Deterministic Shock vs. Stochastic Value-at-Risk – An Analysis of the Solvency II Standard Model Approach to Longevity Risk

Matthias Börger

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Introduction

- **Longevity risk** = risk of insureds on average surviving longer than expected
  - Significant risk for pension funds and annuity providers
  - **Systematic** and **non-hedgeable** risk
    → Explicitly accounted for under Solvency II

- **General concept** for Solvency Capital Requirement (SCR) under Solvency II
  - **SCR** = 99.5% **Value-at-Risk** (VaR) of Available Capital over 1 year
  - “Capital necessary to cover losses over next year with at least 99.5% probability”
  - Stochastic (internal) models required whose implementation is costly and highly sophisticated

- **Solvency II Standard model**
  - Scenario-based rather than stochastic, modular approach
  - Longevity risk: **SCR** = change in Net Asset Value (NAV = assets – best estimate liabilities) due to **longevity shock**
  - Longevity shock is a permanent 25% reduction of mortality rates for all ages
  - Value of 25% is mainly based on what UK insurance companies in 2004 regarded consistent with VaR concept (CEIOPS (2007))
Objective

- Motivation of the standard model longevity stress is rather poor
  - UK insurance companies regarded shock between 5% and 35% as appropriate
  - 25% longevity shock could significantly misjudge the true risk
  - Analysis of the longevity stress is required
  - Comparison with VaR for longevity risk

- Questions regarding structure and calibration:
  - Is a constant shock for all ages and maturities reasonable?
    - QIS4 participants question whether trend risk is appropriately accounted for (CEIOPS (2008b))
  - Is the shock magnitude of 25% adequate?
    - QIS4 participants regard shock as very high, internal models required significantly less capital (CEIOPS (2008b))
  - How can the standard model longevity stress possibly be improved?
Agenda

- Introduction
- Forward mortality model
- Model setup
- Comparison of SCR formulas for longevity risk
- Modification of standard model longevity stress
- Risk Margin
  - Approximations
  - Cost of capital rate
  - Insights into longevity risk pricing
- Conclusion
Computation of VaR requires stochastic modeling of mortality

We use slightly modified version of forward model of Bauer et al. (2008, 2009)

Advantage of forward model: no nested simulations are required

Model is specified in Forward Mortality Framework (for details see Bauer et al. (2008))

\[
\mu_t(T, x_0) = -\frac{\partial}{\partial T} \log \left\{ E_P \left[ T p_{x_0} | \mathcal{F}_t \right] \right\}
\]

Dynamics: \( d\mu_t(T, x_0) = \alpha(t, T, x_0) dt + \sigma(t, T, x_0) dW_t, \quad \mu_0(T, x_0) > 0 \)

Drift condition: \( \alpha \) fully specified by volatility \( \sigma \)

Here: \( \sigma \) deterministic, \( W \) finite dimensional Brownian motion

SCR/VaR is computed empirically based on 50,000 paths for the liabilities
Model Setup

- Reference company situated in the UK, t=0 in 2007
- Risk-free interest rates: QIS4 term structure for UK
- Mortality rates: UK Life Office Pensioners in 2007
- Standard contracts:
  - Life annuities with yearly payments of fixed amount in arrears
  - No options or guarantees, no fees, no surplus participation
- Company’s asset strategy:
  - Risk-free assets only → no equity risk, credit risk etc.
  - Asset cash flows coincide with liability cash flows
  - Complete hedge against changes in interest rates
    → no interest rate risk and future interest rates are known
Comparison of SCR Formulas - Basis Case

- Life annuity for a 65-year old paying GBP 1000 yearly in arrears

<table>
<thead>
<tr>
<th></th>
<th>$L_0$</th>
<th>$L_1 - CF_1$</th>
<th>SCR</th>
<th>SCR/$L_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shock approach</td>
<td>12619.28</td>
<td>14238.81</td>
<td>869.87</td>
<td>6.9%</td>
</tr>
<tr>
<td>VaR approach</td>
<td>12619.28</td>
<td>14050.62</td>
<td>691.59</td>
<td>5.5%</td>
</tr>
</tbody>
</table>

- Shock approach demands about 26% more capital
- This corresponds to 1.4% of the liabilities

→ The deviation in SCRs is significant and the standard model longevity stress might overestimate the true risk

Obvious questions:
- Does the deviation in SCRs change with age?
- For which maturities/durations do deviations occur?
Comparison of SCR Formulas - Different Ages

