Deterministic Shock vs. Stochastic Value-at-Risk –
An Analysis of the Solvency II Standard Model Approach to Longevity Risk

Matthias Börger

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Introduction

- Longevity risk = risk of insured on average surviving longer than expected
  - Significant risk for pension funds and annuity providers
  - Systematic and non-hedgeable risk
  → Explicitly accounted for under Solvency II

- General concept for Solvency Capital Requirement (SCR) under Solvency II
  - SCR = 99.5% Value-at-Risk (VaR) of Available Capital over 1 year
  - „Capital necessary to cover losses over next year with at least 99.5% probability“
  - Stochastic (internal) models required whose implementation is costly and sophisticated

- Solvency II Standard model
  - Scenario-based rather than stochastic, modular approach
  - Longevity risk: SCR = change in Net Asset Value (NAV) due to longevity shock
  - Longevity shock is a permanent 25% reduction of mortality rates for all ages
Objective

- Motivation of the standard model longevity stress is rather poor
  - Value of 25% is mainly based on what UK insurance companies in 2004 regarded consistent with VaR concept (CEIOPS (2007))
  - UK insurance companies regarded shock between 5% and 35% as appropriate
    → 25% longevity shock could significantly misjudge the true risk

- Analysis of the longevity stress regarding structure and calibration is required
  - Is an equal shock for all ages and maturities reasonable?
  - What should the magnitude of the shock be?
  - How can the standard model longevity stress possibly be improved?

→ Comparison with VaR for longevity risk
Agenda

- Introduction
- Mortality modeling
- Model setup
- Comparison of SCR formulas for longevity risk
- Modification of standard model longevity stress
- Analysis of Risk Margin
- Summary
The Forward Mortality Model

- In 1-year setting, longevity risk consists of two components:
  - Low realized mortality in the one year
  - Decrease in expected future mortality

- A stochastic mortality model must account for both components
  - Well known spot mortality models do not cover possible changes in expected mortality
  - Forward mortality model is required

- We use slightly modified version of forward model of Bauer et al. (2008, 2010)
Model Setup

- Reference company situated in the UK
- Company is solely exposed to longevity risk
- Risk-free interest rates: QIS4 term structure for UK for 2007
- Initial mortality rates: UK Life Office Pensioners in 2007
- Standard contracts:
  - Immediate and deferred life annuities with yearly payments of fixed amount in arrears
  - No options or guarantees, no fees, no surplus participation
Comparison of SCR Formulas - Different Ages

Life annuities paying GBP 1000 yearly in arrears for different ages

<table>
<thead>
<tr>
<th>Age</th>
<th>( L_0 )</th>
<th>( SCR^{\text{Shock}} )</th>
<th>( \frac{SCR^{\text{Shock}}}{L_0} )</th>
<th>( SCR^{\text{VaR}} )</th>
<th>( \frac{SCR^{\text{VaR}}}{L_0} )</th>
<th>( \Delta SCR )</th>
<th>( \frac{\Delta SCR}{SCR^{\text{VaR}}} )</th>
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</thead>
<tbody>
<tr>
<td>55</td>
<td>15671.10</td>
<td>657.23</td>
<td>4.2%</td>
<td>729.88</td>
<td>4.7%</td>
<td>-10.0%</td>
<td>-0.5%</td>
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<tr>
<td>65</td>
<td>12619.28</td>
<td>869.87</td>
<td>6.9%</td>
<td>691.59</td>
<td>5.5%</td>
<td>25.8%</td>
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<tr>
<td>75</td>
<td>8941.83</td>
<td>1009.81</td>
<td>11.3%</td>
<td>513.27</td>
<td>5.7%</td>
<td>96.7%</td>
<td>5.6%</td>
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<tr>
<td>85</td>
<td>4940.13</td>
<td>1003.43</td>
<td>20.3%</td>
<td>304.89</td>
<td>6.2%</td>
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<tr>
<td>95</td>
<td>2549.75</td>
<td>818.58</td>
<td>32.1%</td>
<td>214.38</td>
<td>8.4%</td>
<td>281.8%</td>
<td>23.7%</td>
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<tr>
<td>105</td>
<td>1413.19</td>
<td>646.23</td>
<td>45.7%</td>
<td>180.79</td>
<td>12.8%</td>
<td>257.4%</td>
<td>32.9%</td>
</tr>
</tbody>
</table>

- Deviation becomes enormous for old ages
- 25% shock seems to overestimate longevity risk significantly
- Sole adjustment of shock magnitude does not seem appropriate

→ Structural shortcoming of the standard model longevity stress:
   Age-dependent shock magnitude seems more appropriate
Decomposition of annuity in series of endowment contracts for a 65-year old paying GBP 1000 at maturity $T$

- Absolute SCRs are rather similar up to $T=20$
- Thereafter, shock approach demands significantly more capital (larger shocks)
- Relative deviations in SCRs vary considerably

→ **Structural shortcoming of the standard model longevity stress:**
  Maturity-dependent shock (magnitude) seems more appropriate
Modified Standard Model Longevity Stress

- Current standard model longevity stress does not seem to reflect the true longevity risk

- Modified stress according to volatility in forward model
  - Keep structure of one-off shock (integration in standard model remains the same)
  - Shock T-year survival probabilities by setting them to individual 99.5% quantile
  - Application of shock by multiplying best estimate survival probabilities by factors
  - A matrix of shock factors would have to be provided by supervisory authorities (complexity basically unchanged)

- Any diversification effects are neglected
  - Additional SCR between 5% and 10% for reasonable portfolios
  - Acceptable shortcoming given the enormous structural improvements
  - Standard model is to be conservative
Analysis of Risk Margin

- Technical Provisions („market value“ of liabilities) = Best Estimate Liabilities + Risk Margin
- Risk Margin = capital required to guarantee run-off of a portfolio in case of insolvency (cost of capital approach)
- 4 main findings (future SCRs computed based on 25% shock and modified shock):
  1. Risk Margin approximations yield wide range of values
     - Variation of up to 30% for reasonable portfolios
     - Limited comparability and undesired incentives!
  2. Popular assumption of future SCRs being proportional to future liabilities is not adequate in general
     - Ratios typically increase over time → Risk is underestimated!
  3. Cost of capital rate of 6% does not seem overly conservative compared to hypothetical market prices for longevity risk
     - Survival probabilities are adjusted for risk according to a time-constant Sharpe ratio
     - Sharpe ratios between 8% and 19% yield the same markup for reasonable portfolios
  4. Sharpe ratios can be starting point for pricing longevity derivatives
Summary

- **Structural shortcomings in the current standard model longevity stress**
  - Possibly significant overestimation or underestimation of true risk
  - Age and maturity dependent longevity stress required

- **Proposition of modified shock**
  - Simple in structure (one-off shock)
  - Age and maturity dependent
  - Conservative due to waiving of diversification effects

- **Several findings regarding the Risk Margin**
  - Approximations yield wide range of values
  - Assumption of SCR proportional to liabilities in general not appropriate
  - Cost of capital rate of 6% does not seem overly conservative
  - Solvency requirements can provide valuable insights into pricing of longevity derivatives
References

Matthias Boerger

Institute of Insurance, Ulm University & Institute for Finance and Actuarial Sciences (ifa), Ulm
Helmholtzstraße 22, 89081 Ulm, Germany
Phone: +49 731 50-31257, Fax: +49 731 50-31239
Email: m.boerger@ifa-ulm.de