^{(r_g - c)T}}{1 - e^{(r_g - c)\Delta t}} + c_0 \, \frac{T}{\Delta t} \right)$$

Now we consider the limit for $k \to \infty$, respectively $\Delta t \to 0$, and obtain (by applying l'Hôpital's rule) the total amount of charges $C = \beta + \lim_{k\to\infty} \sum_{i=0}^{k-1} C_i$

- for $z \neq c + mc_A$ as

$$C = \beta + F_0 \frac{-c}{r_g - c}\left(1 - e^{(r_g - c)T}\right) + \kappa \begin{pmatrix} \frac{-(c + mc_A)}{r_g - c}\left(1 - e^{(r_g - c)T}\right) + \\ c_0 \frac{-(c + mc_A)}{z - (c + mc_A)}\left(1 - e^{(z-(c+mc_A))T}\right) \end{pmatrix}$$

- for $z = c + mc_A$ as

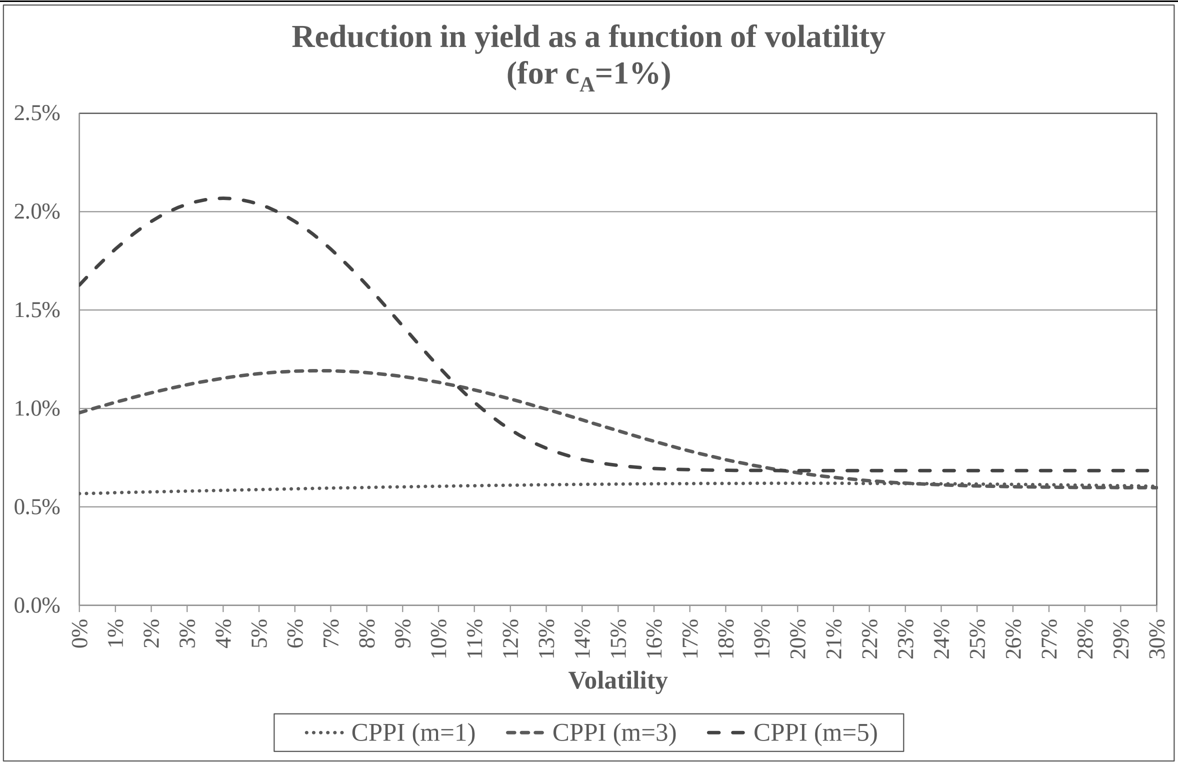
$$C = \beta + F_0 \frac{-c}{r_g - c}\left(1 - e^{(r_g - c)T}\right) + \kappa \left(\frac{-(c + mc_A)}{r_g - c}\left(1 - e^{(r_g - c)T}\right) + c_0 (c + mc_A) T \right)$$

Fig 1 shows the reduction in yield RIY and the total amount of charges C as a function of volatility σ_A for different CPPI-products with varying multiplier m, further assuming $c_A = 1\%$ and the parameter set as summarized in Tab 3.

Parameter	G_T	T	r_g	c	β
value	100%	30	$\ln(1+0.9\%)$	0.25%	5%

Tab 3 Parameter set for CPPI-products

Note, this specification of the riskless rate of return r_g equals the (maximum) technical interest rate currently valid in Germany for new business when traditional with-profit products are considered. This technical interest rate is set by law and currently specified as 0.9%. However this specification is set on a discretely compounded basis and hence transformed to a continuous figure $\ln(1 + 0.9\%)$ in this analysis.



