It Takes Two:
Why Mortality Trend Modeling is more than Modeling one Mortality Trend

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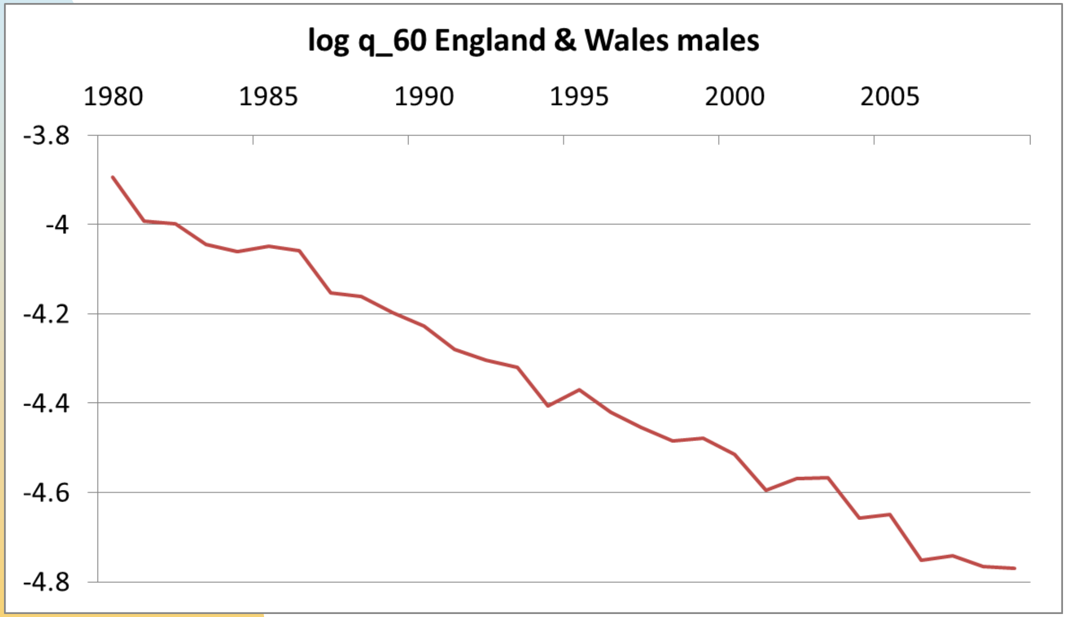
Introduction

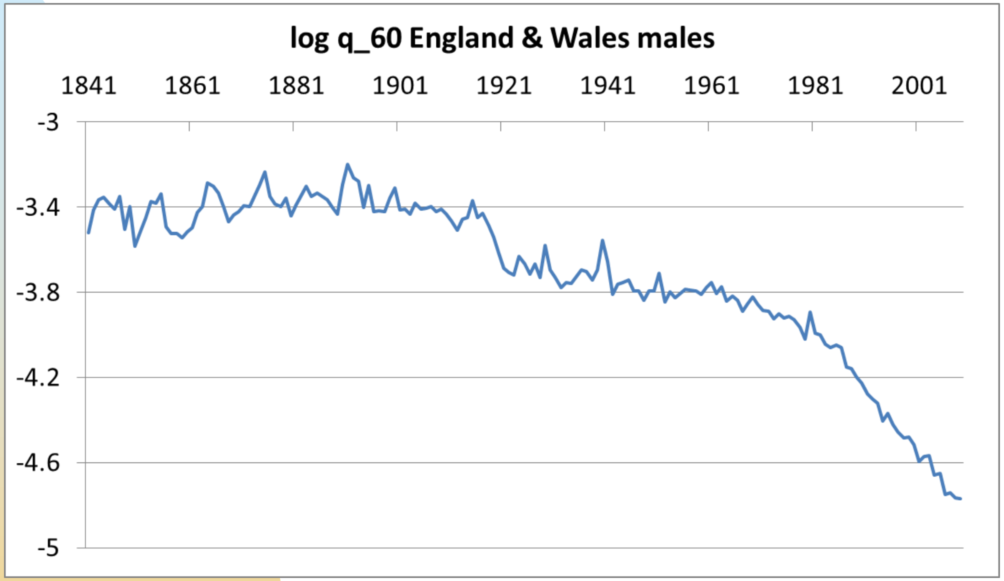
- In almost every country, life expectancies increase and mortality rates decrease
- The decrease in log mortality rates often appears linear:



- Log mortality is usually projected as random walk with drift
- Drift coincides with historically observed slope

Introduction

- **What if we look further into the past?**



- **Trend in log mortality appears only piecewise linear**
- **Slope of the mortality trend changes**
- **Random walk with drift does not account for trend changes**
  - Finding is not new, see e.g. Sweeting (2011) or Li et al. (2011)

