Deterministic Shock vs. Stochastic Value-at-Risk –
An Analysis of the Solvency II Standard Model Approach to Longevity Risk

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Introduction

- **Longevity risk = risk of insured on average surviving longer than expected**
  - Significant risk for pension funds and annuity providers
  - Systematic and non-hedgeable risk
    → **Explicitly accounted for under Solvency II**

- **General concept for Solvency Capital Requirement (SCR) under Solvency II**
  - SCR = 99.5% Value-at-Risk (VaR) of Available Capital over 1 year
  - „Capital necessary to cover losses over next year with at least 99.5% probability“
  - Stochastic (internal) models required whose implementation is costly and sophisticated

- **Solvency II Standard model**
  - Scenario-based rather than stochastic, modular approach
  - Longevity risk: SCR = change in Net Asset Value (NAV) due to longevity shock
  - Longevity shock is a permanent 25% reduction of mortality rates for all ages

Objective

- **Motivation of the standard model longevity stress is rather poor**
  - Value of 25% is mainly based on what UK insurance companies in 2004 regarded consistent with VaR concept (CEIOPS (2007))
  - UK insurance companies regarded shock between 5% and 35% as appropriate

  → 25% longevity shock could significantly misjudge the true risk

- **Analysis of the longevity stress regarding structure and calibration is required**
  - Is an equal shock for all ages and maturities reasonable?
  - What should the magnitude of the shock be?
  - How can the standard model longevity stress possibly be improved?

→ Comparison with VaR for longevity risk

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Agenda

- Introduction
- Mortality modeling
- Model setup
- Comparison of SCR formulas for longevity risk
- Modification of standard model longevity stress
- Analysis of Risk Margin
- Summary

The Forward Mortality Model

- **In 1-year setting, longevity risk consists of two components:**
  - Low realized mortality in the one year
  - Decrease in expected future mortality
- **A stochastic mortality model must account for both components**
  - Well known spot mortality models do not cover possible changes in expected mortality
  - Forward mortality model is required
- **We use slightly modified version of forward model of Bauer et al. (2008, 2010)**

Model Setup

- **Reference company situated in the UK**
- **Company is solely exposed to longevity risk**
- **Risk-free interest rates: QIS4 term structure for UK for 2007**
- **Initial mortality rates: UK Life Office Pensioners in 2007**
- **Standard contracts:**
  - Immediate and deferred life annuities with yearly payments of fixed amount in arrears
  - No options or guarantees, no fees, no surplus participation

Comparison of SCR Formulas – Different Ages

- **Life annuities paying GBP 1000 yearly in arrears for different ages**

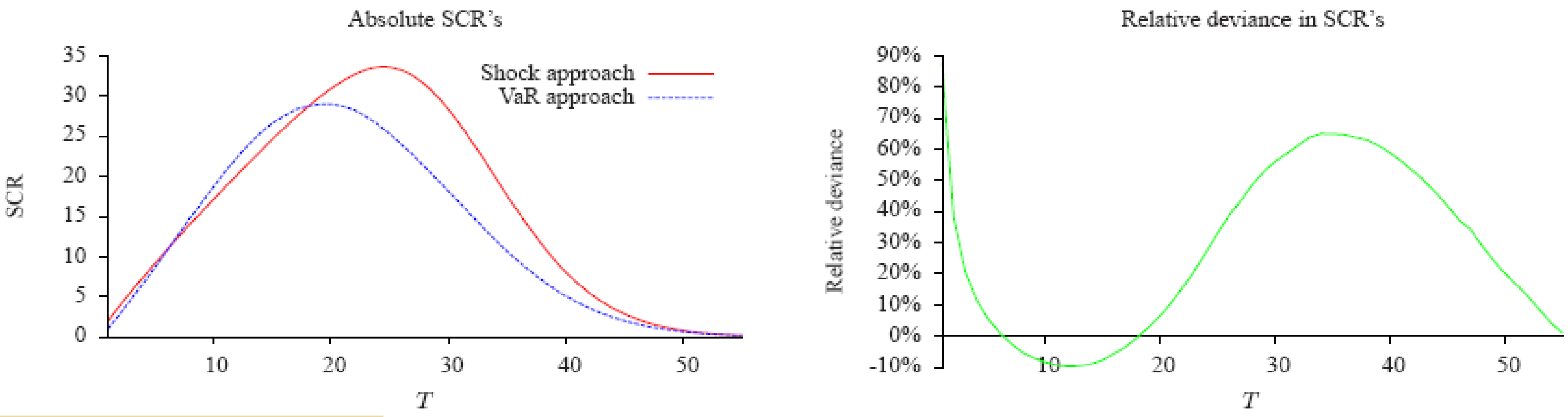
Age	L_0	SCR^{shock}	$\frac{SCR^{shock}}{L_0}$	SCR^{VaR}	$\frac{SCR^{VaR}}{L_0}$	$\frac{\Delta SCR}{SCR^{VaR}}$	$\frac{\Delta SCR}{L_0}$
55	15671.10	657.23	4.2%	729.88	4.7%	-10.0%	-0.5%
65	12619.28	869.87	6.9%	691.59	5.5%	25.8%	1.4%
75	8941.83	1009.81	11.3%	513.27	5.7%	96.7%	5.6%
85	4940.13	1003.43	20.3%	304.89	6.2%	229.1%	14.1%
95	2549.75	818.58	32.1%	214.38	8.4%	281.8%	23.7%
105	1413.19	646.23	45.7%	180.79	12.8%	257.4%	32.9%