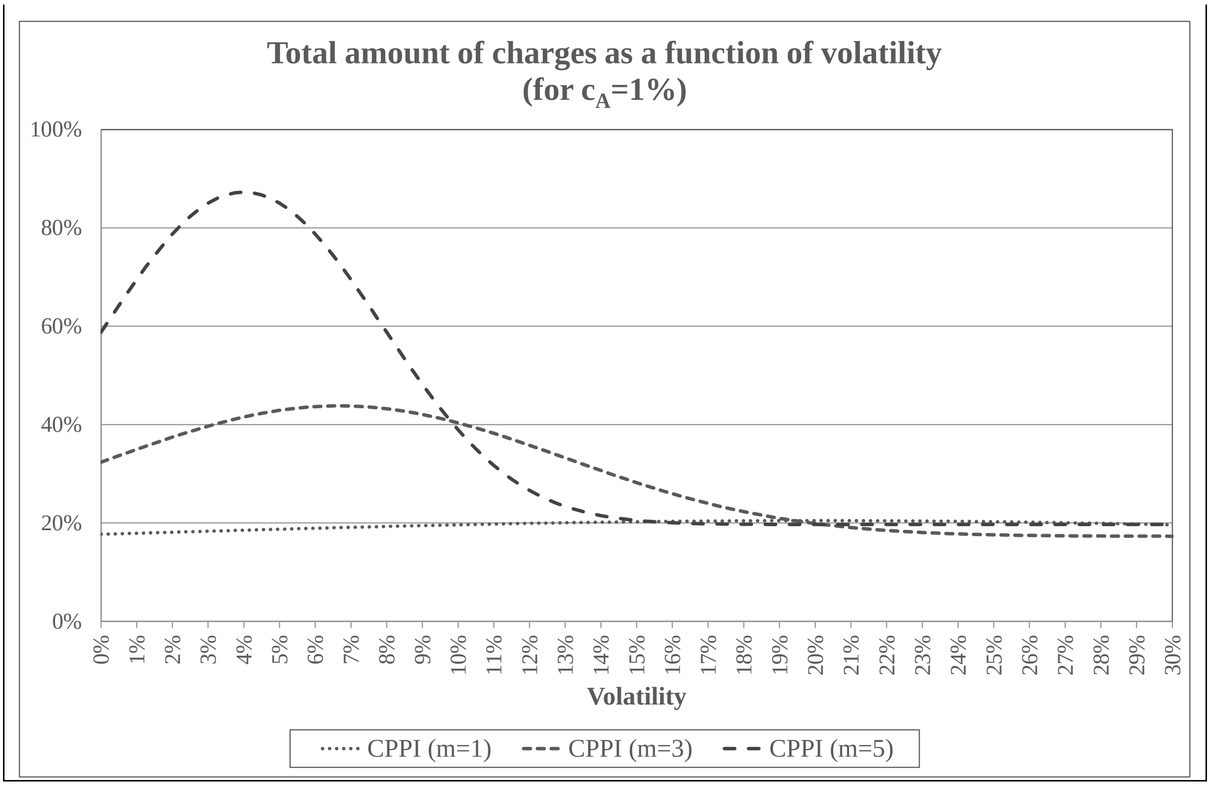


Fig 1 Reduction in yield and total amount of charges as a function of volatility for different multipliers with $c_A = 1\%$

The shape of the reduction in yield and the total amount of charges as a function of volatility shown in Fig 1 is not at all intuitive. The reduction in yield as well as the total amount of charges are increasing in volatility for lower volatilities, reach some maximum and are decreasing from then on. There are different overlapping effects leading to this shape: On the one side, a higher volatility comes along with a higher expected return. A higher expected return c.p. leads to a higher investment portion in the risky asset and therefore higher charges. This effect seems to be the reason for increasing charges at first. At the same time, however, a higher volatility for CPPI products results in more reallocations from the risky asset to the riskless asset and hence more scenarios (or probability mass) where a high portion of the portfolio value needs to be invested into the riskless asset in order to finance the guarantee. Therefore, c.p. a higher volatility comes along with a higher (and cheaper) investment in the riskless asset. This effect is the reason for decreasing charges for higher volatilities.

For different products (i.e. different multipliers), the reduction in yield and the total amount of charges reach their maximum at different levels of the volatility σ_A. Hence, the "worst-case" volatility for which reduction in yield and total amount of charges are maximized is product-specific. Therefore, in the next section, we address this "worst-case" volatility σ_A in more detail and further investigate the corresponding "worst-case" charge c_A as well. By "worst-case" charge c_A we mean the value of c_A that leads to a maximum in the reduction in yield and the total amount of charges.

6.3 Worst-case set of volatility and charge

Fig 1 showed that the reduction in yield and the total amount of charges highly depend on the level of the volatility. Therefore, we now analyze how a "worst-case" assumption set of volatility σ_A on the one side and the respective charge c_A on the other side may be obtained.

First, Section 6.3.1 derives the worst-case volatility such that for any given parameter set (especially for some fixed but arbitrary charge c_A) the moderate scenario, i.e. V_T^{mod} and hence z^{mod} is maximized. Although we were not able to provide a mathematical proof, intensive numerical analyses beyond this study indicate that for a given c_A the reduction in yield and further the total amount of charges are also maximized when the underlying constant rate of return z^{mod} reaches its maximum level. Section 6.3.2 will then further investigate which particular specification of c_A will likely yield to the highest reduction in yield and the highest total amount of total charges when e.g. this worst-case volatility is assumed.

