On the Pricing of Longevity-Linked Securities

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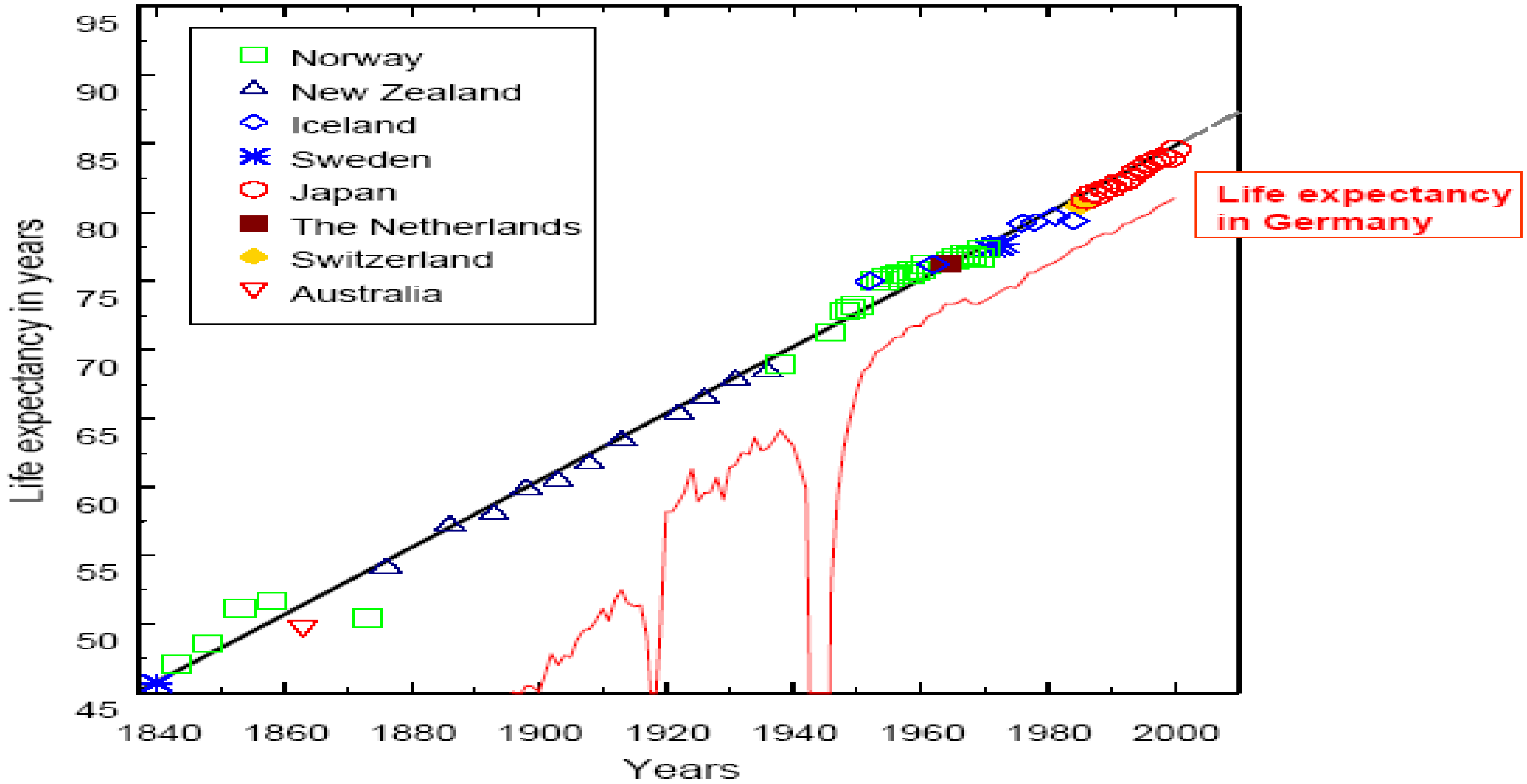
Agenda

- **Introduction and overview**
- **Different Approaches for Pricing Longevity-Linked Securities**
- **Theoretical Comparison of the Approaches**
- **Empirical Comparison of the Approaches**
- **An Option-Type Longevity Derivative**
- **Conclusions**

Introduction

- **Longevity Risk = The Risk that future mortality improvement exceeds today's assumptions**



