A Universal Pricing Framework for Guaranteed Minimum Benefits in Variable Annuities

Daniel Bauer,

Alexander Kling,

Dr. Jochen Ruß

Bergen Norway, May 10th, 2006

Helmholtzstraße 22
D-89081 Ulm
phone +49 (0) 731/50-31230
fax +49 (0) 731/50-31239
email ifa@ifa-ulm.de

ifa
Institut für Finanz- und Aktuarwissenschaften

Agenda

- **Introduction**
- **Types of guarantees**
  - Guaranteed Minimum Death Benefits
  - Guaranteed Minimum Living Benefits
- **Pricing Framework**
- **Numerical Analysis**
- **Results**
- **Summary & Outlook**

ifa
Institut für Finanz- und Aktuarwissenschaften

Introduction

- **Variable Annuity Industry Total US Sales (dollars in billions)**

Year	Variable
1985	$5.3
1986	9.3
1987	12.7
1988	11.8
1989	13.5
1990	17.2
1991	21.5
1992	30.7
1993	46.6
1994	50.2
1995	51.3
1996	74.3
1997	88.2
1998	99.8
1999	123.0
2000	137.3
2001	113.3
2002	115.0
2003	126.4
2004	129.7

Source: NAVA, Morningstar, Inc., and LIMRA Int'l

- variable annuity sales in the US strongly increased over the last years
- During the first half of 2005
  - 28% of VA sales offered a guaranteed minimum accumulation benefit (GMAB)
  - 52% of VA sales offered a guaranteed minimum income benefit (GMIB)
  - 78% of VA sales offered a guaranteed minimum withdrawal benefit (GMWB)

ifa Institut für Finanz- und Aktuarwissenschaften

Agenda

- **Introduction**
- **Types of guarantees**
  - Guaranteed Minimum Death Benefits
  - Guaranteed Minimum Living Benefits
- **Pricing Framework**
- **Numerical Analysis**
- **Results**
- **Summary & Outlook**

ifa
Institut für Finanz- und Aktuarwissenschaften

Types of Guarantees

- **variable annuities as unit-linked deferred annuities**
  - single premium invested in funds
  - In the 90s, insurance companies started to provide additional guarantees
    - Guaranteed Minimum Death Benefits (GMDB)
    - Guaranteed Minimum Living Benefits (GMLB) or Living Benefit Guarantees (LBG)
  - guarantee fee: percentage of the net asset value (NAV) p.a.
  - guarantee provided by the insurance company
  - usual risk management
    - "reinsurance"
    - internal hedging

ifa
Institut für Finanz- und Aktuarwissenschaften

Types of Guarantees: Guaranteed Death Benefits

- **Guaranteed Minimum Death Benefits (GMDB)**
  - death benefit = max {NAV ; guaranteed benefit base}
  - typical forms of guaranteed benefit base
    - premium paid
    - maximum historical NAV of the fund at certain observation dates
      - e.g. once a year → annual ratchet guarantee
    - annually increasing death benefit
      - premium compounded by 5% – 6% p.a.
  - typical guarantee fees: 0.15% - 0.35% of the NAV p.a.

Types of Guarantees: Guaranteed Living Benefits

- **Guaranteed Minimum Accumulation Benefits (GMAB)**
  - survival benefit = max {NAV ; guaranteed benefit base}
  - typical forms of guaranteed benefit base
    - premium paid
    - maximum historical NAV of the fund at certain observation dates
      - e.g. once a year → annual ratchet guarantee
  - typical guarantee fees: 0.25% - 0.75% of the NAV p.a.

