On the Pricing of Longevity-Linked Securities

Daniel Bauer
Matthias Börger
Jochen Ruß

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Agenda

- Introduction and overview
- Different Approaches for Pricing Longevity-Linked Securities
- Theoretical Comparison of the Approaches
- Empirical Comparison of the Approaches
- An Option-Type Longevity Derivative
- Conclusions
Introduction

Longevity Risk = The Risk that future mortality improvement exceeds today’s assumptions
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The previous chart looked very predictable, but…
**Introduction**

Longevity Risk = The Risk that future mortality improvement exceeds today's assumptions

- Important risk factor for annuity providers and pension funds
- **Importance of this risk will increase** in the future
  - reduction of benefits from public pension systems
  - tax incentives for annuitization
- **Securitization is seen as a solution** for managing this risk:
  - In the literature: Survivor bonds; survivor swaps
  - In practice: First attempt to issue a longevity linked security failed.
  - However: There appears to be a consensus that suitable instruments will be available in the near future
- Interesting question: **How to price** such instruments
  - What are suitable (actuarial or economic) methods
  - How can such methodologies be applied (calibration, etc.)
Overview

- Theoretical and empirical survey on proposed pricing approaches for longevity securities including several new ideas
  - Review and comparison of different pricing methods
  - Empirical comparison of different approaches based on UK data
  - Discussion of several financial engineering aspects of longevity securitization
  - Analysis of a specific longevity derivative

- Most of the models proposed so far are spot-force models
  - Simplified: Stochastic versions of period life tables

- In classical actuarial applications, annuities are usually priced based on generational tables
  - Trend is already embedded in the tables
  - Stochastic version of these: Forward mortality models
  - We work in a forward modeling framework
Different Approaches for Pricing Longevity-Linked Securities

- Price of a longevity derivative depends on the estimate of uncertain future mortality trends and the degree of uncertainty of this estimate → Mortality risk premium (MRP)
- Problem: There are no liquidly traded securities → MRP can not be observed in the market
- Consequence: Different pricing methods have been proposed

1) CAPM/CCAPM based model (Friedberg and Webb 2007)
   - MRP suggested by the models is very low (MRP-puzzle similar to equity premium puzzle)
   - Probably limited applicability of this approach

2) Instantaneous Sharpe Ratio (ISR) based model (Milevsky et al. 2005; Bayraktar et al. 2008)
   - Issuer of life contingency requires compensation according to some ISR ($\lambda$)
   - Return in excess of risk free return = $\lambda$ * standard deviation (after diversifiable risk is “hedged”)
   - For large portfolio size this coincides with a change of probability measure ($P \rightarrow Q$) with a constant market price of risk (still open: how to calibrate $\lambda$)
Different Approaches for Pricing Longevity-Linked Securities

- Price of a longevity derivative depends on the estimate of uncertain future mortality trends and the degree of uncertainty of this estimate → Mortality risk premium (MRP)
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   - Adjust the cdf of the future lifetime by a Wang transform to account for risk
   - Calibrate transform parameter to annuity market price
     \[ q_x^O = \Phi(\Phi^{-1}(q_x^P) - \theta) \]  or  \[ q_x^O = Q(\Phi^{-1}(q_x^P) - \theta) \]
   - But: Adequacy of Wang transform (Pelsser, 2008 → see below)
Theoretical Comparison of the Approaches

Our methodology

- Establish the different approaches in a common framework
  - “Forward Mortality Framework” (Details see Bauer et al. (2007))
  - \( \dot{\mu}_t(T, x_0) = -\frac{\partial}{\partial T} \log \{ E_{P_t} \left[ T p_{x_0} \mid \mathcal{F}_t \right] \} \)
  - Dynamics \( d\dot{\mu}_t(T, x_0) = \dot{\alpha}(t, T, x_0) dt + \dot{\sigma}(t, T, x_0) dW_t, \quad \dot{\mu}_0(T, x_0) > 0 \)
  - Here: \( \dot{\sigma} \) deterministic, \( W \) finite dimensional Brownian motion

