On the Pricing of Longevity-Linked Securities

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Agenda

- **Introduction**
- **Different Approaches for Pricing Longevity-Linked Securities**
- **Theoretical Comparison of the Approaches**
- **Empirical Comparison of the Approaches**
- **An Option-Type Longevity Derivative**
- **Conclusion**

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Introduction

- **Longevity risk = The risk that future mortality improvement exceeds today's assumptions**
  - **Important risk factor** for annuity providers and pension funds
  - **Importance of this risk will increase** in the future
    - reduction of benefits from public pension systems
    - tax incentives for annuitization
  - **Securitization is seen as a solution** for managing this risk:
    - In the literature: Survivor bonds; survivor swaps, longevity bonds,…
    - In practice: First attempt to issue a longevity linked security failed.
    - However: There appears to be a consensus that suitable instruments will be available in the near future
  - Interesting question: **How to price** such instruments
    - What are suitable (actuarial or economic) methods?
    - How can such methodologies be applied (calibration, etc.)?

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Different Approaches for Pricing Longevity-Linked Securities

- **Price of a longevity derivative depends on the estimate of uncertain future mortality trends and the degree of uncertainty of this estimate → Mortality risk premium (MRP)**
- **Problem: There are no liquidly traded securities → MRP can not be observed in the market**
- **Consequence: Different pricing methods have been proposed**
- **CAPM/CCAPM based approach (Friedberg and Webb 2007)**
  - MRP suggested by the models is very low (MRP-puzzle similar to equity premium puzzle)
  - → Probably limited applicability of this approach
- **Instantaneous Sharpe Ratio (ISR) based approach (Milevsky et al. 2005; Bayraktar et al. 2008)**
  - Investor in longevity risk requires compensation according to some ISR (λ)
  - Return in excess of risk free return = λ * standard deviation (after diversifiable risk is "hedged")
  - For large portfolio size this coincides with a change of probability measure (P→Q) with a constant market price of risk
- **Wang Transform based approach (Lin and Cox 2005, 2006)**
  - Adjust the cdf of the future lifetime by a Wang transform to account for risk: ${}_tq_x^Q = \Phi(\Phi^{-1}({}_tq_x^P) - \theta)$ or ${}_tq_x^Q = \Psi(\Phi^{-1}({}_tq_x^P) - \theta)$

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Theoretical Comparison of the Approaches

Our methodology

- **Establish the different approaches in a common framework**
  - "Forward Mortality Framework" (Details see Bauer et al. (2007))
  - $\hat{\mu}_t(T, x_0) = -\partial / \partial T \log\left\{E_P\left[{}_T p_{x_0} \middle| \mathfrak{I}_t\right]\right\}$
  - Dynamics $d\hat{\mu}_t(T, x_0) = \hat{\alpha}(t, T, x_0)dt + \hat{\sigma}(t, T, x_0)dW_t, \quad \hat{\mu}_0(T, x_0) > 0$
  - Here: $\hat{\sigma}$ deterministic, *W* finite dimensional Brownian motion

- **Derive Pricing Formulas for "simple" (T,x_0)-Longevity bonds based on different approaches**
  (simple longevity bonds pays ${}_T p_x$ at time *T*, *"longevity zero"*)

1. Wang Transform Approach: $\Pi_0(T, x_0) = B(0,T) \cdot \left(1 - \Phi\left(\Phi^{-1}\left(1 - E_P\left[{}_T p_{x_0}\right]\right) - \theta\right)\right)$

2. Sharpe Ratio Approach: $\Pi_0(T, x_0) = B(0,T) \cdot \exp\left\{\lambda \int_0^T \int_0^s \left\|\hat{\sigma}(u, s, x_0)\right\| du\, ds\right\} \cdot E_P\left[{}_T p_{x_0}\right]$

3. "Generic" model: $\Pi_0(T, x_0) = B(0,T) \cdot \exp\left\{-\int_0^T \int_0^s \hat{\sigma}(u, s, x_0) \cdot \lambda(u)\, du\, ds\right\} \cdot E_P\left[{}_T p_{x_0}\right]$

   (*$\lambda(\cdot)$* is a negative MPR process)

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Theoretical Comparison of the Approaches (ctd.)

