Coherent Projections of Age, Period, and Cohort Dependent Mortality Improvements

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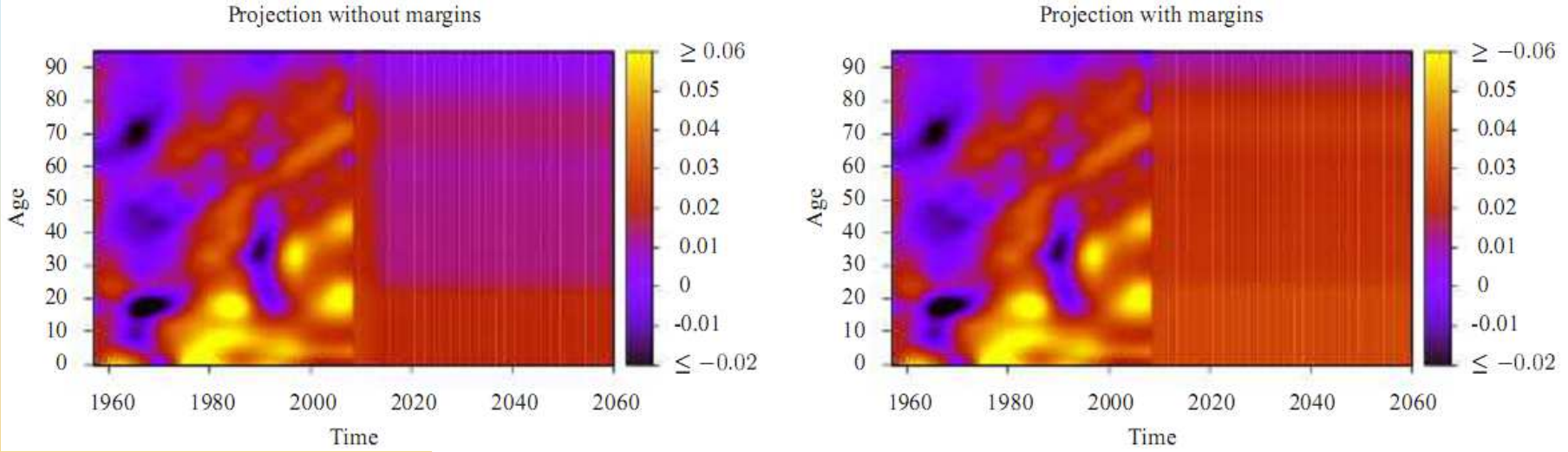
Introduction

- **Some of the currently used standard projections show significant shortcomings**
- **Example: standard projection for German annuity business**