Types of Guarantees: Guaranteed Living Benefits

- **Guaranteed Minimum Income Benefits (GMIB)**
  - guaranteed annuity benefit
    - guaranteed annuity amount in the case of annuitization during an annuitization period
  - during the annuitization period, the policyholder may at any time choose
    - to annuitize the fund NAV at current annuity conversion rates $\ddot{a}_{curr}$
    - to receive the fund NAV as a lump sum payment
    - to annuitize the guaranteed benefit base at guaranteed annuity conversion rates $\ddot{a}_{guar}$
  - typical forms of the guaranteed benefit base
    - maximum historical NAV of the fund
    - annually increasing benefit (by 5% - 6% p.a.)
  - typical guarantee fees: 0.5% - 0.75% of the NAV p.a.

Types of Guarantees: Guaranteed Living Benefits

- **Guaranteed Minimum Withdrawal Benefits (GMWB)**
  - insurer guarantees that
    - the policyholder may withdraw at least the guaranteed withdrawal benefit base over time
    - as long as the annual withdrawal amount is always below some maximum level
    - even if the account value drops to 0
  - example
    - guaranteed withdrawal benefit base: gross premium paid
    - maximum annual withdrawal amount: 7% of gross premium paid
  - great variety of options on the market
  - typical guarantee fees: 0.4% - 0.65% of the NAV p.a.

ifa Institut für Finanz- und Aktuarwissenschaften

Agenda

- **Introduction**
- **Types of guarantees**
  - Guaranteed Minimum Death Benefits
  - Guaranteed Minimum Living Benefits
- **Pricing Framework**
- **Numerical Analysis**
- **Results**
- **Summary & Outlook**

Pricing Framework

- **Define state variables to describe the evolution of the contract and the embedded guarantees**
  - A_t net asset value at time t of the policyholder's account
  - W_t time t value of a hypothetical withdrawals account
  - D_t time t value of a hypothetical death benefit account
  - G_t^D guaranteed minimum death benefit at time t
  - G_t^A guaranteed minimum accumulation benefit at time t
  - G_t^I guaranteed minimum income benefit at time t
  - G_t^W remaining guaranteed minimum withdrawal amount at time t
  - G_t^E maximum year t withdrawal amount
- **state vector**

$$y_t = \left(A_t, W_t, D_t, G_t^A, G_t^I, G_t^D, G_t^W, G_t^E\right)$$

ifa
Institut für Finanz- und Aktuarwissenschaften

Pricing Framework

- **Describe the evolution of the contract and the state variables**
  - if the asset value of the fund changes or
  - if the policyholder
    - withdraws funds as a guaranteed withdrawal of a GMWB option,
    - performs a partial surrender, i.e. withdraws more than the guaranteed withdrawal amount,
    - fully surrenders the contract, or,
    - passes away
- **Development of the state variables completely determined by the asset process and the policyholder's actions**
- **Any variable annuity contract with any combination of guarantees can be described within this context**

Pricing Framework

- **customer strategy**
  - F_t–measurable process *(X)*, which determines the amount E_t to be withdrawn depending on the state y_t of the system

$$\boldsymbol{X}(t, y_t) = E_t$$

- **"Payoff" $L_T(X)$ of the contract following a given strategy *(X)* is then completely determined by the asset process**
- **value of the contract following strategy *(X)* is given by**

$$V_0\left(\boldsymbol{(X)}\right) = \sum_{t=0}^{\omega - x_0} {}_{t-1}p_{x_0} \cdot q_{x_0+t-1} \cdot E_Q\left[e^{-\int_0^T r_s ds} \left(L_T\left(t, \boldsymbol{(X)}\right) + W_T\left(t, \boldsymbol{(X)}\right) + D_T\left(t, \boldsymbol{(X)}\right)\right)\right]$$