➔ **Additional uncertainty and the need for modeling mortality trend changes**

Agenda

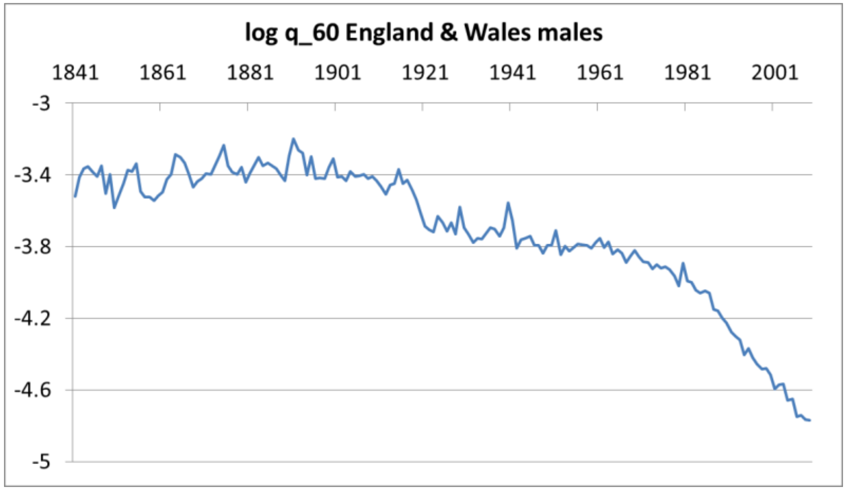
- **Why two mortality trends?**
  - Actual mortality trend (AMT)
  - Expected mortality trend (EMT)
  - Some examples for applications

- **A combined model for both trends**
  - AMT component
  - Stochastic start trend
  - Comparison with other AMT approaches
  - EMT component
  - Comparison with other EMT approaches

- **Conclusion**

Actual Mortality Trend

- **One trend is the actual mortality trend (AMT)**
  - The AMT describes realized future mortality and is the core of most existing mortality models
  - Goal: plausible extrapolation of historically observed mortality
  - Thus, modeling as piecewise linear trend with random changes in the slope plus random fluctuations around the linear trend
  - Time and magnitude of changes in the AMT need to be simulated



- **The AMT is not (fully) observable!**
- **We „know" the historical AMT**
  - Random fluctuations can be filtered out
  - Historical trend changes and slopes of piecewise linear trends are rather obvious
- **We have an idea of the current value of the AMT**
- **But we do not know the current slope**
  - There might be a trend change this year
  - There might have been a trend change over the last years which is covered by random fluctuations

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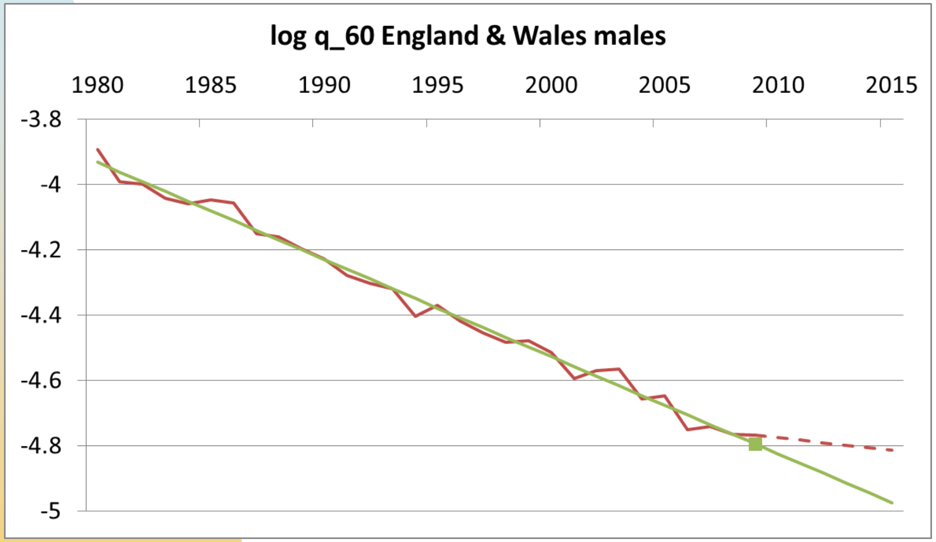
Expected Mortality Trend

- **The second trend is the estimated mortality trend (EMT)**
- **The EMT is the actuary‘s/demographer‘s estimate of the AMT**
  - Current value and current slope of the AMT



- **The EMT is based on the most recent historical mortality evolution and updated as soon as new data becomes available**
- **The EMT is the basis for mortality projections and (generational) mortality tables, e.g., for reserving**

Why Two Mortality Trends?