- **What is a good basis for determining θ and λ?**
  - Loeys et al.: (Sharpe ratio from) **stock markets**
    - **But:** different characteristics
    - Adequacy questionable!
  - Lin & Cox: **Annuity Prices**
    - Strong empirical evidence that there is a mortality risk premium embedded in annuity prices
- **If there is one, which is the better of the two approaches?**
  - Wang transform not coherent with “generic” pricing formula if more than one age cohort is considered.
  - In line with Pelsser, 2008: Inconsistency with arbitrage-free prices
  - Hence, the Sharpe ratio approach is the more general and better approach

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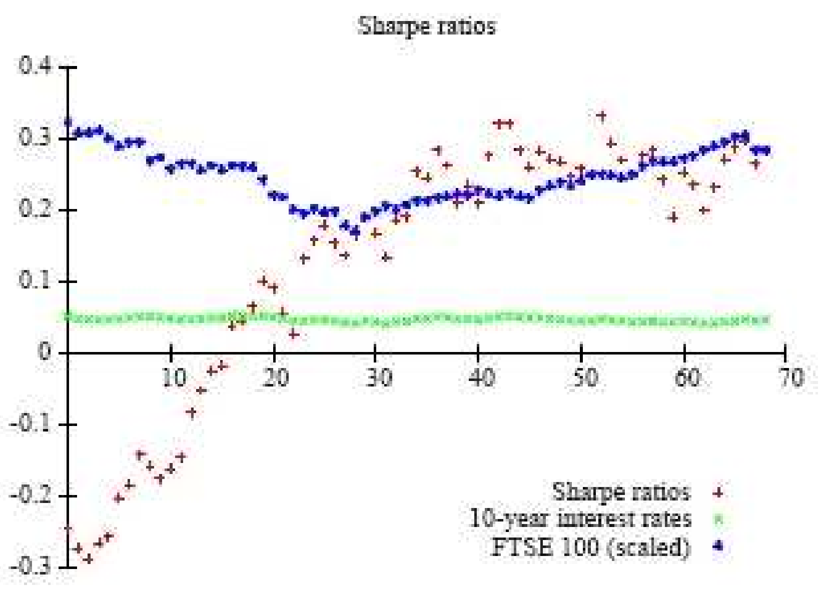
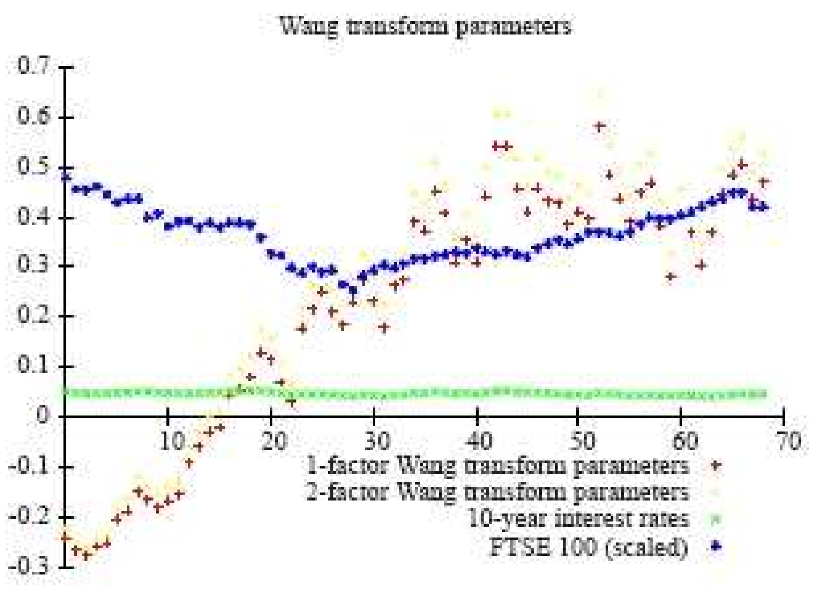
Empirical Comparison of the Approaches

- **We use the "Volatility of Mortality" model from Bauer et al (2007) and recalibrate to UK data**
- **We derive Sharpe Ratios and Wang Transform parameters from monthly UK annuity quotes (November 2000 to July 2006)**





- We find significant correlation between the market price of mortality risk and stock markets / interest rates
  → Assumption of independence between **risk-adjusted** mortality evolution and financial markets seems to be inadequate

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Empirical Comparison of the Approaches (ctd.)