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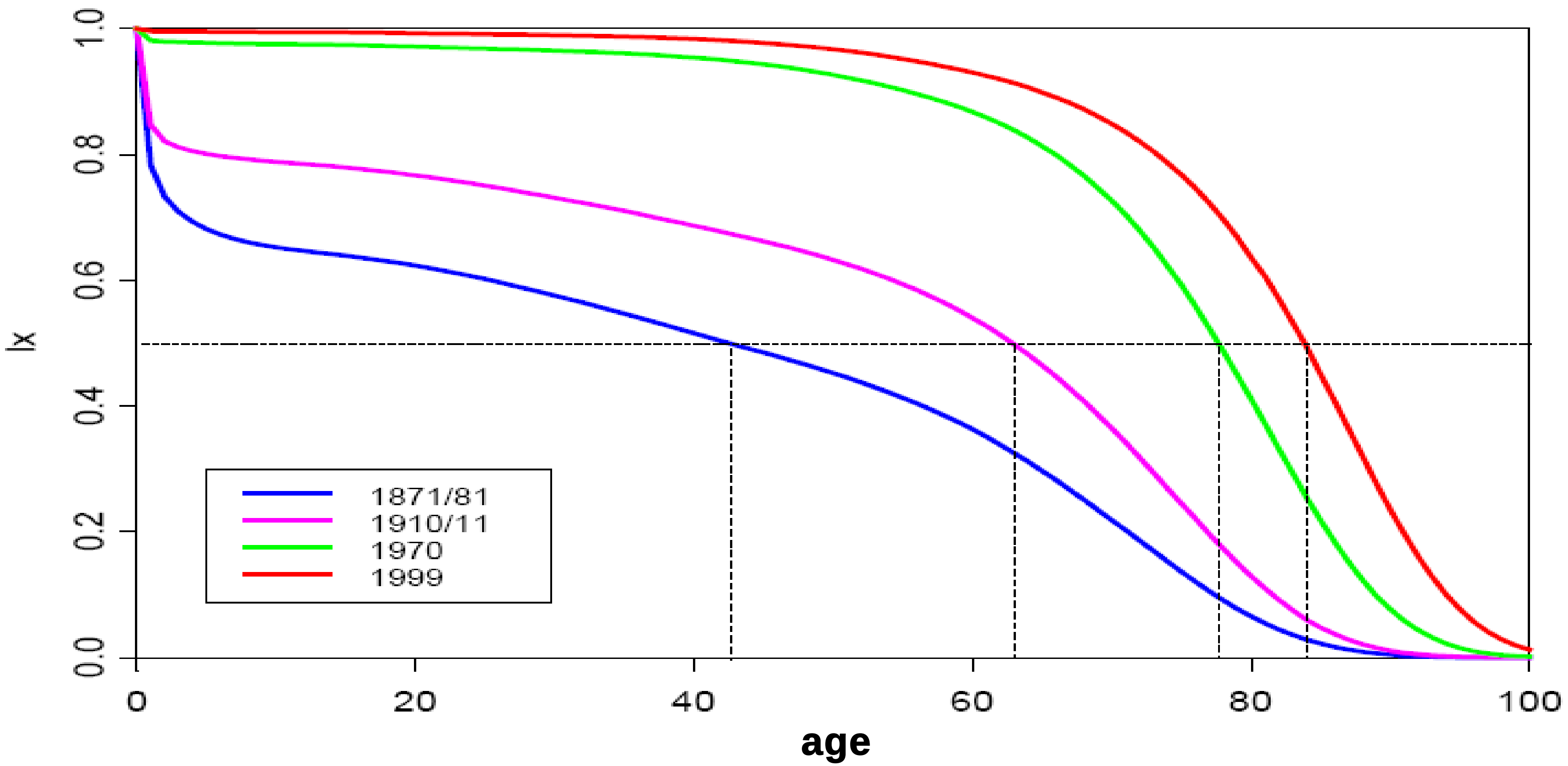
- **Longevity Risk = The Risk that future mortality improvement exceeds today's assumptions**
  - the previous chart looked very predictable, but…



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Introduction

- **Longevity Risk = The Risk that future mortality improvement exceeds today's assumptions**
  - Important risk factor for annuity providers and pension funds
  - **Importance of this risk will increase** in the future
    - reduction of benefits from public pension systems
    - tax incentives for annuitization
  - **Securitization is seen as a solution** for managing this risk:
    - In the literature: Survivor bonds; survivor swaps
    - In practice: First attempt to issue a longevity linked security failed.
    - However: There appears to be a consensus that suitable instruments will be available in the near future
  - Interesting question: **How to price** such instruments
    - What are suitable (actuarial or economic) methods
    - How can such methodologies be applied (calibration, etc.)