- Deviation becomes enormous for old ages
- 25% shock seems to overestimate longevity risk significantly
- Sole adjustment of shock magnitude does not seem appropriate

→ Structural shortcoming of the standard model longevity stress:
Age-dependent shock magnitude seems more appropriate

Comparison of SCR Formulas – Different Maturities

- **Decomposition of annuity in series of endowment contracts for a 65-year old paying GBP 1000 at maturity T**



- Absolute SCRs are rather similar up to T=20
- Thereafter, shock approach demands significantly more capital (larger shocks)
- Relative deviations in SCRs vary considerably

→ Structural shortcoming of the standard model longevity stress: Maturity-dependent shock (magnitude) seems more appropriate

Modified Standard Model Longevity Stress

- **Current standard model longevity stress does not seem to reflect the true longevity risk**

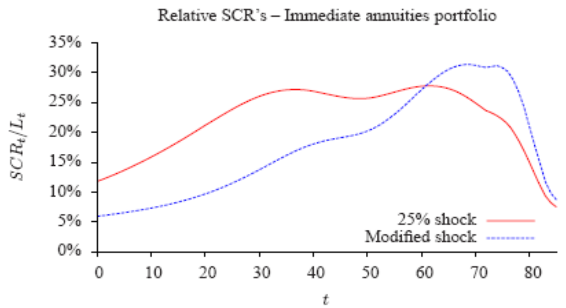
- **Modified stress according to volatility in forward model**
  - Keep structure of one-off shock (→ integration in standard model remains the same)
  - Shock T-year survival probabilities by setting them to individual 99.5% quantile
  - Application of shock by multiplying best estimate survival probabilities by factors
  - A matrix of shock factors would have to be provided by supervisory authorities (→ complexity basically unchanged)

- **Any diversification effects are neglected**
  - Additional SCR between 5% and 10% for reasonable portfolios
  - Acceptable shortcoming given the enormous structural improvements
  - Standard model is to be conservative

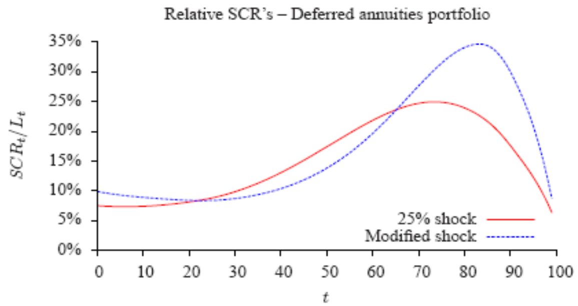
Analysis of Risk Margin

- **Technical Provisions („market value“ of liabilities) = Best Estimate Liabilities + Risk Margin**
- **Risk Margin = capital required to guarantee run-off of a portfolio in case of insolvency (cost of capital approach)**
- **4 main findings (future SCRs computed based on 25% shock and modified shock):**

1. **Risk Margin approximations yield wide range of values**
   - Variation of up to 30% for reasonable portfolios
     → Limited comparability and undesired incentives!



2. **Popular assumption of future SCRs being proportional to future liabilities is not adequate in general**
   - Ratios typically increase over time → Risk is underestimated!



3. **Cost of capital rate of 6% does not seem overly conservative compared to hypothetical market prices for longevity risk**
   - Survival probabilities are adjusted for risk according to a time-constant Sharpe ratio
   - Sharpe ratios between 8% and 19% yield the same markup for reasonable portfolios

4. **Sharpe ratios can be starting point for pricing longevity derivatives**

Summary

- **Structural shortcomings in the current standard model longevity stress**
  - Possibly significant overestimation or underestimation of true risk
  - Age and maturity dependent longevity stress required

- **Proposition of modified shock**
  - Simple in structure (one-off shock)
  - Age and maturity dependent
  - Conservative due to waiving of diversification effects

- **Several findings regarding the Risk Margin**
  - Approximations yield wide range of values
  - Assumption of SCR proportional to liabilities in general not appropriate
  - Cost of capital rate of 6% does not seem overly conservative
  - Solvency requirements can provide valuable insights into pricing of longevity derivatives

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