6.3.1 Worst-case volatility

Let $Y_T := m\int_0^T r(s)ds + m\left(\lambda\frac{\sigma_A}{\sigma_S} - r_g - m0.5\sigma_A^2\right)T + m\sigma_A W^S(T)$ and conclude that $V_T^{mod} = G_T + \overline{C_0}\cdot e^{Y_T^{mod}} e^{-mc_AT}$ where Y_T^{mod}denotes the 50th-percentile of Y_T. Hence – all other parameters fixed – V_T^{mod} obtains its maximum for the volatility σ_A maximizing Y_T^{mod}. Further, Y_T follows a normal distribution with some expectation $Y_{T,\mu}(\sigma_A)$ and variance $Y_{T,\sigma^2}(\sigma_A)$ as a function of the underlying fund's volatility σ_A. Since $Y_T^{mod} = \mathbb{E}[Y_T] = Y_{T,\mu}(\sigma_A)$ we get

$$
\begin{aligned}
Y_{T,\mu}(\sigma_A) &= \mathbb{E}\left[m\int_0^T r(s)ds + m\left(\lambda\frac{\sigma_A}{\sigma_S} - c_A - r - m0.5\sigma_A^2\right)T + m\sigma_A W^S(T)\right] \\
&= \mathbb{E}\left[m\int_0^T r(s)ds + m\left(\lambda\frac{\sigma_A}{\sigma_S} - c_A - r - m0.5\sigma_A^2\right)T\right]
\end{aligned}
$$

which obtains its maximum when $m\left(\lambda\frac{\sigma_A}{\sigma_S} - c_A - r_g - m0.5\sigma_A^2\right)$ is maximized. This leads to the optimal volatility $\sigma_A^\star = \frac{\lambda}{\sigma_S}\frac{1}{m}$. Therefore, $\sigma_A^\star$ is a natural candidate for the maximum reduction in yield and maximum total amount of charges. We conclude that, by increasing the multiplier m, the corresponding volatility $\sigma_A^\star$ decreases. Tab 4 summarizes the resulting worst-case volatilities for the products already considered in Fig 1 when capital market parameters as given by Tab 1 are assumed.[14]

Product	CPPI (m=1)	CPPI (m=3)	CPPI (m=5)
$\boldsymbol{\sigma_A^\star}$	20%	6.67%	4%

Tab 4 Worst-case volatility $\sigma_A^\star$ for different multipliers

Applying this (product-specific) worst-case volatility $\sigma_A^\star$, Fig 2 shows the reduction in yield and the total amount of charges as a function of different charges c_A for the considered products.

[14] For ease of notation, the critical candidate $\sigma_A^\star$ will be referred to the "worst-case volatility" in the remainder of this paper.

