Deterministic Shock vs. Stochastic Value-at-Risk –
An Analysis of the Solvency II Standard Model Approach to Longevity Risk

Matthias Börger

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Helmholtzstraße 22
D-89081 Ulm
phone +49 (0) 731/50-31230
fax +49 (0) 731/50-31239
email ifa@ifa-ulm.de

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Introduction

- **Longevity risk = risk of insureds on average surviving longer than expected**
  - Significant risk for pension funds and annuity providers
  - **Systematic** and **non-hedgeable** risk
    → **Explicitly accounted for under Solvency II**

- **General concept for Solvency Capital Requirement (SCR) under Solvency II**
  - SCR = 99.5% **Value-at-Risk** (VaR) of Available Capital over 1 year
  - „Capital necessary to cover losses over next year with at least 99.5% probability“
  - Stochastic (internal) models required whose implementation is costly and highly sophisticated
- **Solvency II Standard model**
  - Scenario-based rather than stochastic, modular approach
  - Longevity risk: SCR = change in Net Asset Value (NAV = assets – best estimate liabilities) due to **longevity shock**
  - Longevity shock is a permanent 25% reduction of mortality rates for all ages
  - Value of 25% is mainly based on what UK insurance companies in 2004 regarded consistent with VaR concept (CEIOPS (2007))

Objective

- **Motivation of the standard model longevity stress is rather poor**
  - UK insurance companies regarded shock between 5% and 35% as appropriate
  - 25% longevity shock could significantly misjudge the true risk
  - **Analysis of the longevity stress** is required
  - Comparison with VaR for longevity risk

- **Questions regarding structure and calibration:**
- **Is a constant shock for all ages and maturities reasonable?**
  - QIS4 participants question whether trend risk is appropriately accounted for (CEIOPS (2008b))
- **Is the shock magnitude of 25% adequate?**
  - QIS4 participants regard shock as very high, internal models required significantly less capital (CEIOPS (2008b))
- **How can the standard model longevity stress possibly be improved?**

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Agenda

- **Introduction**
- **Forward mortality model**
- **Model setup**
- **Comparison of SCR formulas for longevity risk**
- **Modification of standard model longevity stress**
- **Risk Margin**
  - Approximations
  - Cost of capital rate
  - Insights into longevity risk pricing
- **Conclusion**

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The Forward Mortality Model

- **Computation of VaR requires stochastic modeling of mortality**
- **We use slightly modified version of forward model of Bauer et al. (2008, 2009)**
- **Advantage of forward model: no nested simulations are required**
- **Model is specified in Forward Mortality Framework (for details see Bauer et al. (2008))**
  - $\mu_t(T, x_0) = -\partial/\partial T \log\left\{E_P\left[{}_T p_{x_0} \middle| \mathfrak{I}_t\right]\right\}$
  - Dynamics $d\mu_t(T, x_0) = \alpha(t, T, x_0)dt + \sigma(t, T, x_0)dW_t, \quad \mu_0(T, x_0) > 0$
  - Drift condition: α fully specified by volatility σ
  - Here: σ deterministic, *W* finite dimensional Brownian motion
- **SCR/VaR is computed empirically based on 50,000 paths for the liabilities**

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Model Setup

- **Reference company situated in the UK, t=0 in 2007**
- **Risk-free interest rates: QIS4 term structure for UK**
- **Mortality rates: UK Life Office Pensioners in 2007**
- **Standard contracts:**
  - Life annuities with yearly payments of fixed amount in arrears
  - No options or guarantees, no fees, no surplus participation
- **Company‘s asset strategy:**
  - Risk-free assets only → no equity risk, credit risk etc.
  - Asset cash flows coincide with liability cash flows
  - Complete hedge against changes in interest rates
    → no interest rate risk and future interest rates are known

Comparison of SCR Formulas – Basis Case

- **Life annuity for a 65-year old paying GBP 1000 yearly in arrears**

	L_0	$L_1 - CF_1$	SCR	SCR/L_0
Shock approach	12619.28	14238.81	869.87	6.9%
VaR approach	12619.28	14050.62	691.59	5.5%

  - Shock approach demands about 26% more capital
  - This corresponds to 1.4% of the liabilities

  → **The deviation in SCRs is significant and the standard model longevity stress might overestimate the true risk**

- **Obvious questions:**
  - Does the deviation in SCRs change with age?
  - For which maturities/durations do deviations occur?