Different initial ages for life annuities paying GBP 1000 yearly in arrears

<table>
<thead>
<tr>
<th>Age</th>
<th>$L_0$</th>
<th>$\text{SCR}^{\text{shock}}$</th>
<th>$\frac{\text{SCR}^{\text{shock}}}{L_0}$</th>
<th>$\text{SCR}^{\text{VaR}}$</th>
<th>$\frac{\text{SCR}^{\text{VaR}}}{L_0}$</th>
<th>$\Delta \text{SCR}^{\text{shock}}$</th>
<th>$\Delta \text{SCR}^{\text{VaR}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>55</td>
<td>15671.10</td>
<td>657.23</td>
<td>4.2%</td>
<td>729.88</td>
<td>4.7%</td>
<td>-10.0%</td>
<td>-0.5%</td>
</tr>
<tr>
<td>65</td>
<td>12619.28</td>
<td>869.87</td>
<td>6.9%</td>
<td>691.59</td>
<td>5.5%</td>
<td>25.8%</td>
<td>1.4%</td>
</tr>
<tr>
<td>75</td>
<td>8941.83</td>
<td>1009.81</td>
<td>11.3%</td>
<td>513.27</td>
<td>5.7%</td>
<td>96.7%</td>
<td>5.6%</td>
</tr>
<tr>
<td>85</td>
<td>4940.13</td>
<td>1003.43</td>
<td>20.3%</td>
<td>304.89</td>
<td>6.2%</td>
<td>229.1%</td>
<td>14.1%</td>
</tr>
<tr>
<td>95</td>
<td>2549.75</td>
<td>818.58</td>
<td>32.1%</td>
<td>214.38</td>
<td>8.4%</td>
<td>281.8%</td>
<td>23.7%</td>
</tr>
<tr>
<td>105</td>
<td>1413.19</td>
<td>646.23</td>
<td>45.7%</td>
<td>180.79</td>
<td>12.8%</td>
<td>257.4%</td>
<td>32.9%</td>
</tr>
</tbody>
</table>

- SCR in shock approach first increases and then decreases
  - Reason: structure of the shock (the larger the mortality rates the larger the shocks)
- SCR in VaR approach decreases with age and liabilities which seems more intuitive
- Deviation becomes enormous for old ages
- 25% shock seems far too large
- Sole adjustment of shock magnitude does not seem appropriate

→ Structural shortcoming of the standard model longevity stress:
  Age-dependent shock magnitude seems more appropriate
Decomposition of annuity in series of endowment contracts for a 65-year old paying GBP 1000 at maturity $T$

- Absolute SCRs are rather similar up to $T=20$
- Thereafter, shock approach demands significantly more capital (larger shocks)
- Relative deviations in SCRs vary considerably

→ Structural shortcoming of the standard model longevity stress:

**Maturity-dependent shock** (magnitude) seems more appropriate
Comparison of SCR Formulas - Deferred annuities

Deferred annuities of GBP 1000 in arrears starting at age 65

<table>
<thead>
<tr>
<th>Age</th>
<th>$L_0$</th>
<th>$SCR_{Shock}$</th>
<th>$SCR_{Shock} / L_0$</th>
<th>$SCR_{VaR}$</th>
<th>$SCR_{VaR} / L_0$</th>
<th>$ΔSCR_{VaR}$</th>
<th>$ΔSCR$</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>3205.97</td>
<td>217.90</td>
<td>6.8%</td>
<td>382.66</td>
<td>11.9%</td>
<td>-43.1%</td>
<td>-5.1%</td>
</tr>
<tr>
<td>35</td>
<td>3851.54</td>
<td>268.30</td>
<td>7.0%</td>
<td>428.53</td>
<td>11.1%</td>
<td>-37.4%</td>
<td>-4.2%</td>
</tr>
<tr>
<td>40</td>
<td>4623.92</td>
<td>329.89</td>
<td>7.1%</td>
<td>489.79</td>
<td>10.6%</td>
<td>-32.7%</td>
<td>-3.5%</td>
</tr>
<tr>
<td>45</td>
<td>5549.28</td>
<td>404.85</td>
<td>7.3%</td>
<td>561.71</td>
<td>10.1%</td>
<td>-27.9%</td>
<td>-2.8%</td>
</tr>
<tr>
<td>50</td>
<td>6676.64</td>
<td>495.51</td>
<td>7.4%</td>
<td>631.44</td>
<td>9.5%</td>
<td>-21.5%</td>
<td>-2.0%</td>
</tr>
<tr>
<td>55</td>
<td>8100.16</td>
<td>604.29</td>
<td>7.5%</td>
<td>688.98</td>
<td>8.5%</td>
<td>-12.3%</td>
<td>-1.1%</td>
</tr>
<tr>
<td>60</td>
<td>9978.02</td>
<td>733.27</td>
<td>7.4%</td>
<td>724.06</td>
<td>7.3%</td>
<td>1.3%</td>
<td>0.1%</td>
</tr>
</tbody>
</table>