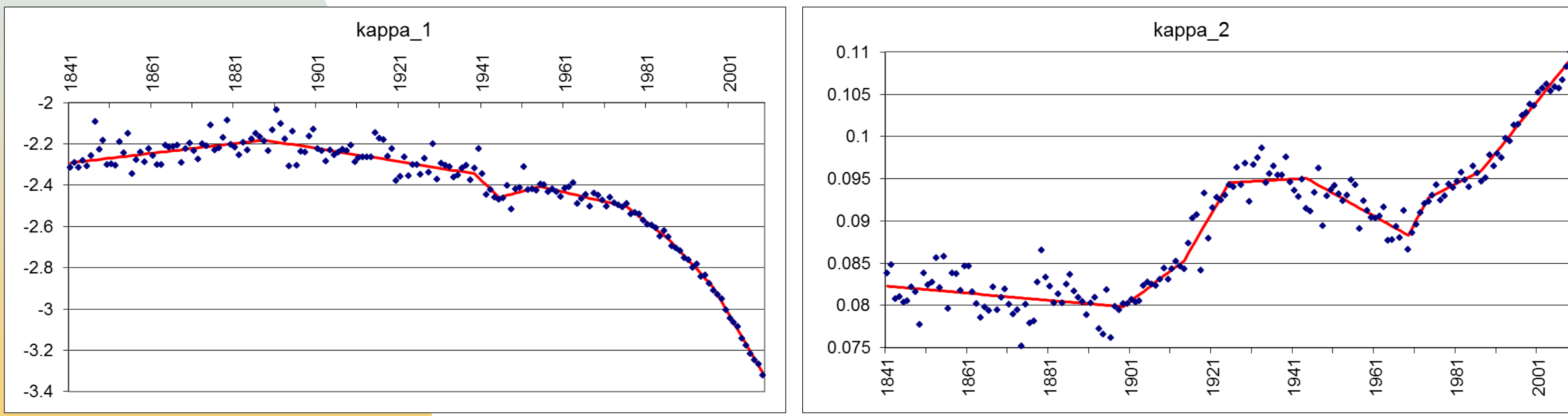
- **The mortality trend to consider depends on the application in view, examples:**
- **Capital for a portfolio run-off ➔ AMT over the run-off**
- **Reserves for the portfolio after 10 years**
  ➔ EMT after 10 years and AMT over the 10 years
  - AMT for the next 10 years is required to be able to compute EMT in 10 years time
- **Payout of a mortality derivative which reduces GAO risk**
  ➔ EMT at maturity and AMT up to maturity
- **Analysis of hedge effectiveness of the derivative**
  ➔ EMT at maturity, AMT also beyond
- **Solvency Capital Requirement: combined 99.5th percentile of actual payments over the next year and changes in the liabilities**
  ➔ AMT for actual payments and EMT for change in liabilities

Combined AMT/EMT Model – AMT Component

- **For the AMT model component, we use the model of Sweeting (2011):**

$$logit(q_{x,t}) := \log\left(\frac{q_{x,t}}{1-q_{x,t}}\right) = \kappa_1(t) + \kappa_2(t)\cdot(x-\bar{x}),$$

- **But in principle, our approach of modeling AMT and EMT could be applied in any model with time process(es)**
- **Model parameters for English and Welsh males aged 60-89:**



- **7 trend changes for both kappa processes ➔ trend change probability p = 7/169**
- **Trend change intensity:** $\lambda_i = S_i \cdot M_i, i = 1{,}2,$
  - $S_i, i = 1{,}2$: sign of trend change, bernoulli distributed with values 1 and -1 and probability 1/2
  - $M_i, i = 1{,}2$: absolute magnitude of trend change, normally distributed with parameters according to sample mean and sample variance

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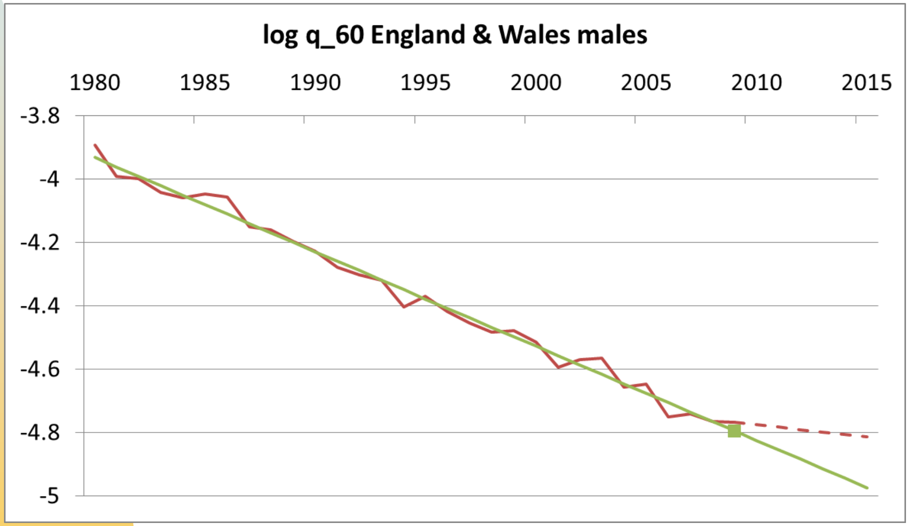
Combined AMT/EMT Model – Stochastic Start Trend

