On the Pricing of Longevity-Linked Securities

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Agenda

- Introduction
- Different Approaches for Pricing Longevity-Linked Securities
- Theoretical Comparison of the Approaches
- Empirical Comparison of the Approaches
- An Option-Type Longevity Derivative
- Conclusion
Introduction

Longevity risk = The risk that future mortality improvement exceeds today’s assumptions

- Important risk factor for annuity providers and pension funds
- Importance of this risk will increase in the future
  - reduction of benefits from public pension systems
  - tax incentives for annuitization
- Securitization is seen as a solution for managing this risk:
  - In the literature: Survivor bonds; survivor swaps, longevity bonds,…
  - In practice: First attempt to issue a longevity linked security failed.
  - However: There appears to be a consensus that suitable instruments will be available in the near future
- Interesting question: How to price such instruments
  - What are suitable (actuarial or economic) methods?
  - How can such methodologies be applied (calibration, etc.)?
Different Approaches for Pricing Longevity-Linked Securities

- Price of a longevity derivative depends on the estimate of uncertain future mortality trends and the degree of uncertainty of this estimate → Mortality risk premium (MRP)
- Problem: There are no liquidly traded securities → MRP can not be observed in the market
- Consequence: Different pricing methods have been proposed

- CAPM/CCAPM based approach (Friedberg and Webb 2007)
  - MRP suggested by the models is very low (MRP-puzzle similar to equity premium puzzle)
  - → Probably limited applicability of this approach

- Instantaneous Sharpe Ratio (ISR) based approach (Milevsky et al. 2005; Bayraktar et al. 2008)
  - Investor in longevity risk requires compensation according to some ISR ($\lambda$)
  - Return in excess of risk free return = $\lambda$ * standard deviation (after diversifiable risk is “hedged”)
  - For large portfolio size this coincides with a change of probability measure ($P \rightarrow Q$) with a constant market price of risk

  - Adjust the cdf of the future lifetime by a Wang transform to account for risk:
    \[ q_x^Q = \Phi(\Phi^{-1}(q_x^P) - \theta) \quad \text{or} \quad q_x^Q = \Psi(\Phi^{-1}(q_x^P) - \theta) \]
Theoretical Comparison of the Approaches

Our methodology

- **Establish the different approaches in a common framework**
  - “Forward Mortality Framework” (Details see Bauer et al. (2007))
  - \( \hat{\mu}_t(T, x_0) = -\frac{3}{\partial T} \log \{ E_p[T p_{x_0} | \mathcal{F}_t] \} \)
  - Dynamics \( d\hat{\mu}_t(T, x_0) = \hat{\alpha}(t, T, x_0)dt + \hat{\sigma}(t, T, x_0)dW_t, \quad \hat{\mu}_0(T, x_0) > 0 \)
  - Here: \( \hat{\sigma} \) deterministic, \( W \) finite dimensional Brownian motion

- **Derive Pricing Formulas for “simple” \((T, x_0)\)-Longevity bonds based on different approaches**
  (simple longevity bonds pays \( T x \) at time \( T \), “longevity zero”)
  1. Wang Transform Approach: \( \Pi_0(T, x_0) = B(0, T) \cdot \left( 1 - \Phi \left( \Phi^{-1} \left( 1 - E_p[T p_{x_0}] \right) - \theta \right) \right) \)
  2. Sharpe Ratio Approach: \( \Pi_0(T, x_0) = B(0, T) \cdot \exp \left\{ \lambda \int_0^T \left\| \hat{\sigma}(u, s, x_0) \right\| du ds \right\} \cdot E_p[T p_{x_0}] \)
  3. “Generic” model: \( \Pi_0(T, x_0) = B(0, T) \cdot \exp \left\{ - \int_0^T \hat{\sigma}(u, s, x_0) \cdot \lambda(u) du ds \right\} \cdot E_p[T p_{x_0}] \)
    (\( \lambda(\cdot) \) is a negative MPR process)
What is a good basis for determining $\theta$ and $\lambda$?

- Loeys et al.: (Sharpe ratio from) *stock markets*
  - **But:** different characteristics
  - Adequacy questionable!

- Lin & Cox: *Annuity Prices*
  - Strong empirical evidence that there is a mortality risk premium embedded in annuity prices

If there is one, which is the better of the two approaches?

- Wang transform not coherent with “generic” pricing formula if more than one age cohort is considered.
- In line with Pelsser, 2008: Inconsistency with arbitrage-free prices
- Hence, the Sharpe ratio approach is the more general and better approach
Empirical Comparison of the Approaches

- We use the “Volatility of Mortality” model from Bauer et al (2007) and recalibrate to UK data.
- We derive Sharpe Ratios and Wang Transform parameters from monthly UK annuity quotes (November 2000 to July 2006).