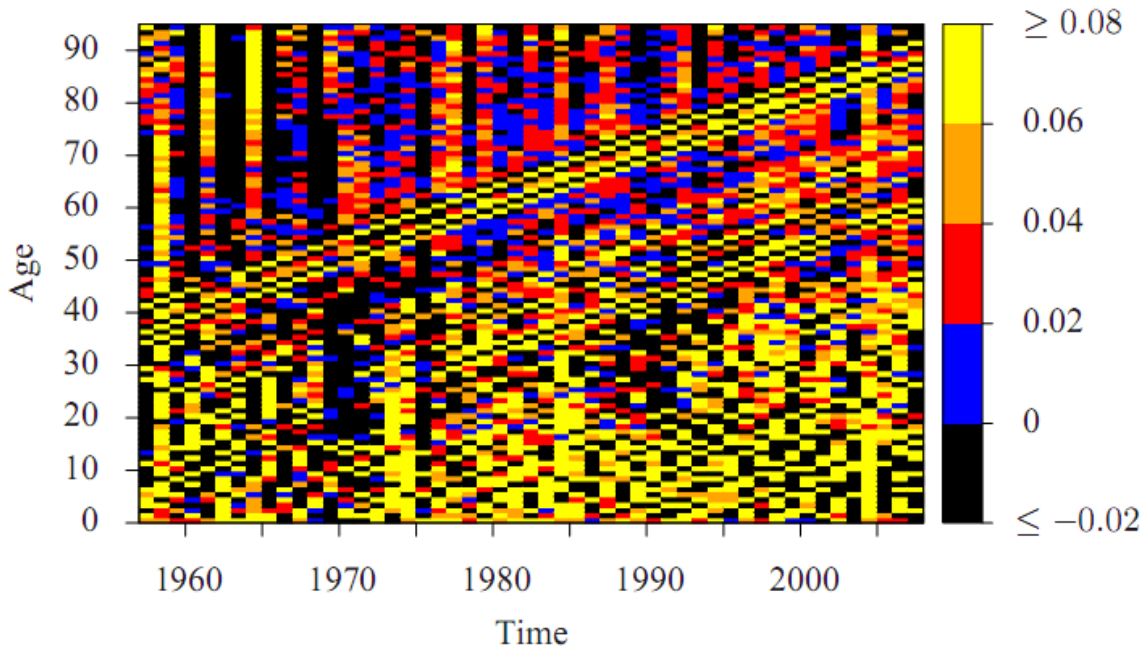


- Structural break between historical and projected improvements
- No cohort effects
- Possibly significant underestimation of future mortality improvements

→ Space and need for improvement of current projections

Model Specification

- **Raw historical mortality improvements for German males**



- **Historical data show period and cohort dependent effects**
- **Mortality improvements have often been shown to be age dependent as well**
- **We model one-year mortality improvements according to the APC model:**

$$v(x,t) = a_x + p_t + c_{t-x}$$

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Model Constraints

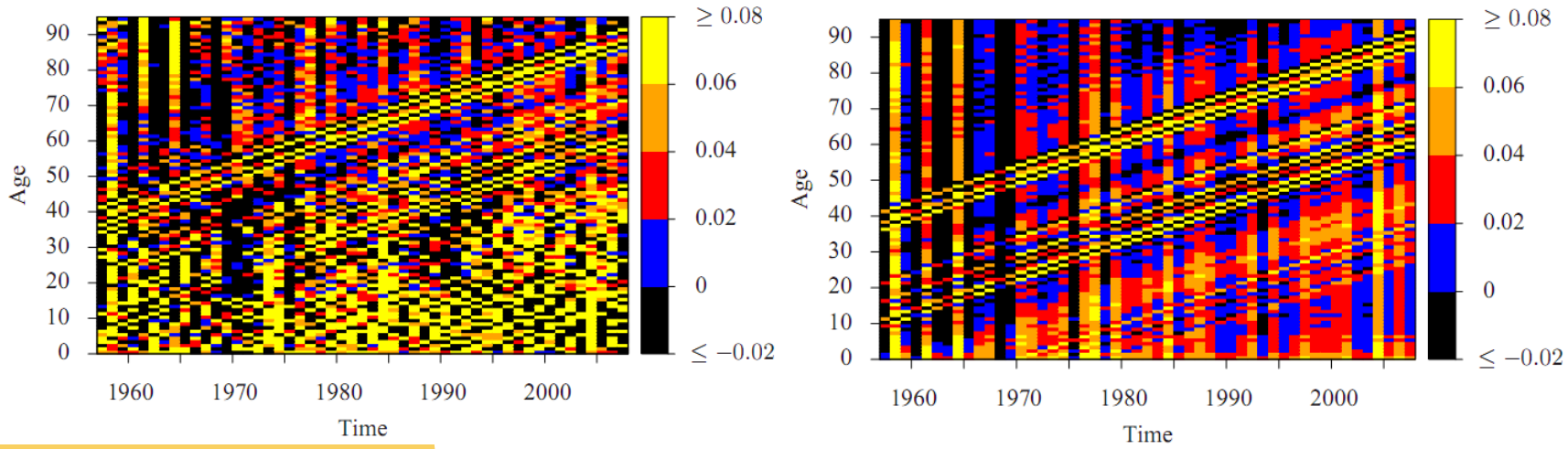
- **Random noise in cohort parameters at the boundaries**
  - Parameters are fitted to only a few data points
  - We set them to their historical average
  - Number of cohort parameters depends on the data set

- **Identifiability problem: APC model calibration is not unique**
  - Period parameters sum up to zero
  - For convenience: Cohort parameters sum up to zero
  - Thus, all "substance" is contained in the age parameters

Model Estimation

- **Model is fitted in iteratively reweighted least squares**
  - Weighting important due to stronger random fluctuations for young ages in particular
  - As weights we use empirical standard deviations from surrounding cells
  - Iteration is stopped when all model parameters change by less than 0.1%