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Overview

- **Theoretical and empirical survey on proposed pricing approaches for longevity securities including several new ideas**
  - **Review and comparison** of different pricing methods
  - **Empirical comparison** of different approaches based on UK data
  - Discussion of several **financial engineering aspects** of longevity securitization
  - Analysis of a **specific longevity derivative**
- **Most of the models proposed so far are spot-force models**
  - Simplified: Stochastic versions of period life tables
- **In classical actuarial applications, annuities are usually priced based on generational tables**
  - Trend is already embedded in the tables
  - Stochastic version of these: **Forward mortality models**
  - → We work in a forward modeling framework

Different Approaches for Pricing Longevity-Linked Securities

- **Price of a longevity derivative depends on the estimate of uncertain future mortality trends and the degree of uncertainty of this estimate → Mortality risk premium (MRP)**
- **Problem: There are no liquidly traded securities → MRP can not be observed in the market**
- **Consequence: Different pricing methods have been proposed**

- **1) CAPM/CCAPM based model (Friedberg and Webb 2007)**
  - MRP suggested by the models is very low (MRP-puzzle similar to equity premium puzzle)
  - → Probably limited applicability of this approach

- **2) Instantaneous Sharpe Ratio (ISR) based model (Milevsky et al. 2005; Bayraktar et al. 2008)**
  - Issuer of life contingency requires compensation according to some ISR (λ)
  - Return in excess of risk free return = λ * standard deviation (after diversifiable risk is "hedged")
  - For large portfolio size this coincides with a change of probability measure (P→Q) with a constant market price of risk (still open: how to calibrate λ)

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- **3) Wang Transform based model (Lin and Cox 2005; 2006)**
  - Adjust the cdf of the future lifetime by a Wang transform to account for risk
  - Calibrate transform parameter to annuity market price

    $${}_tq_x^Q = \Phi(\Phi^{-1}({}_tq_x^P) - \theta) \quad \text{or} \quad {}_tq_x^Q = Q(\Phi^{-1}({}_tq_x^P) - \theta)$$
  - But: Adequacy of Wang transform (Pelsser, 2008 → see below)

Theoretical Comparison of the Approaches

Our methodology

- **Establish the different approaches in a common framework**
  - "Forward Mortality Framework" (Details see Bauer et al. (2007))
  - $\hat{\mu}_t(T,x_0) = -\partial / \partial T \log\left\{E_P\left[{}_T p_{x_0} \middle| \mathfrak{I}_t\right]\right\}$
  - Dynamics $d\hat{\mu}_t(T,x_0) = \hat{\alpha}(t,T,x_0)dt + \hat{\sigma}(t,T,x_0)dW_t, \quad \hat{\mu}_0(T,x_0) > 0$
  - Here: $\hat{\sigma}$ deterministic, *W finite dimensional Brownian motion*

- **Derive Pricing Formulas for "simple longevity bonds" based on different approaches** (simple longevity bonds pays ${}_T p_x$ at time *T, "longevity zero"*)

  1. Lin & Cox (Wang): $\Pi_0(T,x_0) = B(0,T)\cdot\left(1-\Phi\left(\Phi^{-1}\left(1-E_P\left[{}_T p_{x_0}\right]\right)-\theta\right)\right)$
  2. Sharpe Ratio Approach: $\Pi_0(T,x_0) = B(0,T)\cdot\exp\left\{\lambda\int_0^T\int_0^s\left\|\hat{\sigma}(u,s,x_0)\right\|du\,ds\right\}\cdot E_P\left[{}_T p_{x_0}\right]$
  3. "Generic" model: $\Pi_0(T,x_0) = B(0,T)\cdot\exp\left\{-\int_0^T\int_0^s\hat{\sigma}(u,s,x_0)\cdot\lambda(u)\,du\,ds\right\}\cdot E_P\left[{}_T p_{x_0}\right]$