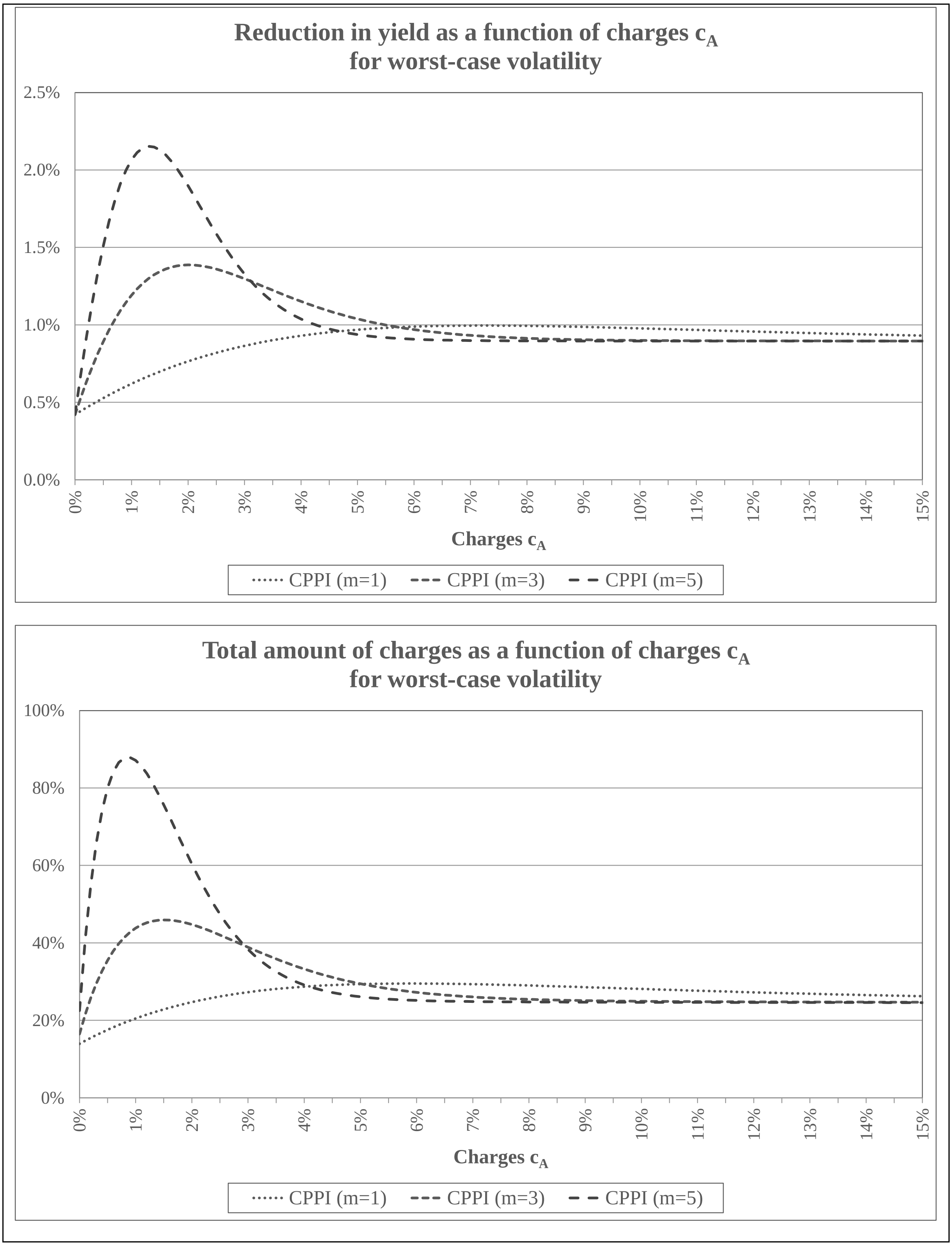


Fig 2 Reduction in yield and total amount of charges as a function of charges for worst-case volatility for different multipliers

The reduction in yield and the total amount of charges are increasing in the charges c_A of the risky asset for lower values of c_A, then reach some maximum and are then decreasing from then on. This, at first sight, seems quite surprising and counterintuitive. The explanation for this effect is that c_A are the charges of the risky asset only. The higher the charges in the risky asset, c.p. the higher is the investment portion in the riskless asset within the CPPI product necessary in order to finance the guarantee. Since the overall charges in the riskless asset are lower than the charges in the risky asset, the total amount of charges and the reduction in yield are decreasing in c_A for higher values of c_A.

Therefore, similar with Fig 1, in addition to the worst-case volatility, a product-specific worst-case charge c_A (which is not "just" given by the highest available charge) seems to exist. Further, note that following Fig 2, the charge maximizing the reduction in yield and the total amount of charges may actually differ.

Therefore, the following section analyzes how to derive candidates for the worst-case charge which maximize the reduction in yield RIY on the one side and the total amount of charges C on the other side when the worst-case volatility $\sigma_A^\star$ is assumed.

6.3.2 Worst-case charges

After having obtained the worst case volatility $\sigma_A^\star$, we are now interested in a candidate $c_A^\star$ maximizing the reduction in yield $RIY(c_A) = z^{mod}(c_A) - r^{mod}(c_A)$ as a function of the underlying fund's charge c_A. A candidate to the solution of this optimization problem is given by the root of $RIY'(c_A)$ with $RIY'(c_A) = \frac{\partial}{\partial c_A}\left(z^{mod}(c_A) - r^{mod}(c_A)\right)$.

Note, since we already had to use a numerical root-finding algorithm to solve for $z^{mod}(c_A)$, an analytical treatment of $RIY'(.)$ is not applicable. Therefore, we rely on numerical differentiation to approximate $RIY'(.)$ and then apply a root-finding algorithm to obtain the root $c_A^\star$ of $RIY'(c_A)$ which will then likely maximize the reduction in yield $RIY(c_A^\star)$.

In addition, when the total amount of charges $C(c_A)$ as a function of c_A is considered, we compute the root $c_A^\star$ of $C'(c_A)$ with

$$C'(c_A) = \frac{\partial}{\partial c_A}\left(\beta + F_0 \frac{-c}{r_g - c}\left(1 - e^{(r_g - c)T}\right) + \kappa \cdot \left(\frac{-(c + mc_A)}{r_g - c}\left(1 - e^{(r_g - c)T}\right) + c_0 \frac{-(c + mc_A)}{z - (c + mc_A)} \cdot \left(1 - e^{(z - (c + mc_A))T}\right)\right)\right)$$

again relying on numerical differentiation.

Assuming our parameter set as given in Tab 3, Tab 5 summarizes the candidates for the worst-case charges $c_A^\star$ for the considered products when the product-specific worst-case volatility $\sigma_A^\star$ (cf. Tab 4) is assumed.

Product	CPPI (m=1)	CPPI (m=3)	CPPI (m=5)
$\boldsymbol{c_A^\star}$ for $\boldsymbol{RIY}$	7.25%	2.02%	1.33%
$\boldsymbol{c_A^\star}$ for $\boldsymbol{C}$	5.90%	1.52%	0.87%

Tab 5 Worst-case charges $c_A^\star$ for the reduction in yield and the total amount of charges for different multipliers assuming the product-specific worst-case volatility $\sigma_A^\star$

Comparing Tab 5 and Fig 2 we conclude that (in this example) the critical candidates $c_A^\star$ indeed yield to the maximum reduction in yield and total amount of charges observed. We note that the higher the multiplier, the lower the required worst-case charge $c_A^\star$ for actually maximizing the reduction in yield and the total amount of charges.

Summarizing all the results in this section, we come to the following conclusion.

For the CPPI-products considered, the most expensive fund in terms of the reduction in yield and the total amount of charges *is generally not* given by the fund portfolio's most expensive fund in terms of its charge c_A, but is rather a function of the fund's underlying volatility and the considered product. Further, different charges c_A actually maximize the reduction in yield and the total amount of charges when the product-specific worst-case volatility is considered.

