On the Pricing of Longevity-Linked Securities

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Agenda

- Introduction
- Different Approaches for Pricing Longevity-Linked Securities
- Theoretical Comparison of the Approaches
- Empirical Comparison of the Approaches
- An Option-Type Longevity Derivative
- Conclusion
Introduction

- Longevity risk = The risk that future mortality improvement exceeds today’s assumptions
  - Important risk factor for annuity providers and pension funds
  - Importance of this risk will increase in the future
    - reduction of benefits from public pension systems
    - tax incentives for annuitization
  - Securitization is seen as a solution for managing this risk:
    - In the literature: Survivor bonds; survivor swaps, longevity bonds,…
    - In practice: First attempt to issue a longevity linked security failed.
    - However: There appears to be a consensus that suitable instruments will be available in the near future
  - Interesting question: How to price such instruments
    - What are suitable (actuarial or economic) methods?
    - How can such methodologies be applied (calibration, etc.)?
Different Approaches for Pricing Longevity-Linked Securities

- Price of a longevity derivative depends on the estimate of uncertain future mortality trends and the degree of uncertainty of this estimate → Mortality risk premium (MRP)

- Problem: There are no liquidly traded securities → MRP can not be observed in the market

- Consequence: Different pricing methods have been proposed

- CAPM/CCAPM based approach (Friedberg and Webb 2007)
  - MRP suggested by the models is very low (MRP-puzzle similar to equity premium puzzle)
  - → Probably limited applicability of this approach

- Instantaneous Sharpe Ratio (ISR) based approach (Milevsky et al. 2005; Bayraktar et al. 2008)
  - Investor in longevity risk requires compensation according to some ISR (λ)
  - Return in excess of risk free return = λ * standard deviation (after diversifiable risk is “hedged”)
  - For large portfolio size this coincides with a change of probability measure (P→Q) with a constant market price of risk

  - Adjust the cdf of the future lifetime by a Wang transform to account for risk:
    
    \[ q^Q_x = \Phi(\Phi^{-1}(q^P_x) - \theta) \text{ or } q^Q_x = \Psi(\Phi^{-1}(q^P_x) - \theta) \]
Theoretical Comparison of the Approaches

Our methodology: Establish the different approaches in a common framework

- “Forward Mortality Framework” (Details see Bauer et al. (2008))
- $\mu_t(T, x_0) = -\frac{\partial}{\partial T} \log \mathbb{E}_\mathbb{P}\left[ \prod_{t} p_{x_0} \right]$ 
- Dynamics $d\hat{\mu}_t(T, x_0) = \hat{\alpha}(t, T, x_0)dt + \hat{\sigma}(t, T, x_0)dW_t, \quad \hat{\mu}_0(T, x_0) > 0$
- Drift condition: $\hat{\alpha}$ only depends on volatility (as in HJM forward interest rate modeling)
- Here:
  - $W$ finite dimensional Brownian motion
  - $\hat{\sigma}$ and market price of risk deterministic
  - Volatilities and hence dynamics under measures $\mathbb{P}$ and $\mathbb{Q}$ coincide
- Initial “risk-adjusted” forward mortality curves derived from best estimate curve using both pricing methods
If there is one, which is the better of the two approaches?

- Wang transform not coherent with a “generic” pricing model in the forward framework if more than one age cohort is considered.
- In line with Pelsser (2008): Inconsistency with arbitrage-free prices
- Hence, the Sharpe ratio approach is the more general and better approach

What is a good basis for determining $\theta$ and $\lambda$?

- Loeys et al.: (Sharpe ratio from) stock markets
  - But: different characteristics
  - Adequacy questionable!
- Lin & Cox: Annuity Prices
  - Strong empirical evidence that there is a significant mortality risk premium embedded in annuity prices
  - Possibly, there are also risk premiums for other sources of risk (e.g. non-systematic mortality risk)
  - Hence, annuity prices provide at least an upper bound for risk premiums in longevity derivative pricing
Empirical Comparison of the Approaches

- We use the “Volatility of Mortality” model from Bauer et al (2008) and recalibrate to UK data
- We derive Sharpe Ratios and Wang Transform parameters from monthly UK annuity quotes (January 2000 to December 2006)

- We find significant correlation between the market price of mortality risk and stock markets / interest rates
  → Assumption of independence between risk-adjusted mortality evolution and financial markets seems to be inadequate
Empirical Comparison of the Approaches (ctd.)