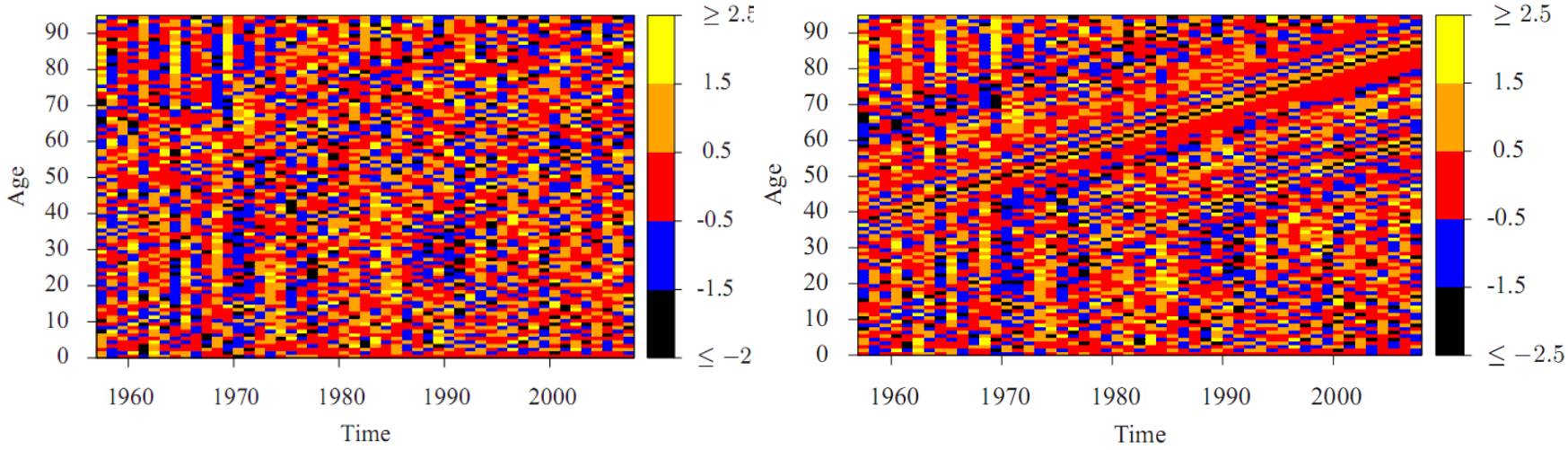
- **Raw and fitted mortality improvements**



  - Very similar in structure
  - Model just seems to eliminate random noise

Model Estimation (ctd.)

- **Standardized residuals for full model and model without cohort parameters**



- **Residuals contain almost no structure**
  → Model specification seems adequate