- **We then apply different pricing methodologies to the EIB/BNP-Bond**
  - Sharpe Ratio calibrated to UK annuity quotes
  - Sharpe Ratio from stock markets
  - 1 factor Wang Transform calibrated to UK annuity quotes
  - 1 factor Wang Transform calibrated to US annuity quotes (Calibration from Lin and Cox 2005)
  - 2 factor Wang Transform calibrated to UK annuity quotes
  - 2 factor Wang Transform calibrated to US annuity quotes (Calibration from Lin and Cox 2006)
- **Design of the EIB/BNP-Bond**
  - Notional = GBP 50m; Pays annual coupons for 25 years
  - Coupons depend on mortality experience of English and Welsh males aged 65 in 2003
- **The EIB/BNP-Bond is therefore essentially equivalent to a portfolio of (T,65)-Longevity Bonds for T=1,2,…,25**
- **The EIB/BNP-Bond was offered at GBP 540m**
  - discounting best estimate coupon payments at LIBOR-35bp
- **EIB's yield curve is about LIBOR-15bp → 20bp can be interpreted as "fee for the longevity hedge"**

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Empirical Comparison of the Approaches (ctd.)

- **Lin and Cox (2006): Risk premium is very high → Bond is unattractive**
  - Conclusion is based on a Wang Transform approach
- **Cairns et al. (2006): Price seems reasonable**
  - Conclusion is based on an approach similar to an Instantaneous Sharpe Ratio approach

- **We "repriced" the bond using the 6 methods above and two hypothetical bonds of the same design but being offered in 2001 and 2006, respectively**

	11/2001	11/2004	7/2006
Actual	*na*	540	*na*
SRUK	487.56	584.40	605.50
SRLOE	540.42	580.60	597.95
1WTUK	482.19	601.02	618.74
1WTLC	530.87	563.42	576.32
2WTUK	480.03	595.77	612.33
2WTLC	516.91	548.27	560.72

- **Significant differences between issue dates**
  - Due to changes in interest rates, mortality projections and Sharpe Ratio / Wang Transform parameter calibrations
- **Significant differences between the 6 models**
- **All models result in a value that exceeds the price → The Bond was a "good deal"**

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Empirical Comparison of the Approaches (ctd.)

- **If all 6 pricing models state that the EIB/BNP-Bond was a good deal, 2 questions arise:**
  - Why did Lin & Cox regard the Bond as too expensive?
    - They used a different yield curve and survival rates based on realized mortality rates in 2003 as opposed to projections
  - Why was it not successfully placed?
    - Based on population as opposed to inureds (basis risk)
    - Fixed maturity of the bond → tail risk is not hedged
    - Capital intensive hedge

- **→ We conclude that the financial engineering and not the pricing was the reason for the failure of the EIB/BNP-Bond.**
  - Therefore, in the final section, we analyzed a call-option-type longevity derivative

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An Option-Type Longevity Derivative