- Derive Pricing Formulas for “simple longevity bonds” based on different approaches (simple longevity bonds pays \( T p_x \) at time \( T \), “longevity zero”)

  1. Lin & Cox (Wang):
     \[ \Pi_0(T, x_0) = B(0, T) \cdot \left( 1 - \Phi(\Phi^{-1}(1 - E_{P_0} \left[ T p_{x_0} \right]) - \theta) \right) \]

  2. Sharpe Ratio Approach:
     \[ \Pi_0(T, x_0) = B(0, T) \cdot \exp \left\{ \int_0^T \sigma(u, s, x_0) dW_s \right\} \cdot E_{P_0} \left[ T p_{x_0} \right] \]

  3. “Generic” model:
     \[ \Pi_0(T, x_0) = B(0, T) \cdot \exp \left\{ \int_0^T \dot{\alpha}(u, s, x_0) \cdot \dot{\sigma}(u, s, x_0) du ds \right\} \cdot E_{P_0} \left[ T p_{x_0} \right] \]

(\( \dot{\lambda}(\cdot) \) is a negative MPR process)
There is a P-HJM condition similar to the Q-HJM condition

**BUT:** P–model (risk analysis) is different than Q–model (pricing)

- A model calibrated under P can be used to determine expected payoffs, percentiles, etc. of mortality contingent claims
- However, it is not immediately suitable as a pricing model

Under the assumption of mortality risk-neutrality, the models coincide

- In the Gaussian case and under the assumption of a deterministic market price of risk, the volatilities of the P and Q models coincide
- The initial risk-neutral forward plane, the Q-HJM condition, and a P-calibrated volatility structure yield a pricing model

In general, in order to specify a corresponding pricing model, the market price of risk has to be specified
Theoretical Comparison of the Approaches (ctd.)

- Wang transform not coherent with “generic” pricing formula if more than one age cohort is considered.
  - In line with Pelsser, 2008: Inconsistency with arbitrage-free prices
- **What is a good basis for determining $\theta$, $\lambda$, $\lambda(\cdot)$?**
  - Loeys et al.: (Sharpe ratio from) stock markets
    - **But:** different characteristics
    - Link to individual/aggregate consumption different than stock market!
    - Adequacy questionable!
- **Lin & Cox: Annuity Prices**
  1. Strong empirical evidence that there is a mortality risk premium embedded in annuity prices
  2. Questionable whether there is also risk premium for non-systematic mortality risk (several reasons against)
  - **If only 1.**, annuity prices should provide good basis
  - **If 1. & 2.**, annuity prices should at least yield upper bound
Empirical Comparison of the Approaches

- We use the “Volatility of Mortality” model from Bauer et al (2007) and recalibrate to UK data
- We derive Sharpe Ratios and Wang Transform parameters from UK annuity quotes (November 2000 to July 2006)

We find significant correlation between the market price of mortality risk and stock markets / interest rates $\rightarrow$ independence between risk-adjusted mortality evolution and financial markets seems to be inadequate
Empirical Comparison of the Approaches

- We then apply different pricing methodologies to the EIB/BNP-Bond
  - Sharpe Ratio calibrated to UK annuity quotes
  - Sharpe Ratio from stock markets
  - 1 factor Wang Transform calibrated to UK annuity quotes
  - 1 factor Wang Transform calibrated to US annuity quotes (Calibration from Lin and Cox 2005)
  - 2 factor Wang Transform calibrated to UK annuity quotes
  - 2 factor Wang Transform calibrated to US annuity quotes (Calibration from Lin and Cox 2006)

- Design of the EIB/BNP-Bond
  - Notional = GBP 50m; Pays annual coupons for 25 years
  - Coupons depend on mortality experience of English and Welsh males aged 65 in 2003

