The Actuarial Profession
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Pensions, benefits and social security colloquium 2011

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Modeling Mortality Trend Risk

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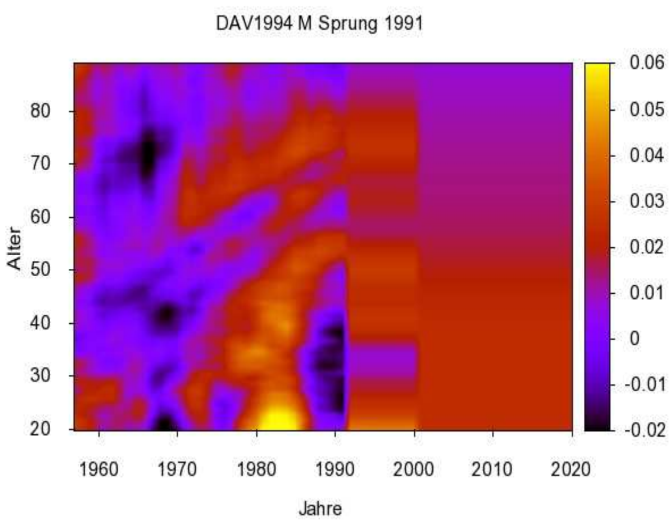
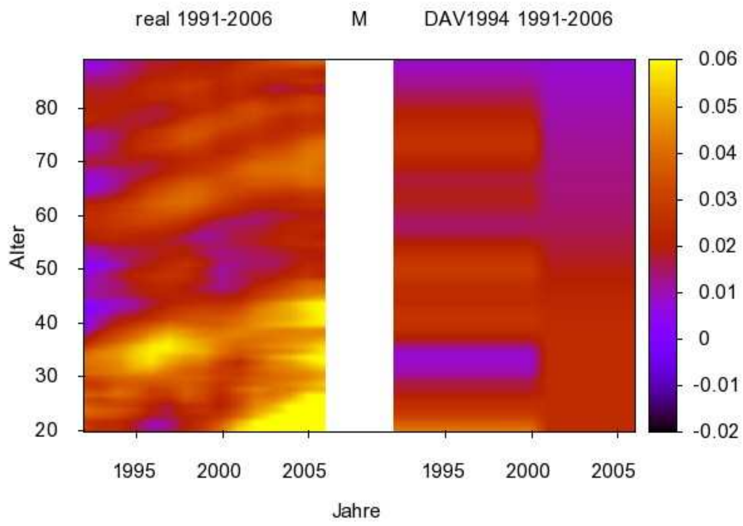
Introduction

What is longevity risk?





→ The risk of underestimating mortality improvements

- Trend risk
- Systematic and non-hedgeable

Mortality Trend Model Requirements

Goal: Mortality model for solvency purposes with the following properties

- Simultaneous modeling of mortality and longevity risk
- Full age range (20 to 105)
- Consideration of several populations at the same time, e.g. males and females
- Quantification of risk over limited time horizons
  - 1 year for Solvency II or several years for strategic planning
  - Risk in realized mortality evolution and changes in long-term assumptions
  - Stochastic mortality trend
- Plausible tail scenarios
- Conservative calibration
- Epidemiological and demographic input