- **value of the contract assuming a rational policyholder is given by**

$$V_0 = \sup_{\boldsymbol{(X)} \in \Xi} V_0\left(\boldsymbol{(X)}\right)$$

while Ξ is the set of all admissible customer strategies

ifa Institut für Finanz- und Aktuarwissenschaften

Agenda

- **Introduction**
- **Types of guarantees**
  - Guaranteed Minimum Death Benefits
  - Guaranteed Minimum Living Benefits
- **Pricing Framework**
- **Numerical Analysis**
- **Results**
- **Summary & Outlook**

ifa
Institut für Finanz- und Aktuarwissenschaften

Numerical Analysis

- **use Geometric Brownian motion for the underlying assets**

$$\frac{dS_t}{S_t} = r dt + \sigma dZ_t \,,\; S_0 = 1$$

- **numéraire process** $$\frac{dB_t}{B_t} = r\,dt\,,\; B_0 > 0$$
- **use Monte-Carlo-Simulation to calculate the contract value *$V_0(X)$* for any given strategy *(X)***
- **use a multidimensional discretization approach to calculate the contract value *V_0* under rational policyholder behavior**
  - generalizing Tanskanen and Lukkarinen (2004):
    - determination of a quasi-analytic solution
    - discretization of the problem via a finite mesh
  - similar to Bauer et al. (presented earlier)

ifa
Institut für Finanz- und Aktuarwissenschaften

Numerical Analysis

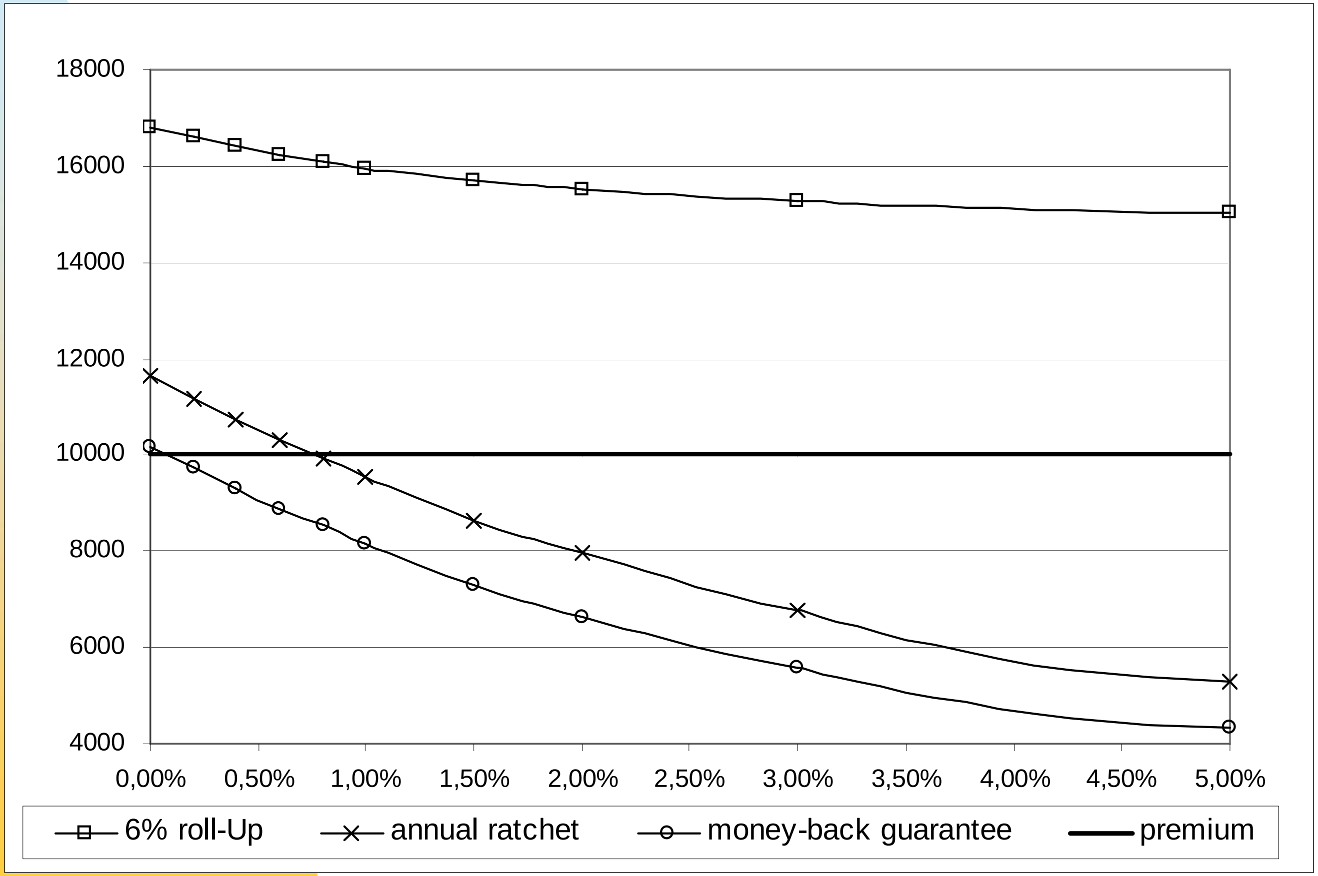
- **What's different?** **compared to Bauer et al.**
  - High dimensionality
    - adequate interpolation scheme
    - runtime/complexity:
      - adequate grids
  - Policyholder's strategy
    - in earlier presentation: surrender vs. not surrender
    - here: arbitrary withdrawals (algorithm finds 'optimal' strategy)
      ➔ adequate discretization needed, additional level of complexity
  - details see paper