In a practical application of CPPI products when only a limited number of investment options can actually be analyzed, the following approach sounds therefore promising to set up suitable synthetic funds: Based on the results of Section 6.3, identify the worst-case volatility $\sigma_A^\star$ of the CPPI-like product linked to its multiplier. Given this worst-case volatility, identify the worst case charges applying the algorithms introduced. The synthetic funds specified with this worst-case volatility and these worst-case charges may provide a suitable upper bound ("most expensive fund") in the required range. A prudent estimate for the lower bound ("cheapest investment fund") may be given by the synthetic fund specified with the worst-case volatility and the cheapest charge c_A of the available investment options.[15]

Note, when more complex products are considered in practice – such as products with regular premium payments or CPPI-products with some constraint on possible leveraging – these results may still be used as an "educated guess" for a specification of synthetic funds which yield to a suitable range within the generic key information document.

7 OBPI-products

Next, we consider an OBPI-product equipped with a guaranteed maturity benefit G_T. Similar with the CPPI-product this product is invested into an equity fund $A(t)$ with volatility σ_A, but in contrast to the CPPI-product the issued investment guarantee is not ensured by continuously rebalancing risky and riskless assets, but is rather "just" guaranteed by the product provider instead.[16] Assuming charges of $(c + c_A)$ on the client's account value and an initial charge of β on the premium paid therefore yields the maturity benefit V_T as

$$V_T = \max\left(G_T, (1-\beta)A_T e^{-(c+c_A)T}\right).$$

The following sections will now derive analytical solutions of the reduction in yield and the total amount of charges, respectively and will similar with the previous section identify those volatilities and charges which likely maximize the required disclosure of costs.

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[15] "Prudent" in this sense means that combining the worst-case volatility $\sigma_A^\star$ and the lowest available charge c_A yields the highest disclosure of RIY and C for the presumably cheapest fund in terms of its charge c_A.

[16] In practice some parts of the fund's charges c_A or an additional guarantee fee would be used to finance some hedge portfolio such that this hedge will deliver $(G_T - A_T)^+$ at maturity and hence the guaranteed payment would not be "for free".

7.1 Calculation of reduction in yield and the total amount of charges

As introduced in Section 2, European Commission (2017) requires deriving the reduction in yield and the total amount of charges assuming the moderate performance scenario. However, especially for OBPI-products it is in our view not entirely clear if – according to European Commission (2017) – the cost disclosure shall actually be based on the client's account value before the guaranteed benefit payment might kick in – i.e. $(1-\beta)A_T e^{-(c+c_A)T}$ – or after it had been possibly accounted for – i.e. V_T. Throughout this section we assume that the disclosure of charges shall be based on the moderate scenario of V_T, hence after potentially allowing for G_T. If in contrast one would disclose the costs based on $(1-\beta)A_T e^{-(c+c_A)T}$ instead, the corresponding treatment would be a special case of the results derived in this section by further setting $G_T = 0$.

For any specified parameter combination, the moderate performance scenario V_T^{mod}, i.e. the 50th-percentile of V_T, is either given by the 50th-percentile of $(1-\beta)A_T e^{-(c+c_A)T}$ or the guaranteed maturity benefit G_T. Therefore, in what follows let A_T^{mod} denote the 50th-percentile of A_T. Following the requirements of Section 4.3 to derive the reduction in yield and the total amount of charges, we first need to calculate the constant (deterministic) rate z^{mod} by solving for

$$V_T^{mod} = (1-\beta)e^{\left(z^{mod}-(c+c_A)\right)T}$$

which yields $z^{mod} = \frac{1}{T}\ln\left(\frac{V_T^{mod}}{(1-\beta)}\right) + (c + c_A)$. Note that when $A_T^{mod} > \frac{G_T}{1-\beta}\, e^{(c+c_A)T}$ holds, we obtain $z^{mod} = \frac{1}{T}\ln\left(A_T^{mod}\right)$ which is then independent of c and c_A. However, in case $A_T^{mod} \leq \frac{G_T}{1-\beta} \cdot e^{(c+c_A)T}$, we obtain $V_T^{mod} = G_T$ and hence $z^{mod} = \frac{1}{T} \cdot \ln\left(\frac{G_T}{1-\beta}\right) + (c + c_A)$ which then depends on c and c_A.

Reduction in yield

Based on these derivations further taking $r^{mod} = \frac{1}{T}\ln\left(V_T^{mod}\right)$ into account we finally obtain the reduction in yield as $RIY = z^{mod} - r^{mod} = -\frac{1}{T}\ln(1-\beta) + c + c_A$ and hence conclude the following for OBPI-products when the reduction in yield is assessed:

In contrast to the CPPI-products (cf. Section 6) the reduction in yield RIY reaches its maximum for the most expensive fund, i.e. the fund equipped with highest charge c_A. Further, in contrast to the previous results for the CPPI-products, the RIY is now independent of the fund's underlying volatility σ_A.

Total amount of charges

By setting $z = z^{mod}$ for ease of notation, similar derivations as in Section 6.2 yield the total amount of charges C as

$$C = \begin{cases} \beta + (1-\beta)\dfrac{-(c+c_A)}{z-(c+c_A)}\left(1-e^{(z-(c+c_A))T}\right), z \neq c + c_A \\ \beta + (1-\beta)(c+c_A)T \,, z = c + c_A \end{cases}$$

Again numerical analyses indicate that C is increasing in z and hence similar analyses as in Section 6.3.1 can be performed to obtain worst-case volatilities and worst-case charges.