     (*λ(·)* is a negative MPR process)

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Backup – Some Results in the Forward Mortality Framework

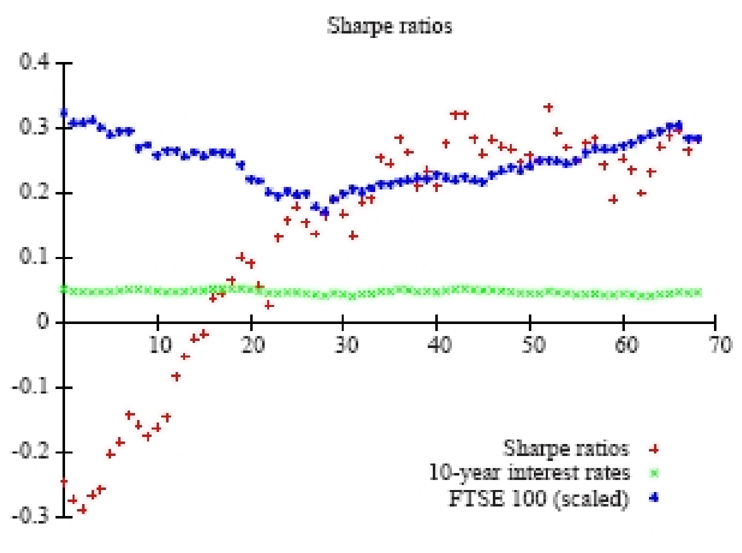
- **There is a P-HJM condition similar to the Q-HJM condition**
- **BUT: P–model (risk analysis) is different than Q–model (pricing)**
  - A model calibrated under P can be used to determine expected payoffs, percentiles, etc. of mortality contingent claims
  - However, it is not immediately suitable as a pricing model
- **Under the assumption of mortality risk-neutrality, the models coincide**
- **In the Gaussian case and under the assumption of a deterministic market price of risk, the volatilities of the P and Q models coincide**
  - The initial risk-neutral forward plane, the Q-HJM condition, and a P-calibrated volatility structure yield a pricing model
- **In general, in order to specify a corresponding pricing model, the market price of risk has to be specified**

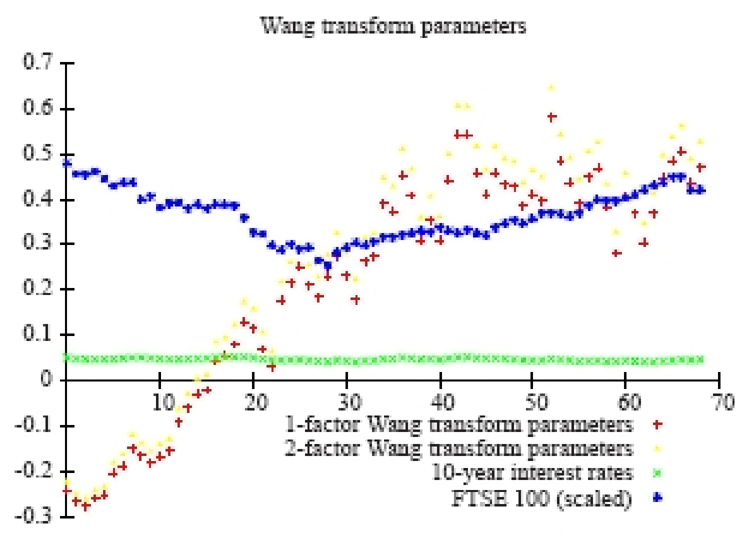
Theoretical Comparison of the Approaches (ctd.)