- **Model simplification obviously not feasible**

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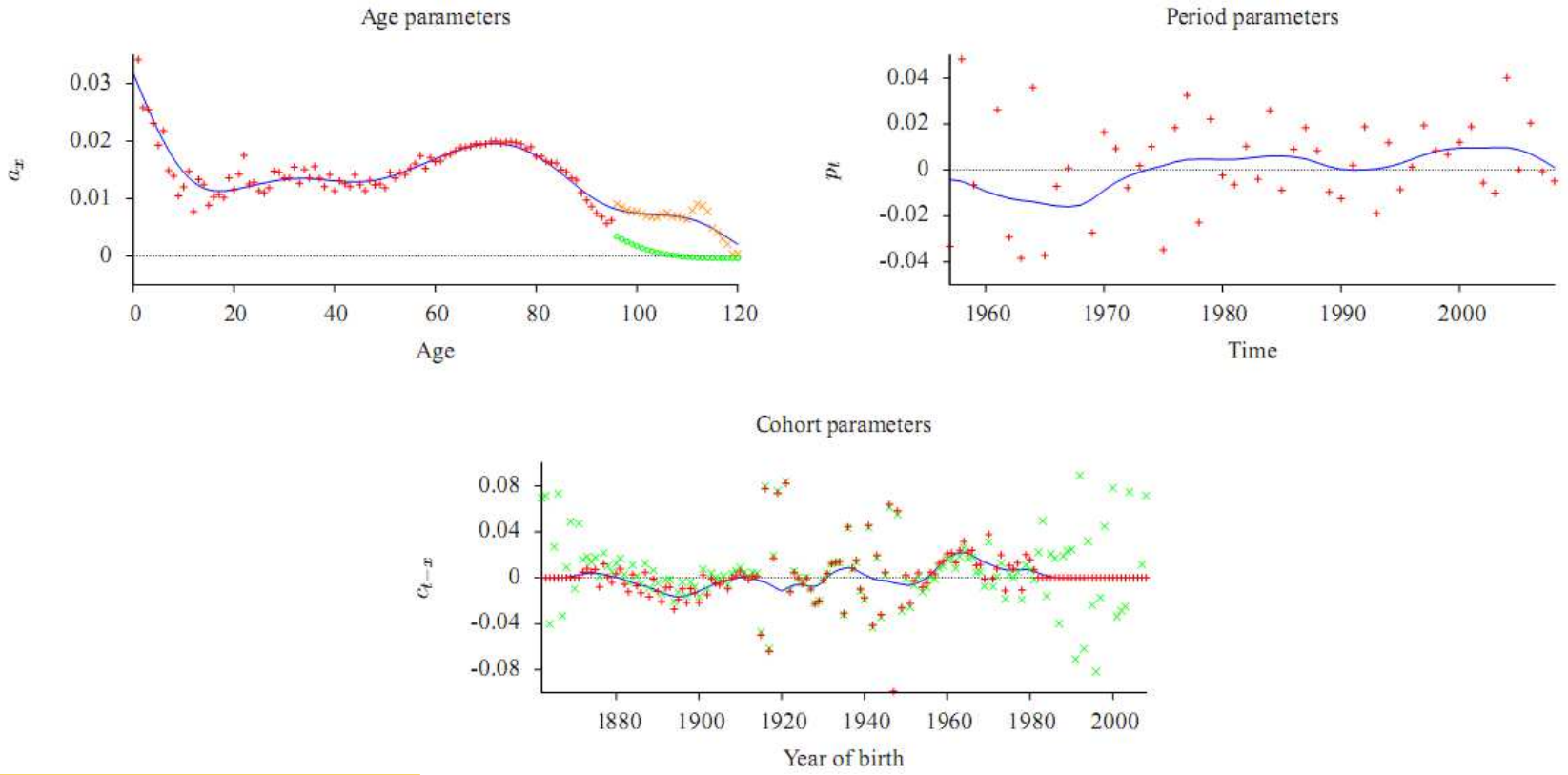
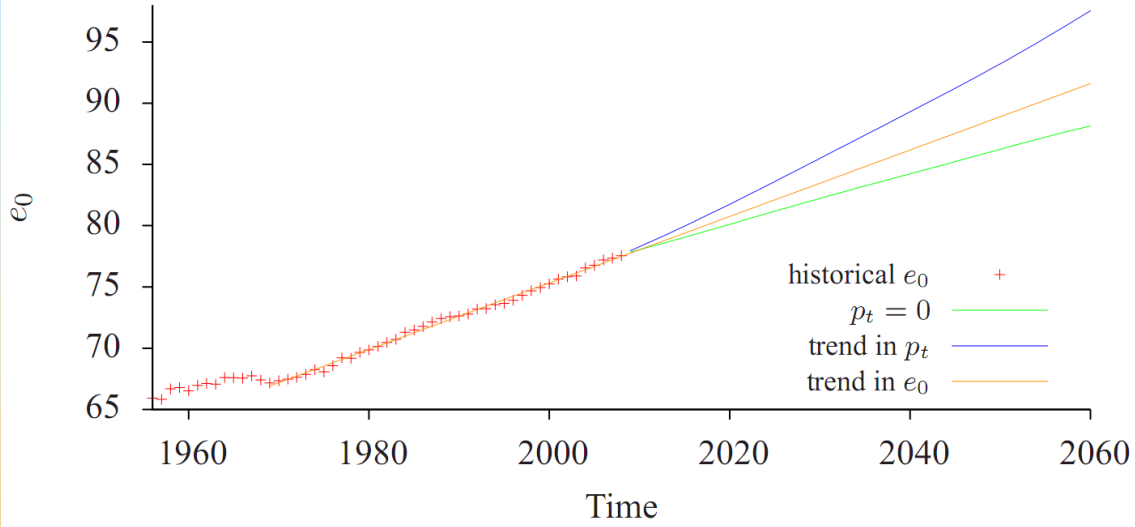
Estimation Results



- **Age parameters starting from age 96 are based on extrapolated mortality rates**
  - Period-wise (green) and cohort-wise (orange) Kannisto extrapolations
  - Period-wise extrapolations yield negative and thus implausible age parameters
  - We consider age parameters from cohort-wise extrapolations