7.2 Worst-case set of volatility and charge

Recall that $RIY = -\frac{1}{T}\ln(1-\beta) + (c + c_A)$ is independent of the fund's underlying volatility σ_A and hence the volatility will only affect the total amount of charges C and not the reduction in yield RIY. Further, the reduction in yield reaches its maximum for the highest available charge c_A. Hence, we will now only be concerned with a treatment of the maximum total amount of charges C in what follows.

When $A_T^{mod} > \frac{G_T}{1-\beta} \cdot e^{(c+c_A)T}$ and hence $V_T^{mod} = (1-\beta)A_T^{mod}e^{-(c+c_A)T}$ holds, the total amount of charges will likely reach its maximum value for the volatility $\sigma_A^\star$ which maximizes A_T^{mod}. Section 6.3.1 showed that this volatility for the OBPI-product (with "a multiplier equal to 1") is given by $\sigma_A^\star = \frac{\lambda}{\sigma_S}$.

In addition, a critical candidate for the charge $c_A^\star$ maximizing the total amount of charges for any fixed volatility σ_A is given by the root of $C'(c_A)$ with

$$C'(c_A) = \frac{\partial}{\partial c_A}\left(\beta + (1-\beta)\frac{-(c+c_A)}{z-(c+c_A)}\left(1-\exp\left(\left(z-(c+c_A)\right)T\right)\right)\right).$$

When $A_T^{mod} > \frac{G_T}{1-\beta} \cdot e^{(c+c_A)T}$ holds, z is independent of c_A as shown above and hence this derivative may be calculated analytically. Setting $u \coloneqq c + c_A$ the first derivative $C'(c_A)$ is then equivalent to

$$\begin{aligned} &\frac{\partial}{\partial u}\left(\beta + (1-\beta)\frac{u}{u-z}\left(1-e^{(z-u)T}\right)\right) \\ &= (1-\beta)\left(\frac{\partial}{\partial u}\left(\frac{u}{u-z}\right)\cdot\left(1-e^{(z-u)T}\right) + \frac{u}{u-z}Te^{(z-u)T}\right) \\ &= (1-\beta)\left(-\frac{z}{(u-z)^2}\left(1-e^{(z-u)T}\right) + \frac{u}{u-z}Te^{(z-u)T}\right). \end{aligned}$$

A numerical algorithm to find the root for u resp. c_A can then be applied in order to obtain a candidate for the optimal specification of $c_A^\star$ likely maximizing the total amount of charges in this case.

If in contrast $A_T^{mod} \leq \frac{G_T}{1-\beta} e^{(c+c_A)T}$ holds, we obtain $V_T^{mod} = G_T$ and hence $z = \frac{1}{T} \ln\left(\frac{G_T}{1-\beta}\right) + (c + c_A)$. Setting $\frac{G_T}{1-\beta} = e^{r_0 T}$ then yields

$$
\begin{aligned}
C &= \beta + (1-\beta) \frac{-(c+c_A)}{\frac{1}{T}\ln\left(\frac{G_T}{1-\beta}\right)} \left(1 - \exp\left(\left(\frac{1}{T}\ln\left(\frac{G_T}{1-\beta}\right)\right) T\right)\right) \\
&= \beta + (1-\beta) \frac{-(c+c_A)}{r_0} \left(1 - \frac{G_T}{1-\beta}\right) \\
&= -\frac{(1-\beta-G_T\ )}{r_0} c_A - \frac{(1-\beta-G_T)}{r_0} c + \beta
\end{aligned}
$$

which is linearly increasing in c_A. Therefore in this setting (i.e. $A_T^{mod} \leq \frac{G_T}{1-\beta} \cdot e^{(c+c_A)T}$) the maximum available charge c_A of the considered fund portfolio and not the critical candidate $c_A^\star$ will yield the highest amount of total charges.

Fig 3 and Fig 4 illustrate these results by showing the total amount of charges when different guaranteed benefit payments G_T for the OBPI-product are assumed as a function of volatility given the underlying fund's charge as $c_A = 1\%$.