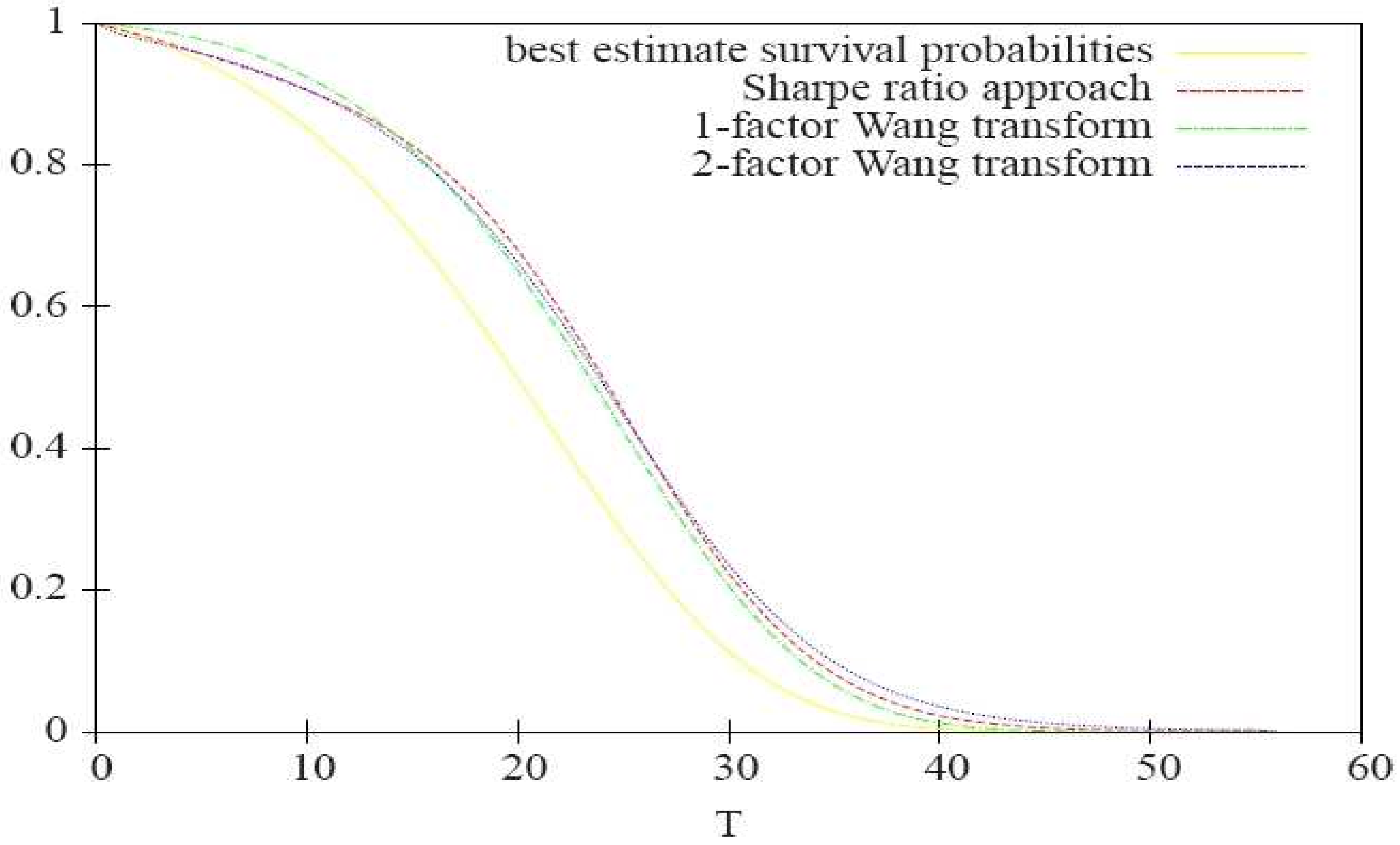
- **Payoff:** $C_T = \left({}_T p_{x_0} - K(T)\right)^+$ **with strike** $K(T) = (1+a)E_P\left[{}_T p_{x_0}\right], a > 0$
- **By suitable adjustment of the strike (choice of the parameter a), the insurer can decide, which portion of the risk to keep**
  - Example: No hedge against small deviation of actual/expected longevity. Hedge only against a deviation of more than, say, 10%
- **Such derivatives can be priced within our framework with a Black-type formula (Bauer 2007)**

a		$T=5$	$T=10$	$T=15$	$T=20$	$T=25$	$T=30$
2%	SRUK	0.00486	0.03678	0.07483	0.09079	0.07223	0.03827
	SRLOE	0.00443	0.03357	0.06752	0.08071	0.06316	0.03277
	1WTUK	0.01532	0.04780	0.07357	0.08069	0.06447	0.03430
	1WTLC	0.00634	0.02669	0.04235	0.04386	0.03127	0.01442
	2WTUK	0.00580	0.03838	0.07034	0.08423	0.07168	0.04237
	2WTLC	0.00075	0.01694	0.03545	0.04008	0.03066	0.01771
5%	SRUK	0.00049	0.02639	0.06665	0.08573	0.06977	0.03744
	SRLOE	0.00043	0.02372	0.05973	0.07589	0.06081	0.03197
	1WTUK	0.00342	0.03582	0.06545	0.07587	0.06210	0.03348
	1WTLC	0.00076	0.01816	0.03630	0.04030	0.02957	0.01386
	2WTUK	0.00066	0.02772	0.06239	0.07933	0.06922	0.04151
	2WTLC	0.00003	0.01075	0.03003	0.03671	0.02899	0.01708
10%	SRUK	0.00025	0.01366	0.05422	0.07768	0.06578	0.03606
	SRLOE	0.00022	0.01195	0.04800	0.06826	0.05703	0.03066
	1WTUK	0.00215	0.02013	0.05313	0.06824	0.05829	0.03216
	1WTLC	0.00041	0.00855	0.02762	0.03488	0.02693	0.01296
	2WTUK	0.00035	0.01454	0.05038	0.07154	0.06525	0.04010
	2WTLC	0.00001	0.00445	0.02240	0.03159	0.02637	0.01608