Agenda

- **Introduction**
- **Types of guarantees**
  - Guaranteed Minimum Death Benefits
  - Guaranteed Minimum Living Benefits
- **Pricing Framework**
- **Numerical Analysis**
- **Results**
- **Summary & Outlook**

Results

- **Contract value (GMAB) as a function of the annual guarantee fee**



Results

- **Fair guarantee fee for contracts with GMDB under different customer behavior**

contract / strategy	Money-back guarantee	Ratchet benefit base	6 % roll-up benefit base
1: no withdrawals or surrenders	0.01%	0.04%	0.14%
2: "typical" deterministic surrender probability	< 0%	< 0%	0.05%

- fair guarantee fee for all the GMDB contracts analyzed is rather low
- the fair guarantee fee strongly decreases if policyholders surrender
  - customers have paid fees before surrendering but will not receive any benefits from the corresponding options
  - surrender fees can be used to finance the guarantees of the clients who do not surrender
- market charges for GMDBs are higher than the fair guarantee fee

ifa
Institut für Finanz- und Aktuarwissenschaften

Results

- **Fair guarantee fee for contracts with GMAB under different customer behavior**

contract / strategy	Money-back guarantee		Ratchet benefit base		6% roll-up benefit base	
	w/o DB	with DB	w/o DB	with DB	w/o DB	with DB
1: no withdrawals or surrenders	0.07%	0.23%	0.76%	0.94	---	---
2: deterministic surrender probability	< 0%	0.12%	0.57%	0.74%	---	---

- fair guarantee fees for the contracts differ significantly
  - no fair guarantee fee for a 6% roll-up benefit base
- additional fee necessary to back the additional death benefit exceeds the fair guarantee fee of the pure death benefit guarantee shown above
- rational customer behavior merely increases the risk-neutral value of the contracts compared to strategy 1
  - surrender fee 5%

ifa Institut für Finanz- und Aktuarwissenschaften

Results

- **Fair guarantee fee for contracts with GMIB under different customer behavior ($\ddot{a} = \ddot{a}_{curr} / \ddot{a}_{guar}$)**