- **Wang transform not coherent with "generic" pricing formula if more than one age cohort is considered.**
  - In line with Pelsser, 2008: Inconsistency with arbitrage-free prices
- **What is a good basis for determining θ, λ, $\lambda(\cdot)$?**
  - Loeys et al.: (Sharpe ratio from) **stock markets**
    - **But:** different characteristics
    - Link to individual/aggregate consumption different than stock market!
    - Adequacy questionable!
  - Lin & Cox: **Annuity Prices**
    1. Strong empirical evidence that there is a mortality risk premium embedded in annuity prices
    2. Questionable whether there is also risk premium for non-systematic mortality risk (several reasons against)
    - **If only 1.**, annuity prices should provide good basis
      If 1. & 2., annuity prices should at least yield upper bound

Empirical Comparison of the Approaches

- **We use the "Volatility of Mortality" model from Bauer et al (2007) and recalibrate to UK data**
- **We derive Sharpe Ratios and Wang Transform parameters from UK annuity quotes (November 2000 to July 2006)**





- We find significant correlation between the market price of mortality risk and stock markets / interest rates → independence between **risk-adjusted** mortality evolution and financial markets seems to be inadequate

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Empirical Comparison of the Approaches

- **We then apply different pricing methodologies to the EIB/BNP-Bond**
  - Sharpe Ratio calibrated to UK annuity quotes
  - Sharpe Ratio from stock markets
  - 1 factor Wang Transform calibrated to UK annuity quotes
  - 1 factor Wang Transform calibrated to US annuity quotes (Calibration from Lin and Cox 2005)
  - 2 factor Wang Transform calibrated to UK annuity quotes
  - 2 factor Wang Transform calibrated to US annuity quotes (Calibration from Lin and Cox 2006)
- **Design of the EIB/BNP-Bond**
  - Notional = GBP 50m; Pays annual coupons for 25 years
  - Coupons depend on mortality experience of English and Welsh males aged 65 in 2003
- **The EIB/BNP-Bond is therefore essentially equivalent to a portfolio of (T,65)-Longevity Bonds for T=1,2,…,25**
- **The EIB/BNP-Bond was offered at GBP 540m**
  - discounting best estimate coupon payments at LIBOR-35bp
- **EIB's yield curve is about LIBOR-15bp → 20bp can be interpreted as "fee for the longevity hedge"**

Empirical Comparison of the Approaches

- **Lin and Cox (2006): Risk premium is very high → Bond is unattractive**
  - Conclusion is based on a Wang Transform approach
- **Cairns et al. (2006): Price seems reasonable**
  - Conclusion is based on an approach similar to an Instantaneous Sharpe Ratio approach
- **We "repriced" the bond using the 6 methods above and two virtual bonds of the same design but being offered in 2001 and 2006, respectively**

	11/2001	11/2004	7/2006
Actual	*na*	540	*na*
SRUK	487.56	584.40	605.50
SRLOE	540.42	580.60	597.95
1WTUK	482.19	601.02	618.74
1WTLC	530.87	563.42	576.32
2WTUK	480.03	595.77	612.33
2WTLC	516.91	548.27	560.72

- **Significant differences between issue dates**
  - Due to changes in interest rates, mortality projections and Sharpe Ratio / Wang Transform parameter calibrations
- **Significant differences between the 6 models**
- **All models result in a value that exceeds the price → The Bond was a "good deal"**

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Empirical Comparison of the Approaches

- **If all 6 pricing models state that the EIB/BNP-Bond was a good deal, 2 questions arise**
  - Why did Lin & Cox regard the Bond as too expensive
    - They used a different yield curve and mortality rates based on realized mortality rates as opposed to projections
  - Why was it not successfully placed?
    - Based on population as opposed to inured (basis risk)
    - Fixed maturity of the bond → tail risk is not hedged
    - Capital intensive hedge (Bond vs. Swap)

- **→ We conclude that the financial engineering and not the pricing was the reason for the failure of the EIB/BNP-Bond.**
  - Therefore, in the final Section, we analyzed a call-option-type longevity derivative