- **As expected: ➚➘ in T**
- **As expected: ➘ in a**
- **Wang prices higher for short maturities and vice versa**
- **Sometimes large differences despite calibration to the same data**

An Option-Type Longevity Derivative

- **The risk premium allocations differ considerably between the pricing approaches**



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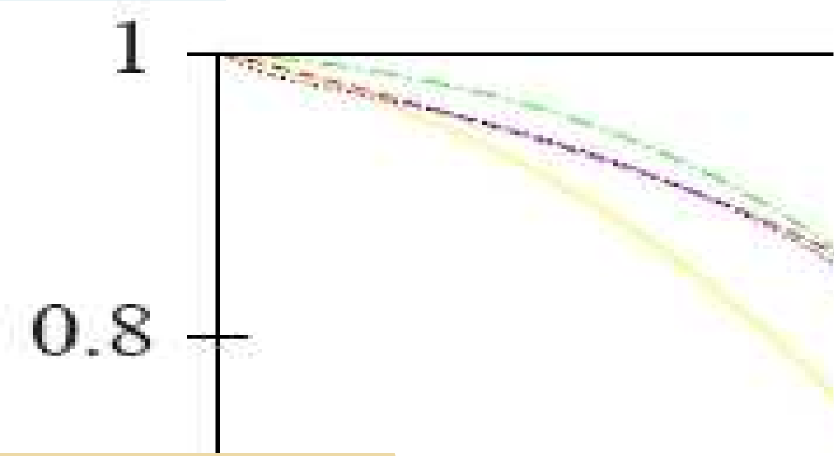
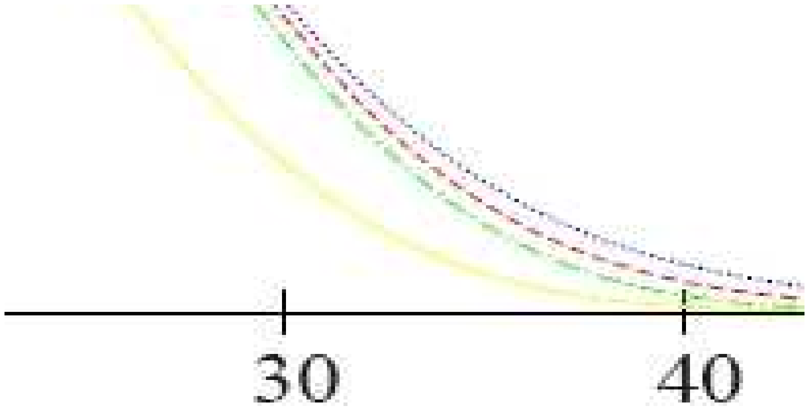
An Option-Type Longevity Derivative

- **The risk premium allocations differ considerably between the pricing approaches**

short maturities



large maturities



  - red: Sharpe ratio approach
  - green: 1-factor Wang transform approach
  - blue: 2-factor Wang transform approach

- **Sharpe ratio approach: risk premium proportional to aggregated risk**
- **Wang Transform: risk premium allocation independent of actual risk**
- **→ Adequacy of the Wang Transform again questionable**

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Conclusion

- **Overview and comparison of different pricing approaches**
- **Risk premium implied by the Wang Transform induces inconsistencies if securities on different ages are traded**
  - Even if just one security is traded, the “risk premium allocation” appears questionable
- **We conclude that currently a “market price of longevity risk” should be derived from annuity quotes**
  - Adopting Sharpe Ratios from equity markets appears to have weaknesses
- **We identify significant correlation between the market price of longevity risk and stock markets / interest rates**
  - Assuming independence between **risk-adjusted** mortality evolution and financial markets seems to be inadequate
- **The EIB/BNP-Bond appears to have been offered at a “good price”**
  - Reason for failure was financial engineering rather than pricing

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