- The EIB/BNP-Bond is therefore essentially equivalent to a portfolio of (T,65)-Longevity Bonds for T=1,2,…,25

- The EIB/BNP-Bond was offered at GBP 540m
  - discounting best estimate coupon payments at LIBOR-35bp

- EIB’s yield curve is about LIBOR-15bp $\rightarrow$ 20bp can be interpreted as “fee for the longevity hedge”
Empirical Comparison of the Approaches

- **Lin and Cox (2006):** Risk premium is very high → Bond is unattractive
  - Conclusion is based on a Wang Transform approach
- **Cairns et al. (2006):** Price seems reasonable
  - Conclusion is based on an approach similar to an Instantaneous Sharpe Ratio approach

- We “repriced” the bond using the 6 methods above and two virtual bonds of the same design but being offered in 2001 and 2006, respectively

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- **Significant differences between issue dates**
  - Due to changes in interest rates, mortality projections and Sharpe Ratio / Wang Transform parameter calibrations
- **Significant differences between the 6 models**
- All models result in a value that exceeds the price → The Bond was a “good deal”
Empirical Comparison of the Approaches

- If all 6 pricing models state that the EIB/BNP-Bond was a good deal, 2 questions arise
  - Why did Lin & Cox regard the Bond as too expensive
    - They used a different yield curve and mortality rates based on realized mortality rates as opposed to projections
  - Why was it not successfully placed?
    - Based on population as opposed to inured (basis risk)
    - Fixed maturity of the bond → tail risk is not hedged
    - Capital intensive hedge (Bond vs. Swap)

→ We conclude that the financial engineering and not the pricing was the reason for the failure of the EIB/BNP-Bond.
  - Therefore, in the final Section, we analyzed a call-option-type longevity derivative
An Option-Type Longevity Derivative

- Payoff: \( C_T = (T \ p^{(T)}_{x_0} - K(T))^+ \) with strike \( K(T) = (1 + a)E_{\rho} \left[ T \ p^{(T)}_{x_0} \right] \), \( a > 0 \)

- By suitable adjustment of the strike (choice of the parameter \( a \)), the insurer can decide, which portion of the risk to keep
  - Example: No hedge against small deviation of actual/expected longevity. Hedge only against a deviation of more than, say, 10%

- Such derivatives can be priced within our framework with a Black-type formula (Bauer 2007)

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<th>( \alpha )</th>
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<th>( T = 10 )</th>
<th>( T = 15 )</th>
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\[ \text{PV annuity} = 14.96 \]

- As expected: \( \uparrow \downarrow \) in \( T \)
- As expected: \( \downarrow \) in \( a \)
- Wang prices higher for short maturities and vice versa
- Option based hedge rather expensive
- Maybe combination of call with short put?
The risk premium allocations differ considerably between the pricing approaches
Backup: Allocation of risk premium

The risk premium allocations differ considerably between the pricing approaches:

- Sharpe ratio approach: risk premium proportional to aggregated risk
- Wang Transform: risk premium allocation independent of actual risk

→ Adequacy of the Wang Transform again questionable
Conclusions

- Overview and comparison of different pricing approaches

- Risk premium implied by the Wang Transform induces inconsistencies if securities on different ages are traded
  - Even if just one security is traded, the “risk premium allocation” appears questionable

- We conclude that currently a “market price of longevity risk” should be derived from annuity quotes
  - Adopting Sharpe Ratios from equity markets appears to have weaknesses

- We identify significant correlation between the market price of mortality risk and stock markets / interest rates
  - Assuming independence between risk adjusted mortality evolution and financial markets seems to be inadequate

- The EIB/BNP-Bond appears to have been offered at a “good price”
  - Reason for failure was financial engineering rather than pricing
www.mortalityrisk.org

- Exchange platform for latest papers and results on mortality/longevity risk and modeling
- Run by a Research Training Group at Ulm University
- You are encouraged to submit your papers!
- submission@mortalityrisk.org
References