Model Specification

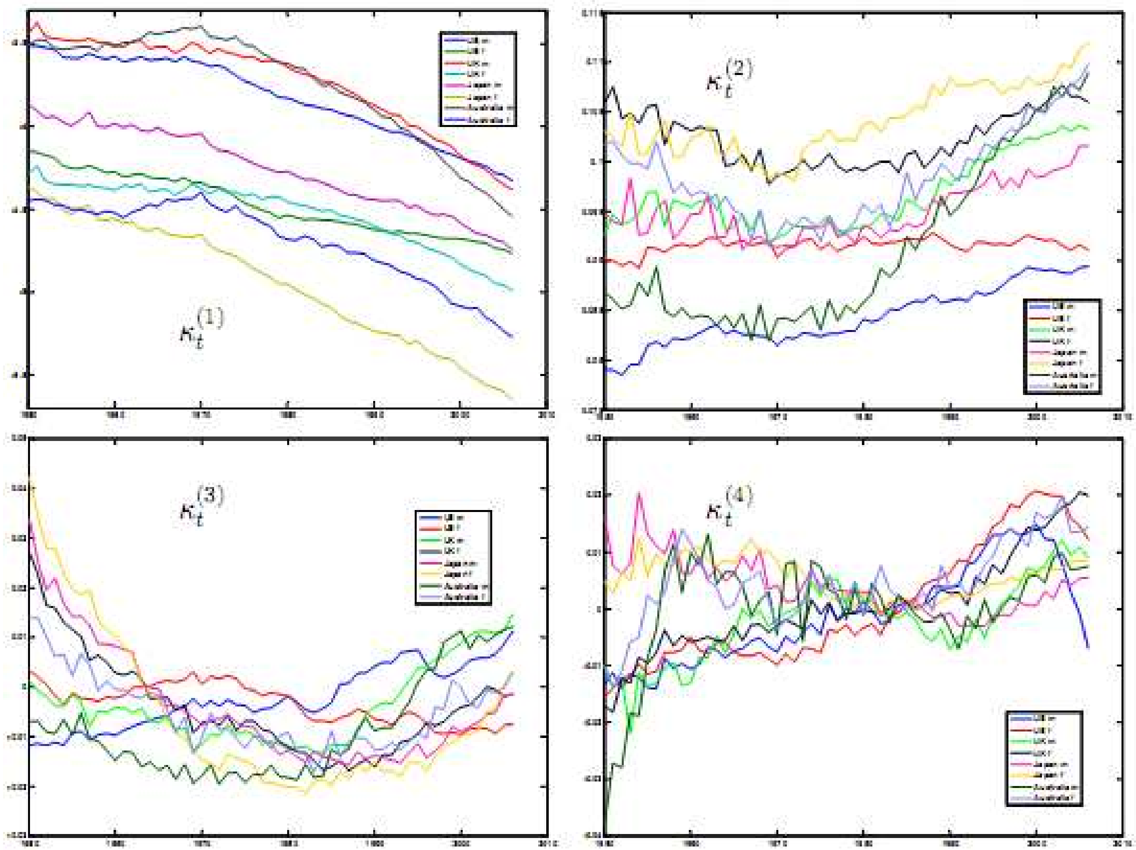
We model the logit of mortality rates

- $\text{logit}(q_{x,t}) = \alpha_x + \kappa_t^{(1)} + \kappa_t^{(2)}(x - x_{center}) + \kappa_t^{(3)}(x_{young} - x)^+ + \kappa_t^{(4)}(x - x_{old})^+ + \gamma_{t-x}$
- $x_{center} = 60$, $x_{young} = 55$, $x_{old} = 85$
- $\kappa_t^{(1)}$ describes the general level of mortality
- $\kappa_t^{(2)}$ is the slope of the mortality curve
- $\kappa_t^{(3)}$ and $\kappa_t^{(4)}$ describe additional effects in young and old age mortality, respectively
  - $\kappa_t^{(3)}$ can be omitted if older ages are considered only