Projection

- **Projection uncertainty can be massive for individual countries**



- **Information from other populations can reduce projection uncertainty**
- **Goal: Coherent Projections between**
  - Males and females in the same country (e.g. Germany)
  - Populations in different but related countries (e.g. European countries)

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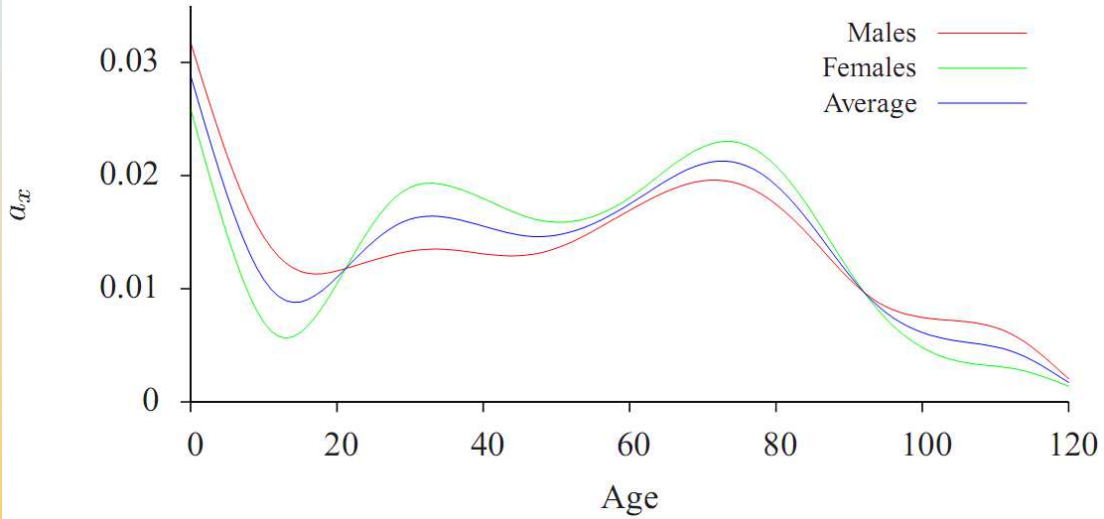
Projection of Age Parameters

- **Different age parameters between males and females lead to diverging mortality rates**
- **Thus, the age parameters should be equal in the long run**
- → **Projection for both genders according to average of age parameters**
  - Appropriate if parameter values are similar for males and females



- **Analogous approach may be reasonable for age parameters for populations from different countries**

Projection of Cohort Parameters

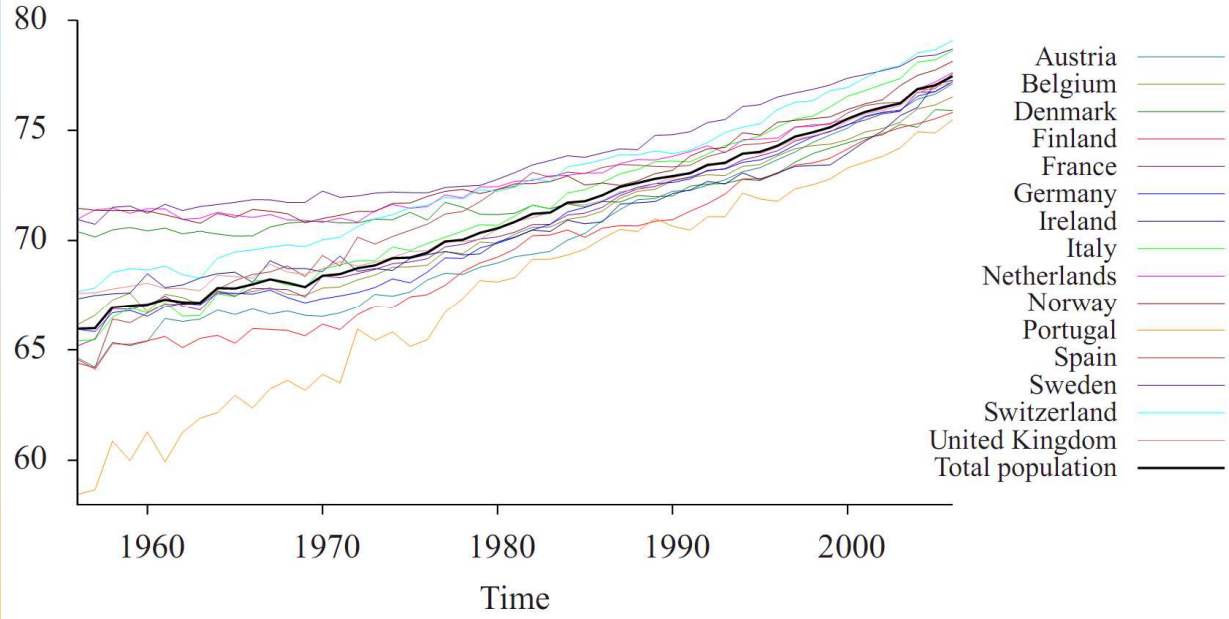
- **Cohort effects are only temporary and thus, do not affect the long-term coherence of mortality projections**
- **Even in the same country, cohort effects for males and females are not necessarily correlated (cf. MacMinn and Weber (2011, Select birth cohorts))**