- We then apply different pricing methodologies to the EIB/BNP-Bond
  - Best estimate valuation
  - Sharpe Ratio calibrated to UK annuity quotes
  - Sharpe Ratio from stock markets
  - 1 factor Wang Transform calibrated to UK annuity quotes
  - 1 factor Wang Transform calibrated to US annuity quotes (Calibration from Lin and Cox 2005)
  - 2 factor Wang Transform calibrated to UK annuity quotes
  - 2 factor Wang Transform calibrated to US annuity quotes (Calibration from Lin and Cox 2006)

- Design of the EIB/BNP-Bond
  - Notional = GBP 50m; Pays annual coupons for 25 years
  - Coupons depend on mortality experience of English and Welsh males aged 65 in 2003
- The EIB/BNP-Bond was offered at GBP 540m
Empirical Comparison of the Approaches (ctd.)

- **Lin and Cox (2006): Risk premium is very high → Bond is unattractive**
  - Conclusion is based on a Wang Transform approach

- **Cairns et al. (2006): Price seems reasonable**
  - Conclusion is based on an approach similar to an Instantaneous Sharpe Ratio approach

- We “repriced” the bond using the 7 methods above and two hypothetical bonds of the same design but being offered in November 2002 and November 2006, respectively

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- Significant differences between issue dates and 7 pricing models
  - Due to changes in interest rates, mortality projections and Sharpe Ratio / Wang Transform parameter calibrations

- All “risk-adjusting” models result in values that exceed the quoted price

- Quoted price in the middle of best estimate and risk-adjusted valuation
  → The Bond seems to have been a “good deal” or at least fairly priced
Empirical Comparison of the Approaches (ctd.)

- If the EIB/BNP-Bond was a fair if not good deal, two questions arise:
  - Why did Lin & Cox regard the Bond as too expensive?
    - They used a different yield curve and survival rates based on realized mortality rates in 2003 as opposed to projections
  - Why was it not successfully placed?
    - Based on population as opposed to inureds (basis risk)
    - Fixed maturity of the bond $\Rightarrow$ tail risk is not hedged
    - Capital intensive hedge

- We conclude that the financial engineering and not the pricing was the reason for the failure of the EIB/BNP-Bond.
  - Therefore, in the final section, we analyzed a call-option-type longevity derivative
An Option-Type Longevity Derivative

**Payoff:** \( C_T = \left( T \, p_{x_0} - K(T) \right)^+ \) with strike \( K(T) = (1 + a)E_p \left[ T \, p_{x_0} \right], \ a > 0 \)

By suitable adjustment of the strike (choice of the parameter \( a \)), the insurer can decide, which portion of the risk to keep

Such derivatives can be priced within our framework with a Black-type formula (Bauer 2007)

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As expected: \( \mathcal{N} \) in \( T \)

As expected: \( \mathcal{N} \) in \( a \)

Sometimes large differences despite calibration to the same data

2 questions:

- Where do these differences come from?
- Which approach yields the “correct” price?
An Option-Type Longevity Derivative

- The risk premium allocations differ considerably between the pricing approaches.

![Graph showing best estimate probabilities for different approaches over time.](image)
An Option-Type Longevity Derivative

The risk premium allocations differ considerably between the pricing approaches:

- **short maturities**
  - red: Sharpe ratio approach
  - green: 1-factor Wang transform approach
  - blue: 2-factor Wang transform approach

- **large maturities**

  Sharpe ratio approach: risk premium proportional to aggregated risk

  Wang Transform: risk premium allocation independent of actual risk

  → Adequacy of the Wang Transform again questionable
Conclusion

- Overview and comparison of different pricing approaches

- Risk premium implied by the Wang Transform induces inconsistencies if securities on different ages are traded
  - Even if just one security is traded, the “risk premium allocation” appears questionable

- We conclude that currently a “market price of longevity risk” should be derived from annuity quotes
  - Adopting Sharpe Ratios from equity markets appears to have weaknesses

- We identify significant correlation between the market price of longevity risk and stock markets / interest rates
  - Assuming independence between risk-adjusted mortality evolution and financial markets seems to be inadequate

- The EIB/BNP-Bond appears to have been offered at a fair if not good price
  - Reason for failure was financial engineering rather than pricing
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- Run by a Research Training Group at Ulm University
- Please feel encouraged to submit your papers!
- submission@mortalityrisk.org
References