An Option-Type Longevity Derivative

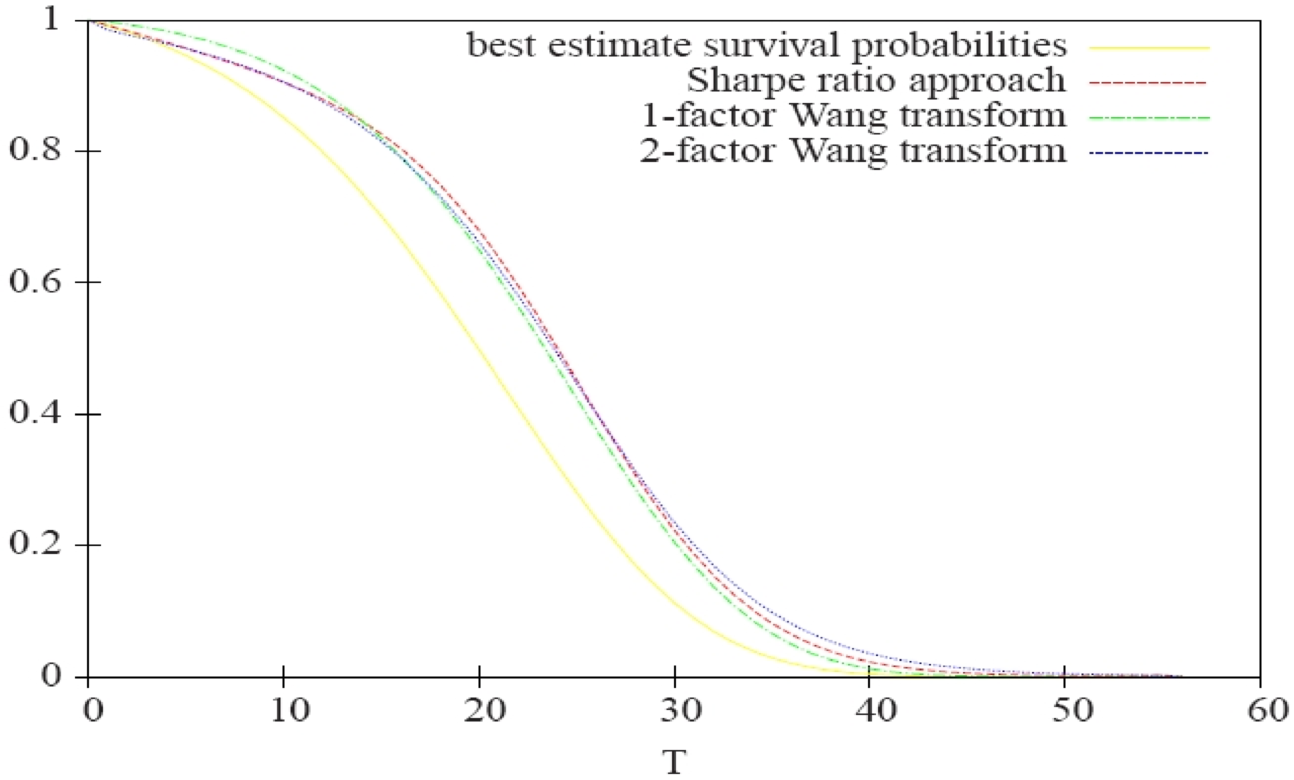
- **Payoff:** $C_T = \left({}_T p_{x_0}^{(T)} - K(T)\right)^+$ **with strike** $K(T) = (1+a)E_P\left[{}_T p_{x_0}^{(T)}\right],\ a > 0$
- **By suitable adjustment of the strike (choice of the parameter a), the insurer can decide, which portion of the risk to keep**
  - Example: No hedge against small deviation of actual/expected longevity. Hedge only against a deviation of more than, say, 10%
- **Such derivatives can be priced within our framework with a Black-type formula (Bauer 2007)**

a		$T=5$	$T=10$	$T=15$	$T=20$	$T=25$	$T=30$	Portfolio
2%	SRUK	0.00486	0.03678	0.07483	0.09079	0.07223	0.03827	1.69716
	SRLOE	0.00443	0.03357	0.06752	0.08071	0.06316	0.03277	1.49682
	1WTUK	0.01532	0.04780	0.07357	0.08069	0.06447	0.03430	1.64501
	1WTLC	0.00634	0.02669	0.04235	0.04386	0.03127	0.01442	0.84800
	2WTUK	0.00580	0.03838	0.07034	0.08423	0.07168	0.04237	1.70705
	2WTLC	0.00075	0.01694	0.03545	0.04008	0.03066	0.01771	0.78157
5%	SRUK	0.00049	0.02639	0.06665	0.08573	0.06977	0.03744	1.54034
	SRLOE	0.00043	0.02372	0.05973	0.07589	0.06081	0.03197	1.34823
	1WTUK	0.00342	0.03582	0.06545	0.07587	0.06210	0.03348	1.45405
	1WTLC	0.00076	0.01816	0.03630	0.04030	0.02957	0.01386	0.72076
	2WTUK	0.00066	0.02772	0.06239	0.07933	0.06922	0.04151	1.54745
	2WTLC	0.00003	0.01075	0.03003	0.03671	0.02899	0.01708	0.68892
10%	SRUK	0.00025	0.01366	0.05422	0.07768	0.06578	0.03606	1.34206
	SRLOE	0.00022	0.01195	0.04800	0.06826	0.05703	0.03066	1.16231
	1WTUK	0.00215	0.02013	0.05313	0.06824	0.05829	0.03216	1.23298
	1WTLC	0.00041	0.00855	0.02762	0.03488	0.02693	0.01296	0.57846
	2WTUK	0.00035	0.01454	0.05038	0.07154	0.06525	0.04010	1.34837
	2WTLC	0.00001	0.00445	0.02240	0.03159	0.02637	0.01608	0.57466