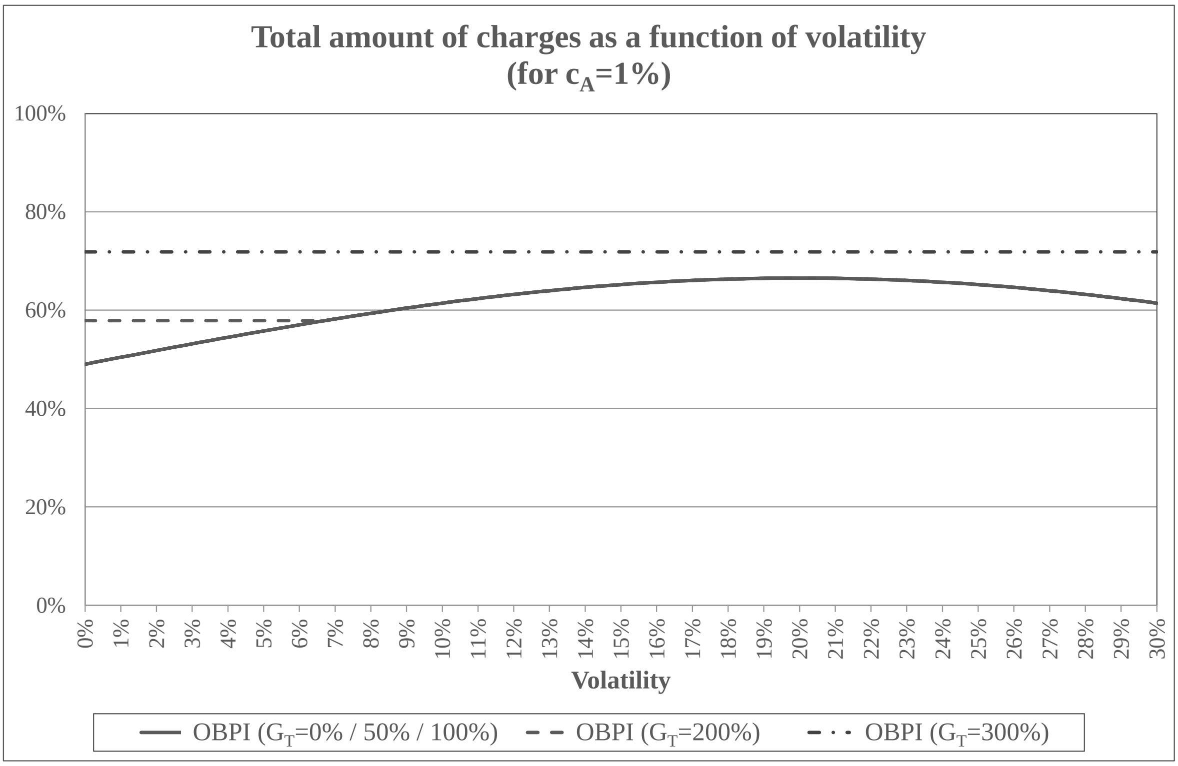

Fig 3 Total amount of charges as a function of volatility for different guaranteed benefit payments G_T with $c_A = 1\%$

We conclude form Fig 3 that – assuming a fixed charge $c_A = 1\%$ – the total amount of charges disclosed highly depends on the guaranteed benefit payment G_T and if this guarantee is "sufficiently low" (e.g. $G_T = 0\%, 50\%, 100\%$ in the example) reaches its maximum at the critical volatility $\sigma_A^\star = 20\%$. When in contrast, the guarantee is "too high" (e.g. $G_T = 300\%$ in this example) the total amount of charges disclosed is actually independent of the underlying fund's volatility since the moderate scenario of V_T will then always coincide with the issued guaranteed benefit G_T and hence the total amount of charges will not vary with the underlying fund's volatility.