In both approaches, the SCRs increase with age
- For young ages, the VaR requires significantly more capital
  - The shock approach seems to underestimate the long-term risk
  - Again, a longevity stress independent of age and maturity seems inadequate
Modified Standard Model Longevity Stress

- Significant structural shortcomings of current standard model longevity stress: Age and maturity-dependent stress seems necessary to appropriately assess longevity risk.

- Modified stress according to volatility in forward model:
  - Keep structure of one-off shock (integration in standard model remains the same).
  - Shock T-year survival probabilities by setting them to individual 99.5% quantiles:
    \[
    E_P \left[ T P_{x_0}^{(T)} | \mathcal{F}_1 \right] = E_P \left[ T P_{x_0}^{(T)} \right] \cdot \exp \left\{ - \int_0^T \alpha(s, u, x_0) ds + \int_0^1 \sigma(s, u, x_0) dW_s du \right\}
    \]
  - A matrix of shock factors would have to be provided by supervisory authorities.
  - Any diversification effects are neglected.
    - Additional SCR between 5% and 10% for reasonable portfolios of immediate and deferred annuities.
    - Acceptable shortcoming given the enormous structural improvements.
    - Standard model is to be conservative.
Risk Margin Approximations

- Technical Provisions ("market value" of liabilities) consist of best estimate liabilities and Risk Margin
- Risk Margin = capital required to guarantee orderly run-off of a portfolio in case of insolvency
- Computation via cost of capital approach (CEIOPS (2009)):
  \[ RM = \sum_{i \geq 0} \frac{CoC \cdot SCR_i}{(1 + i_{i+1})^{i+1}} \]

- Exact computation of Risk Margin practically impossible
- Approximations have been proposed (CEIOPS (2008a)):
  1. Assumption of best estimate mortality evolution ("exact" computation)
    \[ RM^{(I)} = \sum_{i \geq 0} \frac{CoC}{(1 + i_{i+1})^{i+1}} \cdot SCR_i^{BE} \]
  2. Approximation of future SCRs
    \[ RM^{(II)} = \sum_{i \geq 0} \frac{CoC}{(1 + i_{i+1})^{i+1}} \cdot 25\% \cdot q_i^{av} \cdot 1.1^{(dur_i-1)/2} \cdot dur_i \cdot L_i \]
  3. Assumption of constant ratio of SCRs and liabilities over time
    \[ RM^{(III)} = \sum_{i \geq 0} \frac{CoC}{(1 + i_{i+1})^{i+1}} \cdot \frac{SCR_i}{L_i} \cdot L_i \]
  4. Approximation of Risk Margin via modified duration of liabilities
    \[ RM^{(IV)} = CoC \cdot dur_0 \cdot SCR_0 \]
Risk Margin Approximations (ctd.)

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>Method</th>
<th>$L_0$</th>
<th>25% longevity shock</th>
<th>Modified longevity shock</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>$RM$</td>
<td>Rel. dev.</td>
</tr>
<tr>
<td>Immediate</td>
<td>(I)</td>
<td>36394.73</td>
<td>2383.87</td>
<td>6.6%</td>
</tr>
<tr>
<td>annuities</td>
<td>(II)</td>
<td>36394.73</td>
<td>2751.71</td>
<td>15.4%</td>
</tr>
<tr>
<td></td>
<td>(III)</td>
<td>36394.73</td>
<td>1957.24</td>
<td>-17.9%</td>
</tr>
<tr>
<td></td>
<td>(IV)</td>
<td>36394.73</td>
<td>2051.70</td>
<td>-13.9%</td>
</tr>
<tr>
<td>Deferred</td>
<td>(I)</td>
<td>88165.37</td>
<td>11240.74</td>
<td>12.8%</td>
</tr>
<tr>
<td>annuities</td>
<td>(II)</td>
<td>88165.37</td>
<td>10126.91</td>
<td>-9.9%</td>
</tr>
<tr>
<td></td>
<td>(III)</td>
<td>88165.37</td>
<td>10034.70</td>
<td>-10.7%</td>
</tr>
<tr>
<td></td>
<td>(IV)</td>
<td>88165.37</td>
<td>10488.60</td>
<td>-6.7%</td>
</tr>
</tbody>
</table>