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Comparison of SCR Formulas – Different Ages

- **Different initial ages for life annuities paying GBP 1000 yearly in arrears**

Age	L_0	SCR^{shock}	$\frac{SCR^{shock}}{L_0}$	SCR^{VaR}	$\frac{SCR^{VaR}}{L_0}$	$\frac{\Delta SCR}{SCR^{VaR}}$	$\frac{\Delta SCR}{L_0}$
55	15671.10	657.23	4.2%	729.88	4.7%	-10.0%	-0.5%
65	12619.28	869.87	6.9%	691.59	5.5%	25.8%	1.4%
75	8941.83	1009.81	11.3%	513.27	5.7%	96.7%	5.6%
85	4940.13	1003.43	20.3%	304.89	6.2%	229.1%	14.1%
95	2549.75	818.58	32.1%	214.38	8.4%	281.8%	23.7%
105	1413.19	646.23	45.7%	180.79	12.8%	257.4%	32.9%

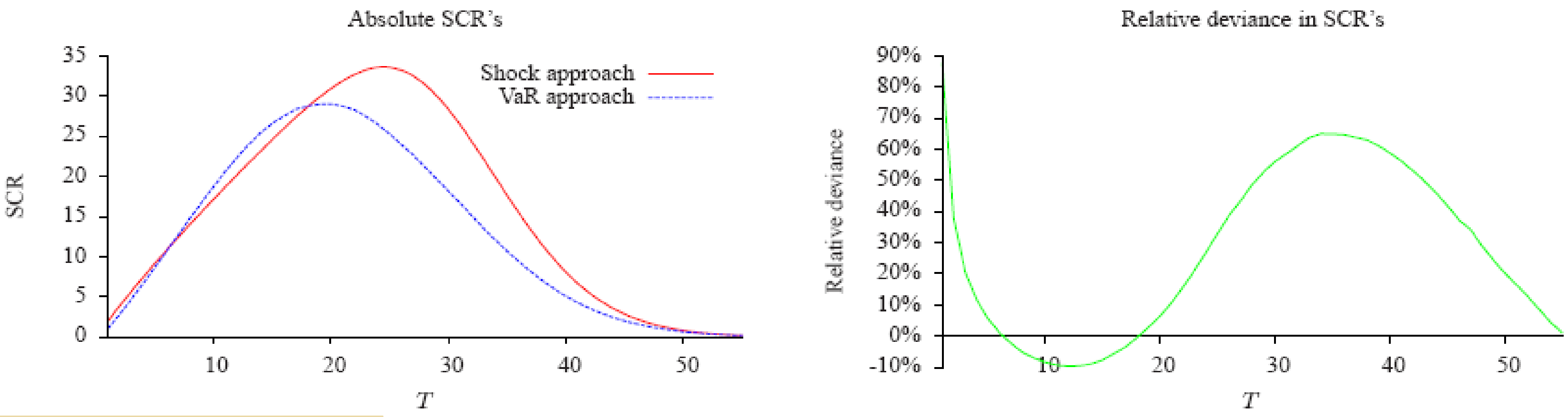
- SCR in shock approach first increases and then decreases
  - Reason: structure of the shock (the larger the mortality rates the larger the shocks)
- SCR in VaR approach decreases with age and liabilities which seems more intuitive
- Deviation becomes enormous for old ages
- 25% shock seems far too large
- Sole **adjustment of shock magnitude does not seem appropriate**

→ **Structural shortcoming of the standard model longevity stress:**

Age-dependent shock magnitude seems more appropriate

Comparison of SCR Formulas – Different Maturities

- **Decomposition of annuity in series of endowment contracts for a 65-year old paying GBP 1000 at maturity T**



  - Absolute SCRs are rather similar up to T=20
  - Thereafter, shock approach demands significantly more capital (larger shocks)
  - Relative deviations in SCRs vary considerably

→ **Structural shortcoming of the standard model longevity stress:**
Maturity-dependent shock (magnitude) seems more appropriate

Comparison of SCR Formulas – Deferred annuities

- **Deferred annuities of GBP 1000 in arrears starting at age 65**

Age	L_0	SCR^{shock}	$\frac{SCR^{shock}}{L_0}$	SCR^{VaR}	$\frac{SCR^{VaR}}{L_0}$	$\frac{\Delta SCR}{SCR^{VaR}}$	$\frac{\Delta SCR}{L_0}$
30	3205.97	217.90	6.8%	382.66	11.9%	-43.1%	-5.1%
35	3851.54	268.30	7.0%	428.53	11.1%	-37.4%	-4.2%
40	4623.92	329.89	7.1%	489.79	10.6%	-32.7%	-3.5%
45	5549.28	404.85	7.3%	561.71	10.1%	-27.9%	-2.8%
50	6676.64	495.51	7.4%	631.44	9.5%	-21.5%	-2.0%
55	8100.16	604.29	7.5%	688.98	8.5%	-12.3%	-1.1%
60	9978.02	733.27	7.4%	724.06	7.3%	1.3%	0.1%

