Modeling Mortality Trend under Modern Solvency Regimes

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Introduction

What is longevity risk?

→ Longevity risk is the risk of underestimating future mortality improvements
  - Trend risk
  - Mortality risk has a trend risk and a catastrophe risk component
  - Systematic and non-hedgeable risk
→ Explicitly accounted for under Solvency II and the Swiss Solvency Test (SST)
Capital Requirements under Solvency II

- General concept for Solvency Capital Requirement (SCR) under Solvency II
  - SCR = 99.5% Value-at-Risk (VaR) of Available Capital over 1 year
  - „Capital necessary to cover losses over next year with at least 99.5% probability“
  - Overall risk is typically split into several modules, individual SCRs are finally aggregated

- Stochastic mortality model is required for mortality/longevity trend risk under Solvency II

- In a 1-year setting, longevity/mortality trend risk consists of two components:
  - Low/high realized mortality in the one year
  - Decrease/increase in expected future mortality, i.e. changes in the long-term mortality trend
Mortality Trend Model Requirements

- Goal: Specification and calibration of a mortality model with the following properties
  - Simultaneous modeling of mortality and longevity risk
    - Exploiting of diversification effects
  - Full age range
    - 20 to 105 in our case
  - Consideration of several populations at the same time
    - Males and females in the same country
    - Populations from different countries
  - Quantification of risk over limited time horizons
    - One-year view of Solvency II and the SST particularly relevant
  - Plausible tail scenarios
    - 99.5% VaR
  - Conservative calibration
We model the logit of mortality rates

\[
\text{logit} \left( q_{x,t} \right) = \alpha_x + \kappa_t^{(1)} + \kappa_t^{(2)} (x - x_{center}) + \kappa_t^{(3)} (x_{young} - x) + \kappa_t^{(4)} (x - x_{old}) + \gamma_{t-x}
\]

- \( x_{center} = 60, \ x_{young} = 55, \ x_{old} = 85 \)
- \( \kappa_t^{(1)} \) describes the general level of mortality, \( \kappa_t^{(2)} \) is the slope of the mortality curve, \( \kappa_t^{(3)} \) and \( \kappa_t^{(4)} \) describe additional effects in young and old age mortality, respectively

Model estimation via Generalized Linear Model Theory

- Logit is canonical link function for Binomial distribution
- Number of deaths is binomially distributed given initial exposures
Multi-Population Setting

- Important note: Even if one is only interested in a single population considering several populations is worthwhile
  - Trend uncertainty can be significantly reduced
  - We generally observe smaller SCRs in the multi-population model compared to the single population model

- There is clearly a common trend in $\kappa_t^{(1)}$
- A model for several populations must account for that
- We apply cointegration and an error correction model for deviations from the common trend
Model Simulation

Projection of $\kappa_{t,\text{total}}^{(1)}$ for the total population

- **Linear trends with breaks in the historical data**
  - Commonly used random walk with drift does not allow for such trend breaks
  - Trend breaks are particularly important under one-year view (change of best estimate trend)

- **Idea:** Each year, fit regression line to historical data and forecast future best estimate mortality as
  \[
  \kappa_{t+1,\text{total}}^{(1)} = l_t(t + 1) + \varepsilon_{t+1}^{(1)}(\sigma^{(1)} + \overline{\sigma}^{(1)})
  \]
  - $\overline{\sigma}^{(1)}$ is a volatility add-on, volatility $\sigma^{(1)}$ may be weighted to stress most recent past
  - Implicit „re-calibration“ of the model with respect to the long-term trend
  - To stress most recent mortality experience, the regression line is fitted with weights
  \[
  w_s = \left(1 + \frac{1}{h}\right)^{s-t}
  \]
Model Simulation (ctd.)

- Weighting parameter $h$ has massive impact
- Plausible one-year and run-off scenarios
- Each run-off scenario is a combination of one-year scenarios
- Disentangling of one-year noise and long-term trend uncertainty
- Possibly more plausible confidence bounds than for a random walk with drift
Model Simulation (ctd.)

Projection of $\kappa_{t,p}^{(1)}$ for individual populations

For each individual population we project as

$\kappa_{t,p}^{(1)} = \kappa_{t,total}^{(1)} + a_p + b_p (\kappa_{t-1,p}^{(1)} - \kappa_{t-1,total}^{(1)}) + \varepsilon_{t,p}$

- $b_p$ denotes the „mean reversion speed“ (absolute value should be smaller than 1)
- $a_p/(1-b_p)$ is the long-term difference between the total population and population $p$

Different approaches of calibrating the long-term difference

- Fitting of an AR(1) process to historical differences
- Weighted/unweighted average of historical differences
- Extrapolation of most recent differences and leveling off after a couple of years
Model Simulation (ctd.)

