Coherent Projections of Age, Period, and Cohort Dependent Mortality Improvements

Marie-Christine Aleksic
Matthias Börger
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Introduction

- Some of the currently used standard projections show significant shortcomings
  - Example: standard projection for German annuity business

  ![Projection without margins](image1)
  ![Projection with margins](image2)

  - Structural break between historical and projected improvements
  - No cohort effects
  - Possibly significant underestimation of future mortality improvements

→ Space for improvement of current projections
Model Specification

- Raw historical mortality improvements for German males

Historical data show period and cohort dependent effects
- Mortality improvements have often been shown to be age dependent as well
- We model one-year mortality improvements according to the APC model:

\[ v(x,t) = a_x + p_t + c_{t-x} \]
Model Constraints

- **Random noise in cohort parameters at the boundaries**
  - Parameters are fitted to only a few data points
  - We set them to their historical average
  - Number of cohort parameters depends on the data set

- **Identifiability problem: APC model calibration is not unique**
  - Period parameters sum up to zero
  - For convenience: Cohort parameters sum up to zero
  - Thus, all “substance” is contained in the age parameters
Model Estimation

- **Model is fitted in iteratively reweighted least squares**
  - Weighting important due to stronger random fluctuations for young ages in particular
  - As weights we use empirical standard deviations from surrounding cells
  - Iteration is stopped when all model parameters change by less than 0.1%

- **Residuals for simplified model versions contain significant structure**
**Estimation Results**

- **Age parameters are extrapolated starting from age 96**
  - Extrapolations of mortality rates by different mortality laws indicate shrinking improvements
  - This observation is in line with findings of other authors, e.g. Gampe (2010)
  - We apply a cubic function which monotonically decreases to zero at age 120
  - Adjustment may be applicable to be more conservative
Projection

- Projection uncertainty can be massive for individual countries

- Information from other populations can reduce projection uncertainty

- Goal: Coherent Projection between
  - Males and females in the same country (e.g. Germany)
  - Populations in different but related countries (e.g. European countries)
Projection of Age Parameters

- Different age parameters between males and females lead to diverging mortality rates
- Thus, the age parameters should be equal in the long run
- Projection for both genders according to average of age parameters
  - Appropriate if parameter values are similar for males and females

- Analogous approach may be reasonable for age parameters for populations from different countries
Cohort effects are only temporary and thus, do not affect the long-term coherence of mortality projections
Cohort effects for males and females are not necessarily correlation (cf. MacMinn and Weber (2009))
We keep cohort parameters as fitted for each population individually
Parameters for new cohorts are set to their long-term average of zero
Projection of Period Parameters

- Common trend in life expectancies in Europe

![Graph showing life expectancy trends in Europe over time]

- Projection approach:
  1. Extrapolate life expectancies for total population (coherent for males and females)
  2. Determine life expectancy extrapolation for individual population relative to total population
  3. If appropriate: Modification of extrapolations to account for model uncertainty, margin,…
  4. Fit period parameters such that these life expectancy extrapolations are met
Projection of Period Parameters (ctd.)

- Life expectancy extrapolations for male and female total populations

- Long-term trend according to average of linear historical trends for males and females
- Difference in life expectancies has been decreasing from the mid-1990’s
  - Extrapolation of this trend in the short run
  - Convergence in lifestyles, e.g. consumption of tobacco/alcohol, employment
  - Luy (2002): Difference in life expectancies between nuns and monks is only about 1 year
- Long-term difference in life expectancies of 3 years
Life expectancies for individual populations

- Extrapolation according to total population not always appropriate, e.g. for Switzerland
  - Assumption of a higher life expectancy also in the long run
- Individual life expectancy projections for Italy and Denmark would not be coherent/plausible
  - A leveling-off at about the current life expectancy difference or a convergence to life expectancies of the total population seems more plausible
- For German males, life expectancies have been about 0.3 years below average in recent decades
  - Projection of individual life expectancies by downward shift by 0.3 years (0.5 years for females)
Projection for Germany

Males (P-spline smoothing)

Males (model smoothing)

Females (P-spline smoothing)

Females (model smoothing)
Contact Details

Matthias Boerger

Institute of Insurance, Ulm University & Institute for Finance and Actuarial Sciences (ifa), Ulm
Helmholtzstraße 22, 89081 Ulm, Germany
Phone: +49 731 50-31257, Fax: +49 731 50-31239
Email: m.boerger@ifa-ulm.de