Model Estimation

Model estimation via Generalized Linear Model theory

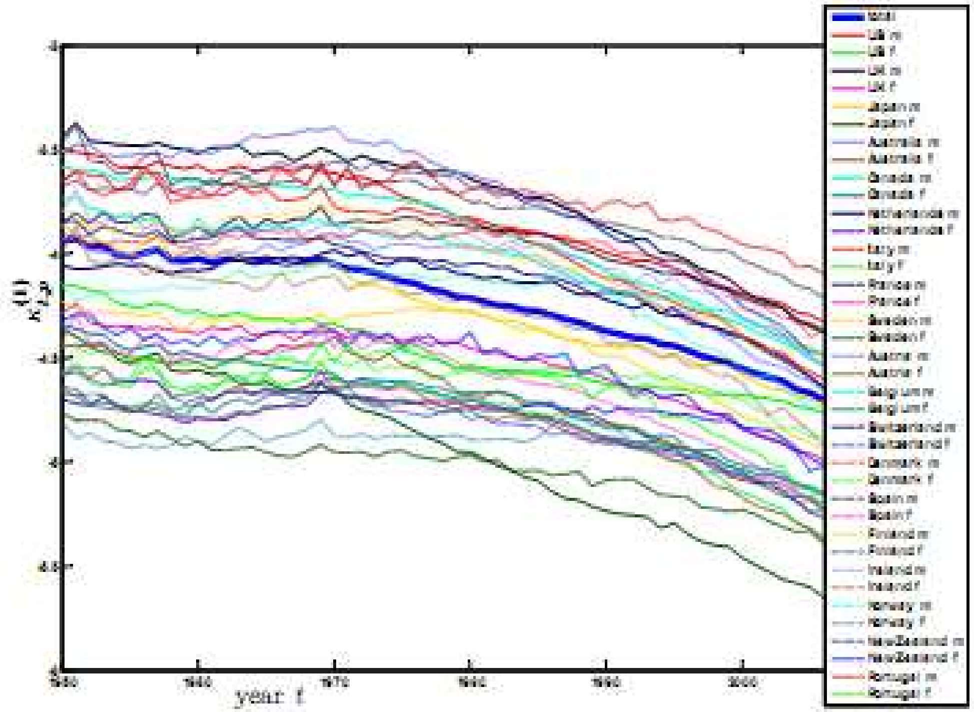
- Logit is canonical link function for Binomial distribution
- Number of deaths is binomially distributed given initial exposures



Multi-population setting

Important note: Even if one is only interested in a single population considering several populations is worthwhile

- Trend uncertainty can be significantly reduced



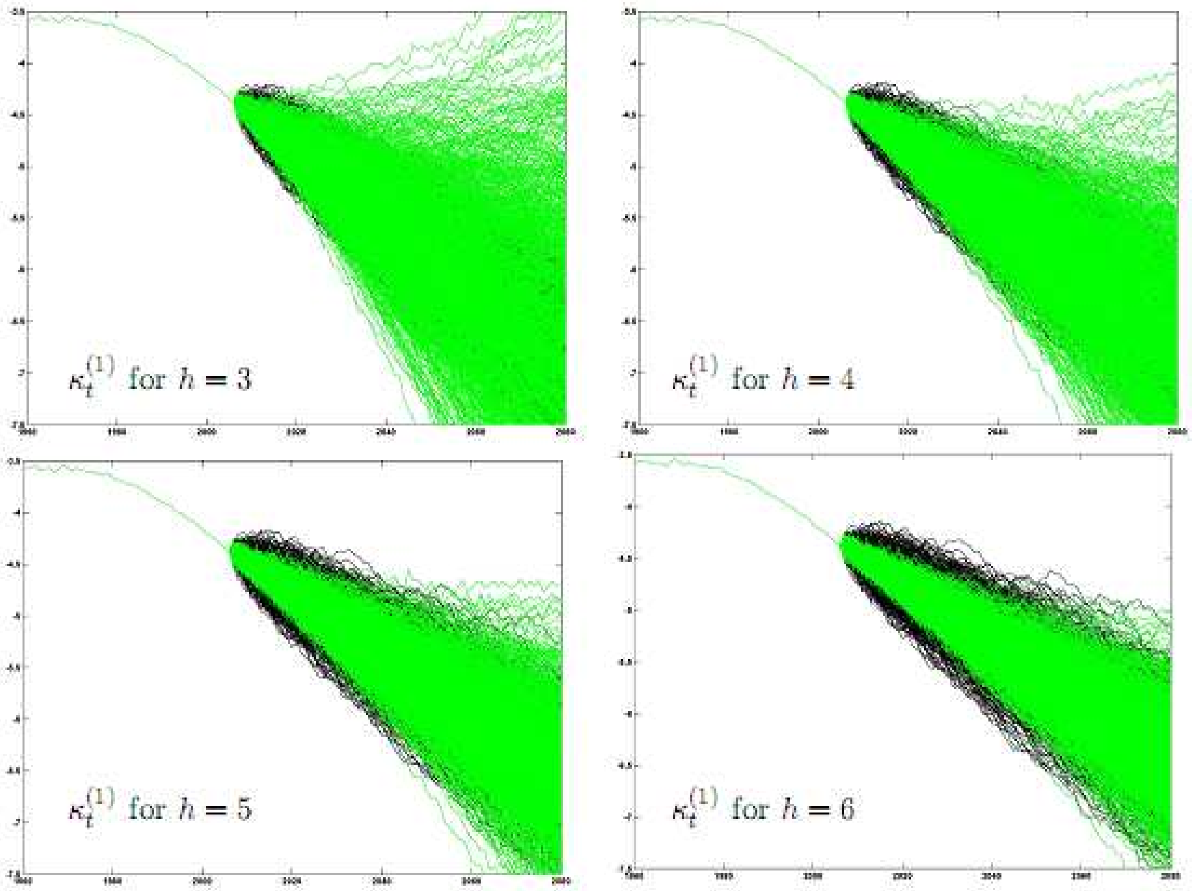
- There is clearly a common trend
- A model for several populations must account for that
- Increment correlations cannot generate such parallel evolutions
- We apply cointegration and an error correction model for deviations from the common trend