Projection of $\kappa_t^{(2)}, \kappa_t^{(3)},$ and $\kappa_t^{(4)}$ for the individual populations
- No substantial trend obvious in the historical data
- Forecast as correlated 3-dimensional random walk
- No substantial correlation with $\kappa_t^{(1)}$
- Volatility add-on $\sigma_t^{(2)}$ for $\kappa_t^{(2)}$
  - The larger the changes in the slope of the mortality, the smaller the correlation between young and old ages
  - Thus the add-on affects diversification between mortality and longevity risk
- Between populations, increments of $\kappa_t^{(1)}$ and $\kappa_t^{(2)}$ are correlated
  - This also implies slight correlation between the $\kappa_t^{(3)}$ and $\kappa_t^{(4)}$
  - Historical correlations should be checked carefully though and possibly adjusted

Projection of $\gamma_{t-x}$
- Cohort parameters should stay around zero
- Forecast as imposed stationary AR(1) process
- Cohort parameters are rather irrelevant for short-term simulations as under Solvency II
Weighting Parameters and Volatility Add-ons

- Parameters $h, \bar{\sigma}^{(1)}$, and $\bar{\sigma}^{(2)}$ have a massive impact on simulation outcomes and thus SCRs.
- Add-on $\bar{\sigma}^{(1)}$ determines possible severity of short-term events.
- Weighting parameter $h$ determines trend changes over one year and width of confidence bounds.

Calibration is difficult but should be conservative:
- Fitting to most severe events/evolutions in the past.
  - Example: Rapid increase in Dutch life expectancy gains starting from about 1970.
  - Question: At which percentile should such extreme evolutions be observed?
  - Note: The parameters interact with each other even though $h$ depends on $\bar{\sigma}^{(1)}$ only weakly.

Calibration of $\bar{\sigma}^{(2)}$:
- The larger the add-on the smaller the correlation between young and old ages thus limiting diversification.
- Choose $\bar{\sigma}^{(2)}$ such that correlation between ages at the boundaries of the age range is (close to) zero for most populations.
Mortality/Longevity Threat Scenarios

- Available data contains only little information on tail scenarios which we are interested in.
- Uncertainty remains whether model outcomes are severe enough.
  → Incorporate epidemiological/demographic expert opinion.

Specification of mortality/longevity threat scenarios
- Shock to mortality projection
- Likely effects of finding of a cure for a certain illness
- Scenarios which the statistical model cannot generate, e.g., diverging mortality trends between countries/regions
- ...

Application of threat scenarios
- Check of model calibration: Adjustment of weighting parameter or volatility add-ons if the model outcomes should cover the threat scenarios but do not.
- Inclusion in SCR computations: SCR as a weighted average of model outcomes and threat scenarios.
Summary

We have specified and calibrated a mortality model with several appealing properties:

- Full age range
- Variability in simulation outcomes due to 5 stochastic drivers
- Clear interpretation of the model parameters
- Non-trivial correlation structure to allow for simultaneous modeling of mortality and longevity risk
- Stochastic trend modeling spares full re-calibration of the model in each scenario (could be applied in other models as well)
- Plausible outcomes in one-year view and run-off view
- Conservative calibration
- Inclusion of expert opinion
- Multi-population model allowing for diversification effects
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Weighting Parameters and Volatility Add-ons

- Parameters $h$, $\bar{\sigma}^{(1)}$, and $\bar{\sigma}^{(2)}$ have a massive impact on simulation outcomes and thus SCRs.
- Most importantly, $\bar{\sigma}^{(1)}$ determines extreme short-term events, $h$ extreme long-term evolutions.
- Calibration is difficult but should be conservative.
  ➔ Fitting to most severe events/evolutions in the past
  - Short-term: Drop in life expectancy in Russia at the beginning of the 1990's.
  - Long-term: Rapid increase in Dutch life expectancy gains starting from about 1970.
Weighting Parameters and Volatility Add-ons (ctd.)

- **Calibration of $\bar{\sigma}^{(1)}$ in the multi-population setting:**
  - Add „non-Western“ countries with data available in the HMD (e.g. Russia)
  - About 50 years of historical data
  - Choose $\bar{\sigma}^{(1)}$ such that most severe event (drop in life expectancy in the 1990’s) is seen e.g. at the 98th percentile

- **Calibration of $h$ in the multi-population setting:**
  - Trend break around 1970 is the most significant long-term change in the available data
  - Several years of data required to observe a trend change
  - Choose $h$ such that trend change is observed, e.g., at the 90th or 95th percentile (measure of conservatism)

- **Note:** The parameters interact with each other even though $h$ depends on $\bar{\sigma}^{(1)}$ only weakly

- **Calibration of $\bar{\sigma}^{(2)}$:**
  - The larger the add-on the smaller the correlation between young and old ages
  - Thus, diversification between mortality and longevity risk is typically reduced
  - Choose $\bar{\sigma}^{(2)}$ such that correlation between ages at the boundaries of the age range is (close to) zero for most populations