→ **We keep cohort parameters as fitted for each population individually**

Parameters for new cohorts are set to their long-term average of zero

Projection of Period Parameters

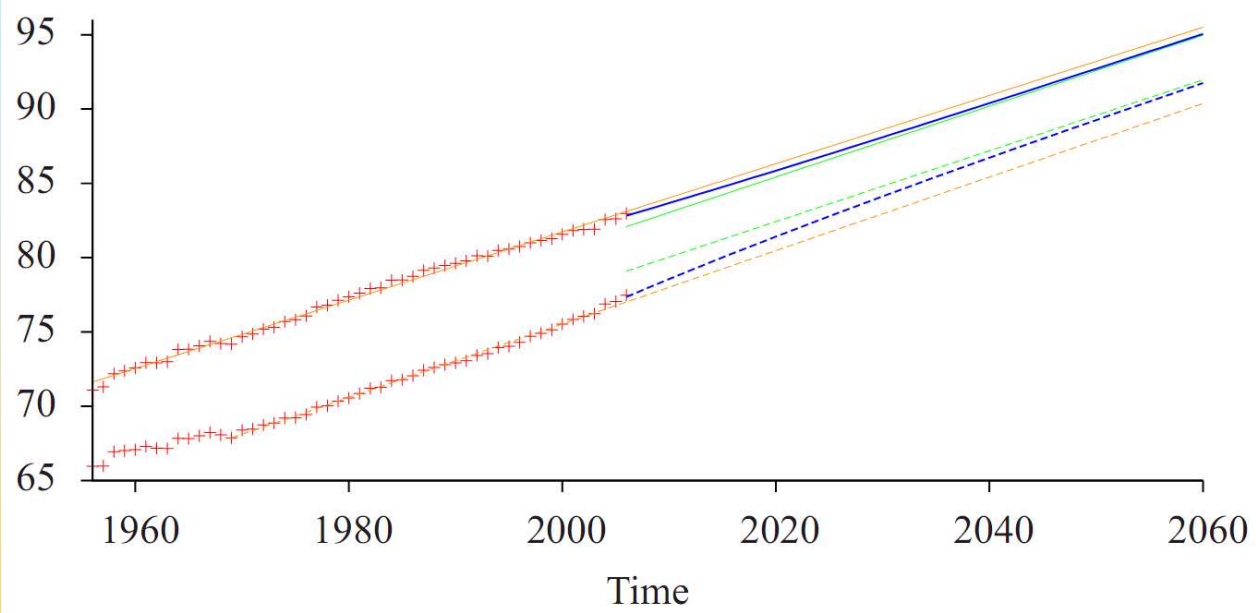
- **Common trend in life expectancies in Europe**



- **Projection approach for period parameters:**
  1. Extrapolate life expectancies for total population (coherent for males and females)
  2. Determine life expectancy extrapolation for individual population relative to total population
  3. If appropriate: Modification of extrapolations to account for model uncertainty, margins,…
  4. Fit period parameters such that these life expectancy extrapolations are met

Projection of Period Parameters (ctd.)