strategy \ contract		Money-back guarantee		Ratchet benefit base		6% roll-up benefit base	
		w/o DB	with DB	w/o DB	w/o DB	with DB	w/o DB
1: no withdrawals or surrenders	$\ddot{a}=1.2$	0.14%	0.31%	1.55%	1.83%	---	---
	$\ddot{a}=1.0$	0.07%	0.23%	0.76%	0.94%	---	---
	$\ddot{a}=0.8$	0.03%	0.18%	0.25%	0.40%	---	---
	$\ddot{a}=0.6$	0.01%	0.16%	0.05%	0.19%	2.32%	3.76%
2: deterministic surrender probability	$\ddot{a}=1.2$	0.04%	0.18%	1.24%	1.40%	---	---
	$\ddot{a}=1.0$	< 0%	0.12%	0.57%	0.74%	---	---
	$\ddot{a}=0.8$	< 0%	0.10%	0.15%	0.29%	> 4%	> 4%
	$\ddot{a}=0.6$	< 0%	0.08%	< 0%	0.11%	1.45%	1.88%

  - value of the guarantee highly depends on $\ddot{a}$
  - surrender highly influences the fair guarantee fee
  - value increases for rational policyholder behavior, e.g. for a 6% roll-up benefit and $\ddot{a}$=0.6, the fair guarantee fee increases from 2.32% to > 4%

Results

- **Fair guarantee fee for contracts with GMWB under different customer behavior**

strategy \ contract	without DB	with DB
1: withdrawals of 700 p.a.	j=1: 0.19%	0.23%
2: withdrawals of 700 if $A_t < G_t^W$ **.**	0.19%	0.28%

  - difference between the two strategies is rather small
  - additional fee for including a GMDB option is significantly lower than for the GMAB and GMIB contracts,
  - fair guarantee fees are lower than the prices of these guarantees
  - however, the fair guarantee fee under rational customer behavior is significantly higher

Results

- **Influence of the capital market parameters *r* and σ on the fair guarantee fee for a contract with GMIB**

risk-free rate / volatility	r=3%	r=4%	r=5%
σ = 10%	0.46%	0.28%	0.20%
σ = 15%	1.09%	0.76%	0.56%
σ = 20%	1.94%	1.40%	1.05%

- fair guarantee fee is a decreasing function of the risk-free rate of interest
  - the risk-neutral value of a guarantee decreases with increasing interest rates
- fair guarantee fee is an increasing function of the asset volatility
  - for any risk-free rate r, the fair guarantee fee for σ = 20% is more than four times as high as the one for σ =10%

Agenda

- **Introduction**
- **Types of guarantees**
  - Guaranteed Minimum Death Benefits
  - Guaranteed Minimum Living Benefits
- **Pricing Framework**
- **Numerical Analysis**
- **Results**
- **Summary & Outlook**

ifa Institut für Finanz- und Aktuarwissenschaften

Summary & Outlook

- **some of these guarantees are underpriced**
  - ➔ insurers assume cross subsidizations from other fees and,
  - ➔ insurers assume their customers to not act rational
    - irrational surrender and withdrawal behavior
    - customers not exercising GMIB-annuitization options even when in the money
- **product calculation based on irrational behavior is risky since**
  - with the increasing discussion about products with embedded guarantees, customers will get more and more educated about their options and how to exercise them in the most beneficial way
  - it is quite possible that market participants specialize in finding arbitrage possibilities and speculating against insurers, e.g. by strategically buying such policies in the secondary market or by consulting policyholders about an optimal behavior

Summary & Outlook

- **future research**
  - different asset model
    - e.g. of Lévy type
    - including stochastic interest rates
  - analysis of an ongoing risk-management of the considered guarantees
    - implementation of efficient hedging strategies
    - sensitivity of the Delta with respect to different policyholder behavior
  - analyze optimal strategies
  - price new features of GMWB contracts

A Universal Pricing Framework for Guaranteed Minimum Benefits in Variable Annuities

Daniel Bauer,

Alexander Kling,

Dr. Jochen Ruß

Bergen Norway, May 10th, 2006

Helmholtzstraße 22
D-89081 Ulm
phone +49 (0) 731/50-31230
fax +49 (0) 731/50-31239
email ifa@ifa-ulm.de

ifa
Institut für Finanz- und Aktuarwissenschaften