- **Risk Margin approximations are rather crude**
  - Wide range of values is problematic:
    - Comparisons of companies' solvency situations can get blurred due to use of different risk margin approximations
    - Companies might choose approximation which yields the smallest value

- **Performance of popular assumption (III) of constant ratios of SCRs and liabilities is rather poor**
  - In general, ratios seem to increase over time
  - Haslip (2008) makes the same observation for non-life insurance
  - Dependence e.g. on average age in portfolio might improve proxi
The Cost of Capital Rate

- Ongoing discussion on adequate calibration of cost of capital rate
  - Calibration is crucial (Risk Margin is linear in cost of capital rate)
  - Currently set to 6%
  - Values between 2% and 8% are regarded as reasonable (see CEIOPS (2009))

- Inferences on calibration by comparison with hypothetical market prices for longevity risk
  - Idea: If there was a market, the Risk Margin should coincide with the markup in this market
  - Forward modeling framework: Risk-adjusted survival probabilities can be derived via a deterministic „market price of longevity risk process“:
    \[
    E_Q \left[ \prod_{T} p_{x_0}^{(T)} | \Omega_T \right] = \exp \left\{ - \int \sigma(u, s, x_0) \cdot \lambda(s) ds \; du \right\} \cdot E_P \left[ \prod_{T} p_{x_0}^{(T)} | \Omega_T \right]
    \]
  - Setup: Risk Margin = risk-adjusted liabilities – best estimate liabilities → Sharpe ratio

<table>
<thead>
<tr>
<th>Portfolio of contracts</th>
<th>25% longevity shock</th>
<th>Modified longevity shock</th>
</tr>
</thead>
<tbody>
<tr>
<td>RM (\lambda)</td>
<td>RM (\lambda)</td>
<td></td>
</tr>
<tr>
<td>Immediate annuities</td>
<td>2383.87 18.6%</td>
<td>1143.08 13.2%</td>
</tr>
<tr>
<td>Deferred annuities</td>
<td>11240.74 8.7%</td>
<td>12159.44 8.9%</td>
</tr>
</tbody>
</table>

- The Sharpe ratios are reasonable but rather small
  → The cost of capital rate of 6% does not seem overly conservative
Market Prices for Longevity Derivatives

- Alternative interpretation of Sharpe ratios: willingness to pay for longevity risk securitization

- Rationale: Company may be indifferent between keeping and transferring longevity risk if price for securitization coincides with Risk Margin
  - Keeping the risk implies the payment of cost of capital to providers of solvency capital: Risk Margin is present value of these cost of capital
  - Transferring risk implies payment of markup above best estimate liabilities

- Influence of other effects, e.g.
  - Expected own cost of capital lower than Risk Margin
  - Diversification with other risks
  - Strategic reasons (e.g. abandonment of line of business)
  - Difficulties in raising solvency capital

- Effects differently relevant for different companies

- Market’s appetite for longevity risk will finally decide on prices
- Nevertheless, the Risk Margin can provide valuable insights in and starting point for pricing of longevity derivatives
Conclusion

- **Structural shortcomings in the current standard model longevity stress**
  - Possibly significant overestimation or underestimation of true risk
  - Age and maturity dependent longevity stress required

- **Proposition of modified shock**
  - Simple in structure (one-off shock)
  - Age and maturity dependent
  - Conservative due to waiving of diversification effects

- **Risk Margin approximations yield wide range of values**
  - Comparison of solvency situations difficult
  - Undesired incentives (minimization of Risk Margin)

- **Assumption of SCR proportional to liabilities in general not appropriate**
  - Risk Margin might be too small due to mostly increasing ratio of SCRs and liabilities

- **Cost of capital rate of 6% does not seem overly conservative**

- **Solvency requirements can provide valuable insights into pricing of longevity derivatives**
  - Capital requirements determine companies' willingness to pay for securitization
References