Model Simulation

Projection of $\kappa_{t,total}^{(1)}$ for the total population

- Linear trends with breaks in the historical data
  - Commonly used random walk with drift does not allow for trend breaks
  - Trend breaks and thus changes of the best estimate trend are crucial when working in finite time horizons
- New idea: Each year, fit regression line to historical data and forecast future best estimate mortality as $\kappa_{t+1,total}^{(1)} = l_t(t+1) + \varepsilon_{t+1}^{(1)}(\sigma^{(1)} + \overline{\sigma}^{(1)})$
  - $\overline{\sigma}^{(1)}$ is a volatility add-on, $\sigma^{(1)}$ is current (best estimate) volatility
  - This trend modeling approach reflects actuarial practice of updating a model (here: the long-term trend) when new data becomes available
  - To stress most recent mortality experience, the regression line is fitted with weights $w_s = \left(1 + \frac{1}{h}\right)^{s-t}$

Model Simulation (ctd.)

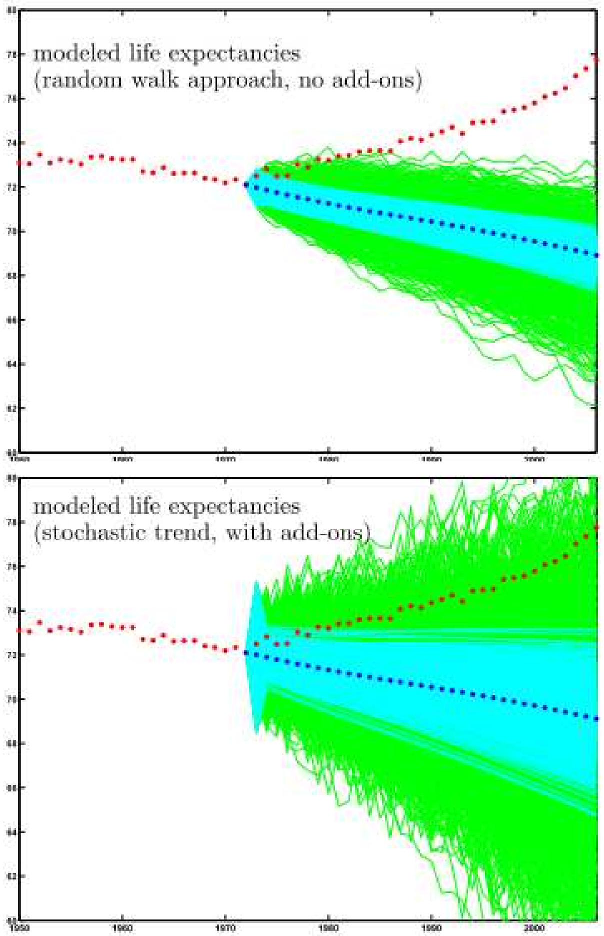


- Weighting parameter h has massive impact
- Plausible one-year and run-off scenarios
- Each run-off scenario is a combination of one-year scenarios
- Disentangling of one-year noice and long-term trend uncertainty
- Possibly more plausible confidence bounds than for a random walk with drift

Model Simulation (ctd.)

Calibration of weighting parameter h

- Adequate parameter calibration is difficult to find and also a question of desired conservatism
- Possible approaches for parameter fitting:
  - Fitting to (most severe) events/evolutions in the past
    - Example: Rapid increase in life expectancies of Dutch males in the 1970's
  - Expert opinion (see mortality/ longevity threat scenarios later)
  - Comparison with confidence bounds in other models (questionable!)



Model Simulation (ctd.)

Projection of $\kappa_{t,p}^{(1)}$ for individual populations

- For each individual population we project as
  - $\kappa_{t,p}^{(1)} = \kappa_{t,total}^{(1)} + a_p + b_p(\kappa_{t-1,p}^{(1)} - \kappa_{t-1,total}^{(1)}) + \varepsilon_{t,p}$
  - b_p denotes the „mean reversion speed“ (absolute value should be smaller than 1)
  - $a_p/(1-b_p)$ is the long-term difference between the total population and population p

- Different approaches of calibrating the long-term difference