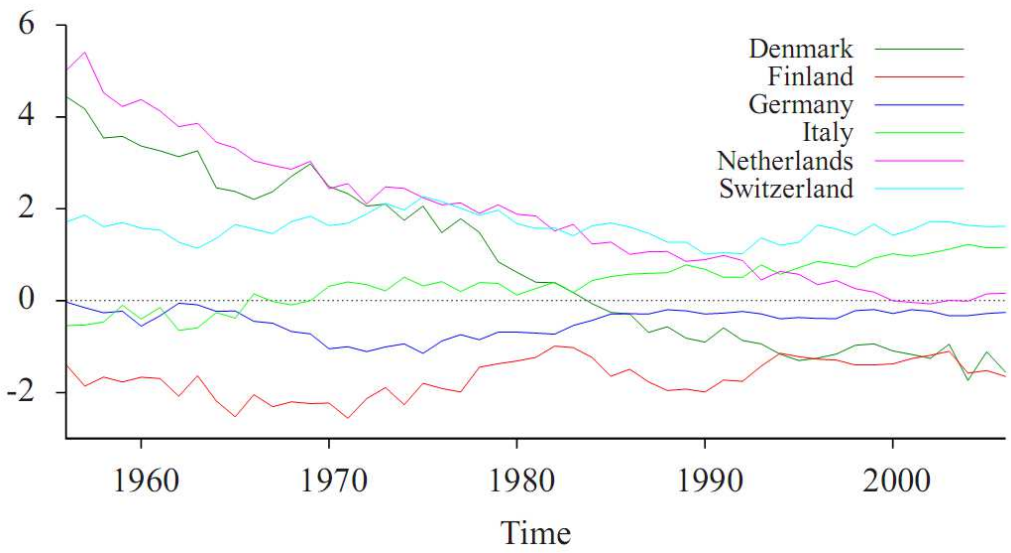
- **Life expectancy extrapolations for male and female total populations**



  - Long-term trend according to average of linear historical trends for males and females
  - Difference in life expectancies has been decreasing since the mid-1990's
    - Extrapolation of this trend in the short run
    - Convergence in lifestyles, e.g. comsumption of tobacco/alcohol, employment
    - Luy (2002): Difference in life expectancies between nuns and monks is only about 1 year
  - We assume a long-term difference in life expectancies of 3 years

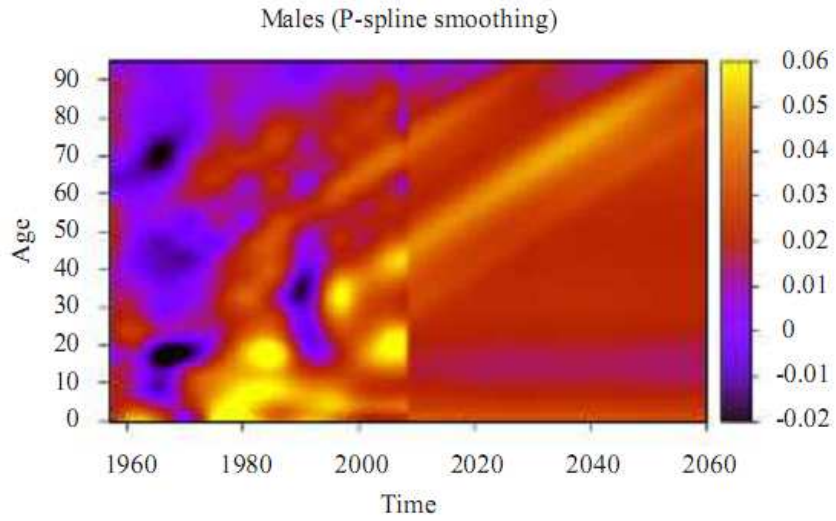
Projection of Period Parameters (ctd.)

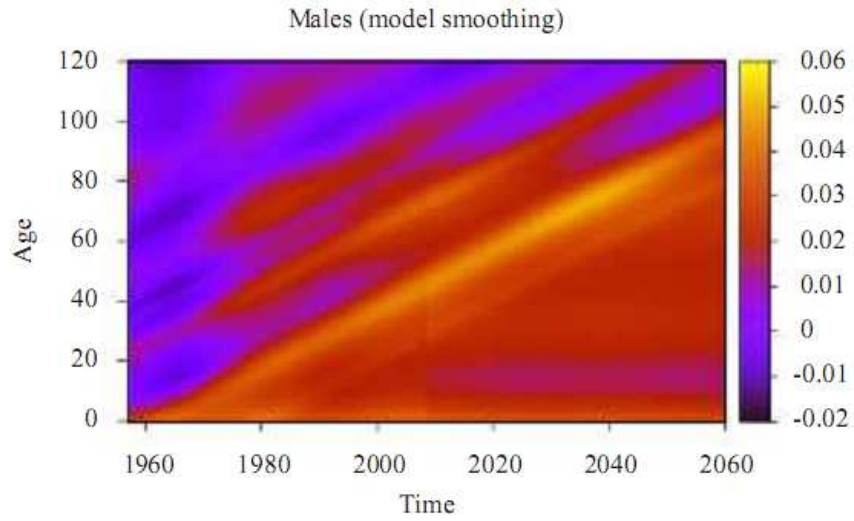
- **Differences in life expectancies between male total population and individual populations**