- In both approaches, the SCRs increase with age
- For young ages, the VaR requires significantly more capital

→ **The shock approach seems to underestimate the long-term risk**

→ **Again, a longevity stress independent of age and maturity seems inadequate**

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Modified Standard Model Longevity Stress

- **Significant structural shortcomings of current standard model longevity stress: Age and maturity-dependent stress seems necessary to appropriately assess longevity risk**

- **Modified stress according to volatility in forward model**
  - Keep structure of one-off shock (→ integration in standard model remains the same)
  - **Shock T-year survival probabilities** by setting them to individual 99.5% quantiles:

$$E_P\left[{}_T\, p_{x_0}^{(T)} \middle| \Im_1\right] = E_P\left[{}_T\, p_{x_0}^{(T)}\right] \cdot \exp\left\{ -\int_0^T \int_0^1 \alpha(s,u,x_0)\,ds + \int_0^1 \sigma(s,u,x_0)\,dW_s\,du \right\}$$

  - A **matrix of shock factors** would have to be provided by supervisory authorities
  - Any diversification effects are neglected
    - Additional SCR between 5% and 10% for reasonable portfolios of immediate and deferred annuities
    - Acceptable shortcoming given the enormous structural improvements
    - Standard model is to be conservative

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Risk Margin Approximations

- **Technical Provisions („market value" of liabilities) consist of best estimate liabilities and Risk Margin**
- **Risk Margin = capital required to guarantee orderly run-off of a portfolio in case of insolvency**
- **Computation via cost of capital approach (CEIOPS (2009)):**

$$RM = \sum_{t\geq 0} \frac{CoC \cdot SCR_t}{(1+i_{t+1})^{t+1}}$$

- **Exact computation of Risk Margin practically impossible**
- **Approximations have been proposed (CEIOPS (2008a)):**
  - Assumption of best estimate mortality evolution („exact" computation)

$$RM^{(I)} = \sum_{t\geq 0} \frac{CoC}{(1+i_{t+1})^{t+1}} \cdot SCR_t^{BE}$$

  - Approximation of future SCRs

$$RM^{(II)} = \sum_{t\geq 0} \frac{CoC}{(1+i_{t+1})^{t+1}} \cdot 25\% \cdot q_t^{av} \cdot 1.1^{(dur_t - 1)/2} \cdot dur_t \cdot L_t$$

  - Assumption of constant ratio of SCRs and liabilities over time

$$RM^{(III)} = \sum_{t\geq 0} \frac{CoC}{(1+i_{t+1})^{t+1}} \cdot \frac{SCR_0}{L_0} \cdot L_t$$

  - Approximation of Risk Margin via modified duration of liabilities

$$RM^{(IV)} = CoC \cdot dur_0 \cdot SCR_0$$

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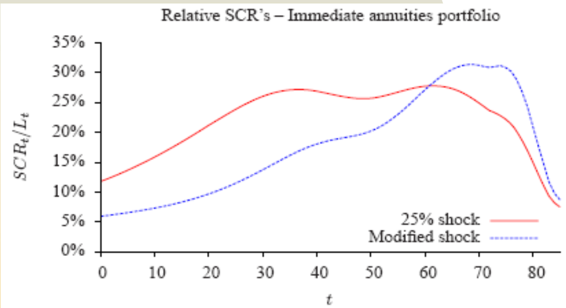
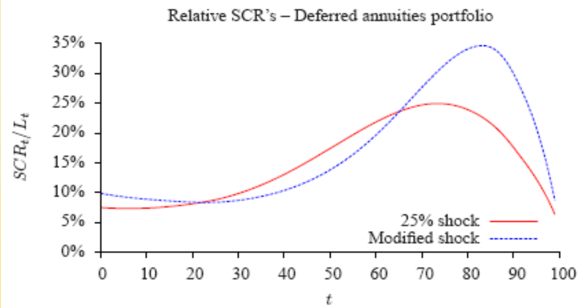
Risk Margin Approximations (ctd.)