- We find significant correlation between the market price of mortality risk and stock markets / interest rates.
  → Assumption of independence between risk-adjusted mortality evolution and financial markets seems to be inadequate.
Empirical Comparison of the Approaches (ctd.)

- We then apply different pricing methodologies to the EIB/BNP-Bond
  - Sharpe Ratio calibrated to UK annuity quotes
  - Sharpe Ratio from stock markets
  - 1 factor Wang Transform calibrated to UK annuity quotes
  - 1 factor Wang Transform calibrated to US annuity quotes (Calibration from Lin and Cox 2005)
  - 2 factor Wang Transform calibrated to UK annuity quotes
  - 2 factor Wang Transform calibrated to US annuity quotes (Calibration from Lin and Cox 2006)

- Design of the EIB/BNP-Bond
  - Notional = GBP 50m; Pays annual coupons for 25 years
  - Coupons depend on mortality experience of English and Welsh males aged 65 in 2003

- The EIB/BNP-Bond is therefore essentially equivalent to a portfolio of (T,65)-Longevity Bonds for T=1,2,…,25

- The EIB/BNP-Bond was offered at GBP 540m
  - discounting best estimate coupon payments at LIBOR-35bp

- EIB’s yield curve is about LIBOR-15bp → 20bp can be interpreted as “fee for the longevity hedge”
Empirical Comparison of the Approaches (ctd.)

- Lin and Cox (2006): Risk premium is very high ⇒ Bond is unattractive
  - Conclusion is based on a Wang Transform approach
- Cairns et al. (2006): Price seems reasonable
  - Conclusion is based on an approach similar to an Instantaneous Sharpe Ratio approach
- We “repriced” the bond using the 6 methods above and two hypothetical bonds of the same design but being offered in 2001 and 2006, respectively

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- Significant differences between issue dates
  - Due to changes in interest rates, mortality projections and Sharpe Ratio / Wang Transform parameter calibrations
- Significant differences between the 6 models
- All models result in a value that exceeds the price ⇒ The Bond was a “good deal”
Empirical Comparison of the Approaches (ctd.)

- If all 6 pricing models state that the EIB/BNP-Bond was a good deal, 2 questions arise:
  - Why did Lin & Cox regard the Bond as too expensive?
    - They used a different yield curve and survival rates based on realized mortality rates in 2003 as opposed to projections
  - Why was it not successfully placed?
    - Based on population as opposed to inureds (basis risk)
    - Fixed maturity of the bond → tail risk is not hedged
    - Capital intensive hedge

- We conclude that the financial engineering and not the pricing was the reason for the failure of the EIB/BNP-Bond.
  - Therefore, in the final section, we analyzed a call-option-type longevity derivative
An Option-Type Longevity Derivative

- Payoff: $C_T = \left( T \ p_{x_0} - K(T) \right)^+ \text{ with strike } K(T) = (1 + a) E_p \left[ T \ p_{x_0} \right], \ a > 0$

- By suitable adjustment of the strike (choice of the parameter $a$), the insurer can decide, which portion of the risk to keep
  - Example: No hedge against small deviation of actual/expected longevity. Hedge only against a deviation of more than, say, 10%

- Such derivatives can be priced within our framework with a Black-type formula (Bauer 2007)

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- As expected: $\mathcal{N}\mathcal{O}$ in $T$
- As expected: $\mathcal{N}O$ in $a$
- Wang prices higher for short maturities and vice versa
- Sometimes large differences despite calibration to the same data
An Option-Type Longevity Derivative

The risk premium allocations differ considerably between the pricing approaches
An Option-Type Longevity Derivative

- The risk premium allocations differ considerably between the pricing approaches.

short maturities

large maturities

- red: Sharpe ratio approach
- green: 1-factor Wang transform approach
- blue: 2-factor Wang transform approach

- **Sharpe ratio approach**: risk premium proportional to aggregated risk
- **Wang Transform**: risk premium allocation independent of actual risk
- → Adequacy of the Wang Transform again questionable
Conclusion

- Overview and comparison of different pricing approaches

- Risk premium implied by the Wang Transform induces inconsistencies if securities on different ages are traded
  - Even if just one security is traded, the “risk premium allocation” appears questionable

- We conclude that currently a “market price of longevity risk” should be derived from annuity quotes
  - Adopting Sharpe Ratios from equity markets appears to have weaknesses

- We identify significant correlation between the market price of longevity risk and stock markets / interest rates
  - Assuming independence between risk-adjusted mortality evolution and financial markets seems to be inadequate

- The EIB/BNP-Bond appears to have been offered at a “good price”
  - Reason for failure was financial engineering rather than pricing
www.mortalityrisk.org

- Exchange platform for latest papers and results on mortality/longevity risk and modeling
- Run by a Research Training Group at Ulm University
- You are encouraged to submit your papers!
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References