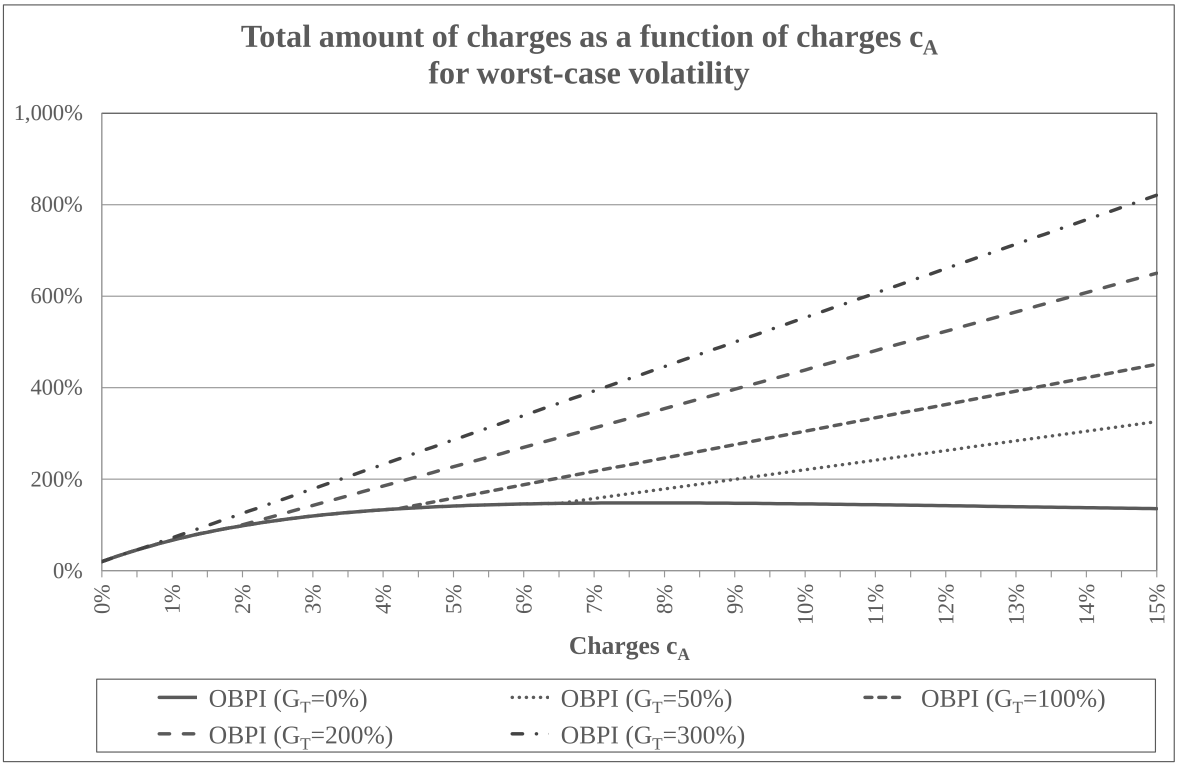

Fig 4 Total amount of charges as a function of c_A for worst-case volatility

If we further increase the underlying charges (cf. Fig 4), it becomes more likely that the moderate scenario leads to the guaranteed payment G_T only. Therefore, in this setting the critical candidate $c_A^\star = 7.76\%$ – obtained by solving for the root of $C'(c_A)$ – only yields the highest total amount of charges for the purely unit-linked product without any investment guarantee (i.e. $G_T = 0\%$). Considering above positive investment guarantees $G_T > 0$, we conclude that the total amount of charges is not maximized at the critical charge $c_A^\star$ anymore, but rather at the highest charge c_A available in the considered fund portfolio. These results are summarized by the following conclusion:

In contrast to the CPPI-products, for the OBPI-products the cheapest and most expensive fund in terms of its reduction in yield are given by the fund portfolio's cheapest and most expensive fund in terms of its charge c_A. In addition, the total amount of charges is maximized assuming the worst-case volatility $\sigma_A^\star = \frac{\lambda}{\sigma_S}$ and some product-specific charge c_A which, depending on the issued investment guarantee G_T, is either given by the critical candidate $c_A^\star$ or by the most expensive available fund.

This conclusion reasons the following approach for defining a limited number of "suitable" synthetic funds in a practical application of OBPI products when only a limited number of investment options can actually be analyzed: Given the worst-case volatility $\sigma_A^\star$, analyze those funds with the highest available charge and the critical candidate $c_A^\star$ obtained by solving the root of $C'(c_A)$ for obtaining the highest total amount of charges. Finally, a prudent guess for the lowest total amount of charges can be obtained by additionally considering the fund with the lowest available charge also assuming the worst-case volatility.

8 Conclusion

This paper has assessed the calculations for a generic key information document when multiple option products of so-called PRIIP category 4 products (cf. European Commission, 2017) are considered. Based on an industry standard developed in the Austrian and German market for a treatment of these products (cf. Section 4), we have analyzed the range of the required summary cost indicator given the available investment options. The summary cost indicator consists of the reduction in yield and the total amount of charges assuming the product's moderate performance scenario (cf. Section 2). It varies with the volatility and the amount of charges of the considered investment options. To derive a range of potential charges for the different investment options, product providers could compute the summary cost indicator for every available investment option and then find the respective range by the minimum and maximum values observed. However, given the considered industry standard, these calculations would typically require the use of Monte-Carlo-simulation for every available investment option. Further taking into account a usually rather broad investment universe, often only some limited number of "synthetic funds" is considered in practice. These synthetic funds are typically specified by a combination of minimum/maximum volatility and respective lowest/highest charges available in the fund portfolio.

Considering analytically tractable versions of different unit-linked products with investment guarantees (cf. Section 3) by constant-proportion-portfolio insurance (CPPI) and option-based-portfolio insurance (OBPI), we have derived closed-form solutions for the reduction in yield and the total amount of charges given an investment option with arbitrary volatility and arbitrary charges (cf. Sections 5, 6 and 7). Further, we have derived "worst-case" combinations of volatility and charges which actually yield candidates to the highest summary cost indicator and thereby conclude that the synthetic funds often applied in practice may significantly underestimate the highest cost indicator observed. These worst-case combinations of volatility and charges are product-specific and especially depend on the issued investment guarantee in the OBPI-products and the applied multiplier in the considered CPPI-products. In addition, we have shown that the reduction in yield and the total amount of charges may be maximized for a different combination of volatility and charges assumed. Therefore, synthetic funds not taking into account the specifications of the considered product may significantly fail to "prudently" assess the required ranges. Therefore, we have additionally provided some guidance on how to set up more suitable synthetic funds for calculation purposes when only a limited number of investment options can be considered in a practical application.

Our analyses focused on single premium payments for analytically tractable products. It would therefore be worthwhile studying how our results also translate to the case of regular premiums and more complex products. Then, we would expect that closed-form solutions for the summary cost indicator could not be derived anymore and hence Monte-Carlo-simulation would be required instead. Further research could therefore investigate whether the derived "worst-case investment options" also hold in this setting and if they could then be used as synthetic funds to derive the required ranges as well.

Finally, the summary cost indicator shall be drawn up assuming the so-called moderate performance scenario of the product (cf. European Commission, 2017). This moderate performance scenario is defined as the median of the product's maturity benefit. However, for deriving the reduction in yield and the total amount of charges, not only the final outcome given the moderate scenario but the whole capital market path which actually led to this maturity benefit has to be specified in addition. For the specification of this path, we – in line with the considered industry standard – derived a (constant) deterministic rate of return which then yields the pre-computed median assuming the product's assets to perform with this constant rate and accounting for the product's allocation mechanism and deduction of charges accordingly. From a mathematical point of view, the requirement of computing the summary cost indicator "given the moderate scenario" may however implicitly yield a more complex definition, e.g. by means of a "conditional expectation" of the summary cost indicator given the product yields its moderate performance scenario at maturity. In our view, this conditional expectation seems challenging both from a theoretical and practical point of view and therefore seems worthwhile studying in some future research.

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