  - Fitting of an AR(1) process to historical differences
  - Weighted/unweighted average of historical differences

Model Simulation for a Single Population (ctd.)

Projection of $\kappa_t^{(2)}, \kappa_t^{(3)}$, and $\kappa_t^{(4)}$

- No substantial trend obvious in the historical data
- Forecast as correlated 3-dimensional random walk
- No substantial correlation with $\kappa_t^{(1)}$
- Volatility add-on $\overline{\sigma}^{(2)}$ for $\kappa_t^{(2)}$ may be appropriate to limit diversification between mortality and longevity risk

- Between populations, increments of $\kappa_t^{(1)}$ and $\kappa_t^{(2)}$ are correlated
  - Historical correlations should be checked carefully and possibly adjusted

Model Simulation for a Single Population (ctd.)

Projection of γ_{t-x}

- Cohort parameters should stay around zero
- Forecast as imposed stationary AR(1) process
- Cohort parameters are rather irrelevant for simulations over short time horizons

Epidemiological and Demographic Expert Opinion

Mortality/Longevity Threat Scenarios

- Mortality data is often very sparse, in particular with respect to tail scenarios
- Thus, stochastic models should be enriched by expert opinion
- Possible derivations of mortality/longevity threat scenarios:
  - Different shocks to mortality projections
  - Likely effects of finding of a cure for certain illnesses
  - Scenarios from cause of death models
  - Scenarios the stochastic model cannot generate due to structural limitations, e.g. diverging mortality trends
- Application of threat scenarios:
  - Calibration/adjustment of model parameters
  - Inclusion in set of model outcomes

Summary

A mortality trend model with several appealing properties

- Large variability in simulation outcomes due to 5 stochastic drivers
- Clear interpretation of the model parameters
- Multi-population setting
  - Coherent mortality scenarios
  - Realistic assessment of diversification and accumulation effects
- Stochastic mortality trend
  - Risk can be quantified over finite time horizons
  - Disentangling of short-term noise and long-term trend uncertainty
  - Plausible outcomes in one-year view and run-off view
  - Trend process could be applied in other models as well
- Inclusion of expert opinion via threat scenarios

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