Portfolio	Method	L_0	25% longevity shock			Modified longevity shock		
			RM	Rel. dev.	RM/L_0	RM	Rel. dev.	RM/L_0
Immediate annuities	(I)	36394.73	2383.87		6.6%	1143.08		3.1%
	(II)	36394.73	2751.71	15.4%	7.6%	n/a	n/a	n/a
	(III)	36394.73	1957.24	-17.9%	5.4%	988.11	-13.6%	2.7%
	(IV)	36394.73	2051.70	-13.9%	5.6%	1035.80	-9.4%	2.9%
Deferred annuities	(I)	88165.37	11240.74		12.8%	12159.44		13.8%
	(II)	88165.37	10126.91	-9.9%	11.5%	n/a	n/a	n/a
	(III)	88165.37	10034.70	-10.7%	11.4%	13206.71	8.6%	15.0%
	(IV)	88165.37	10488.60	-6.7%	11.9%	13804.08	13.5%	15.7%





- **Risk Margin approximations are rather crude**
  - Wide range of values is problematic:
    - Comparisons of companies' solvency situations can get blurred due to use of different risk margin approximations
    - Companies might choose approximation which yields the smallest value
- **Performance of popular assumption (III) of constant ratios of SCRs and liabilities is rather poor**
  - In general, ratios seem to increase over time
  - Haslip (2008) makes the same observation for non-life insurance
  - Dependence e.g. on average age in portfolio might improve proxi

The Cost of Capital Rate

- **Ongoing discussion on adequate calibration of cost of capital rate**
  - Calibration is crucial (Risk Margin is linear in cost of capital rate)
  - Currently set to 6%
  - Values between 2% and 8% are regarded as reasonable (see CEIOPS (2009))
- **Inferences on calibration by comparison with hypothetical market prices for longevity risk**
  - Idea: If there was a market, the Risk Margin should coincide with the markup in this market
  - Forward modeling framework: Risk-adjusted survival probabilities can be derived via a deterministic „market price of longevity risk process":

$$E_Q\left[{}_T p_{x_0}^{(T)} \middle| \Im_t\right] = \exp\left\{-\int_0^T \int_t^u \sigma(u,s,x_0)\cdot\lambda(s)\,ds\,du\right\}\cdot E_P\left[{}_T p_{x_0}^{(T)} \middle| \Im_t\right]$$

  - Setup: Risk Margin = risk-adjusted liabilities – best estimate liabilities → Sharpe ratio

Portfolio of contracts	25% longevity shock		Modified longevity shock	
	RM	λ	RM	λ
Immediate annuities	2383.87	18.6%	1143.08	13.2%
Deferred annuities	11240.74	8.7%	12159.44	8.9%

  - The Sharpe ratios are reasonable but rather small
    → **The cost of capital rate of 6% does not seem overly conservative**

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Market Prices for Longevity Derivatives

- **Alternative interpretation of Sharpe ratios: willingness to pay for longevity risk securitization**

- **Rationale: Company may be indifferent between keeping and transferring longevity risk if price for securitization coincides with Risk Margin**
  - Keeping the risk implies the payment of cost of capital to providers of solvency capital: Risk Margin is present value of these cost of capital
  - Transferring risk implies payment of markup above best estimate liabilities

- **Influence of other effects, e.g.**
  - Expected own cost of capital lower than Risk Margin
  - Diversification with other risks
  - Strategic reasons (e.g. abandonment of line of business)
  - Difficulties in raising solvency capital
- **Effects differently relevant for different companies**

- **Market‘s appetite for longevity risk will finally decide on prices**
- **Nevertheless, the Risk Margin can provide valuable insights in and starting point for pricing of longevity derivatives**

Conclusion

- **Structural shortcomings in the current standard model longevity stress**
  - Possibly significant overestimation or underestimation of true risk
  - Age and maturity dependent longevity stress required

- **Proposition of modified shock**
  - Simple in structure (one-off shock)
  - Age and maturity dependent
  - Conservative due to waiving of diversification effects

- **Risk Margin approximations yield wide range of values**
  - Comparison of solvency situations difficult
  - Undesired incentives (minimization of Risk Margin)

- **Assumption of SCR proportional to liabilities in general not appropriate**
  - Risk Margin might be too small due to mostly increasing ratio of SCRs and liabilities

- **Cost of capital rate of 6% does not seem overly conservative**

- **Solvency requirements can provide valuable insights into pricing of longevity derivatives**
  - Capital requirements determine companies' willingness to pay for securitization

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