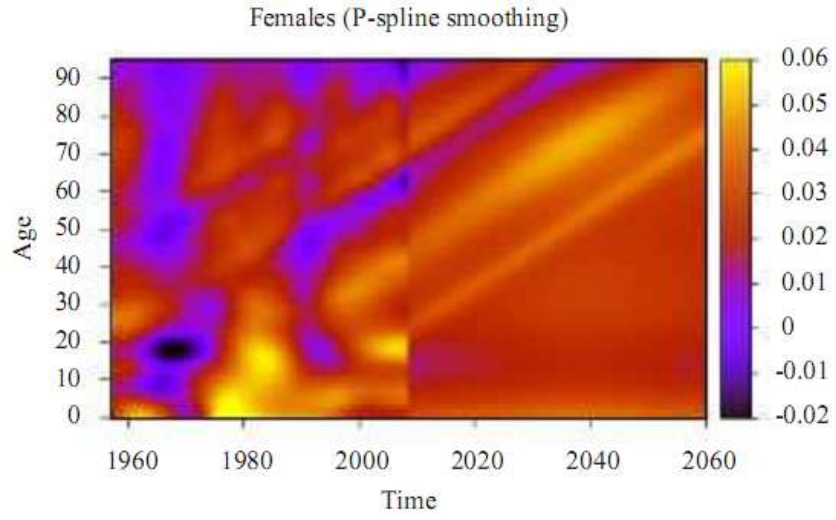


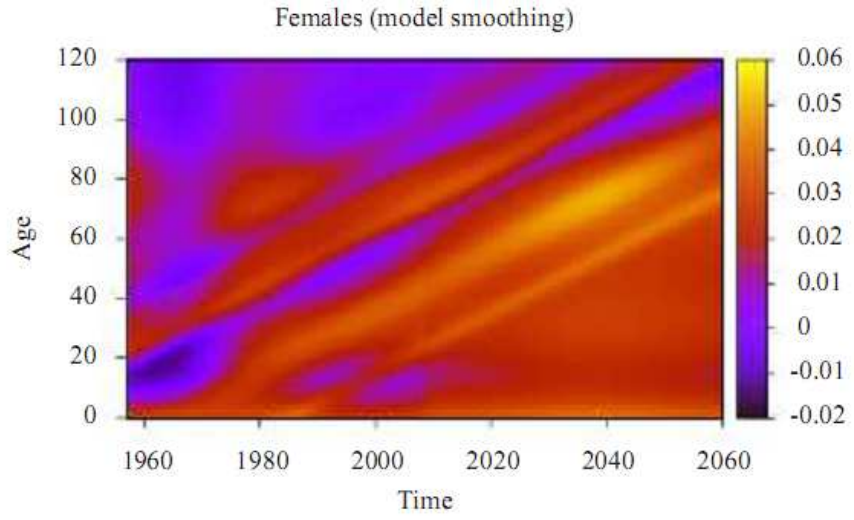
- **Extrapolation according to total population not always appropriate, e.g. for Switzerland**
  - Assumption of a higher life expectancy also in the long run
- **Individual life expectancy projections for Italy and Denmark would not be plausible**
  - A leveling-off at about the current life expectancy difference or a convergence to life expectancies of the total population seems more plausible
- **Life expectancy for German males has been about 0.3 years below average recently**
  - Projection of individual life expectancies by downward shift by 0.3 years
  - For females: downward shift by 0.5 years

Projection for Germany









Projection for Germany (ctd.)

- **Plausible extrapolation of historical structures, in particular cohort effects**
- **Structural break in the P-spline case is due to different degree of smoothinG**
  - Optimal degree of smoothing is unclear
- **Slight structural break in the model smoothing case due to switch to average age parameters**
- **Strong improvements around age 20 in recent years**
  - Model does not detect these as age effects
  - Possibly cohort effects, more observations required
  - Evolution needs to be followed closely over the next years

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