- **As expected: ↗↘ in T**
- **As expected: ↘ in a**
- **Wang prices higher for short maturities and vice versa**
- **PV annuity = 14.96**
- **→ Option based hedge rather expensive**
- **Maybe combination of call with short put?**

Backup: Allocation of risk premium

- **The risk premium allocations differ considerably between the pricing approaches**



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Backup: Allocation of risk premium

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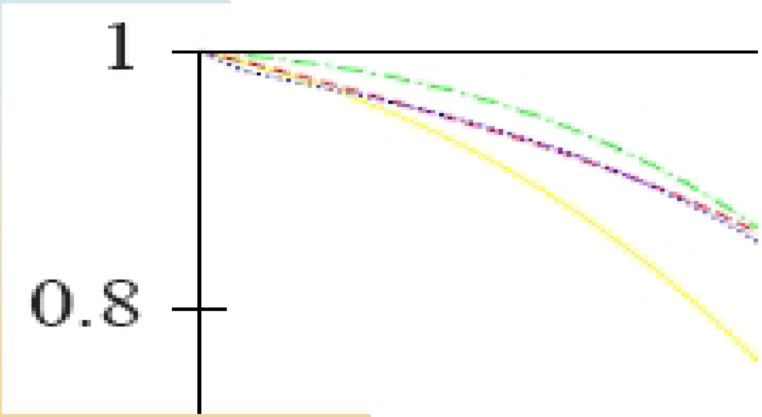
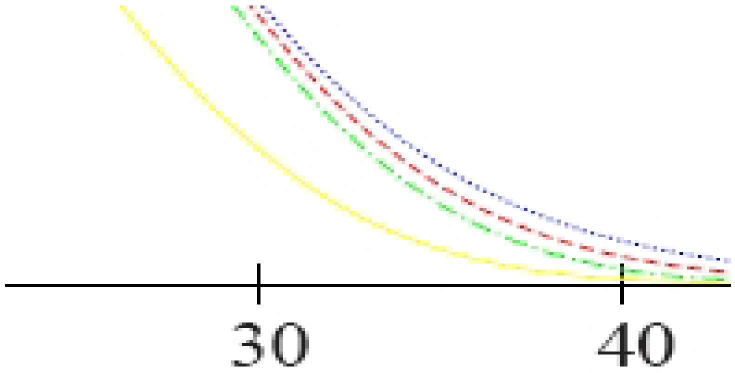
short maturities



large maturities



- Sharpe ratio approach: risk premium proportional to aggregated risk
- Wang Transform: risk premium allocation independent of actual risk

- **→ Adequacy of the Wang Transform again questionable**

Conclusions

- **Overview and comparison of different pricing approaches**
- **Risk premium implied by the Wang Transform induces inconsistencies if securities on different ages are traded**
  - Even if just one security is traded, the “risk premium allocation” appears questionable
- **We conclude that currently a “market price of longevity risk” should be derived from annuity quotes**
  - Adopting Sharpe Ratios from equity markets appears to have weaknesses
- **We identify significant correlation between the market price of mortality risk and stock markets / interest rates**
  - Assuming independence between **risk adjusted** mortality evolution and financial markets seems to be inadequate
- **The EIB/BNP-Bond appears to have been offered at a “good price”**
  - Reason for failure was financial engineering rather than pricing

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