- **The AMT at the start of a simulation is not observable**
  - Typically, the EMT is assumed as the starting AMT
- **But what if another trend change occured after the last significant one?**

  ➔ Additional uncertainty
- **This is more than parameter uncertainty in estimating a linear trend**
  - Example:



- **Uncertainty accounted for by stochastic start trend**
  - We explain how a combined distribution for the current value and the current slope of the AMT can be established

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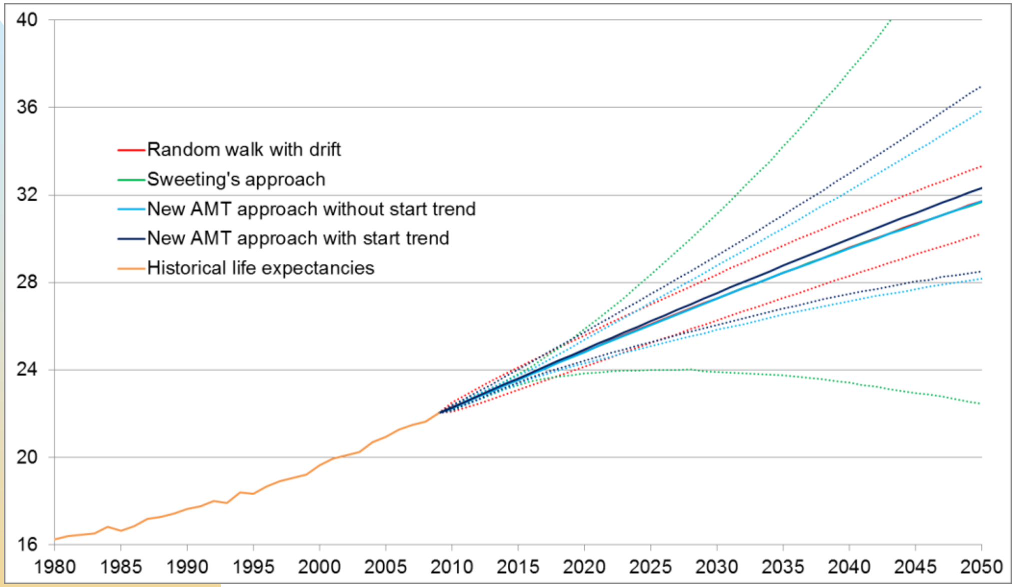
Combined AMT/EMT Model – Comparison of AMT Approaches

- **Remaining period life expectancy for a 60-year old (with 10th and 90th percentiles)**



  - Similar and in every case plausible medians
  - Random walk with drift: first widest and then narrowest confidence bounds, 3.1 years in 2050 seem unrealistically small
  - Sweeting‘s approach: implausibly wide confidence bounds (23.3 years in 2050)
  - New ATM approach: confidence bounds look plausible
  - Stochastic start trend widens confidence bound in 2050 from 7.7 years to 8.5 years

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Combined AMT/EMT Model – The EMT Component

- **The EMT is the best estimate of the AMT at any point in time**
- **In principle, every estimation procedure is feasible for the EMT**

- **„Optimal“ EMT in our setting: Mean of start trend distribution**
  - Start trend distribution is too complex to be established whenever the EMT is required
  - Simpler methods required for the EMT in simulations

- **We propose to compute the EMT by weighted regression**
  - Extrapolation of linear trend in most recent data points
  - Crucial question: How many data points?
    - Too many data points: Delayed reaction to change in the AMT
    - Too little data points: EMT is exposed to random noise in the AMT
  - Weights decrease exponentially going backwards in time
  - Optimal weighting derived by minimizing the MSE between AMT and EMT

Combined AMT/EMT Model – Comparion of EMT Approaches

- **MSE in estimating the cohort life expectancy for a 60-year old in 2050**

EMT estimation method	Mean squared error	Root mean squared error
Optimal weighting	5.136	2.266
Stronger weighting	6.666	2.582
Reduced weighting	5.190	2.278
Unweighted regression on 5 data points	10.188	3.192
Unweighted regression on 10 data points	5.801	2.409
Unweighted regression on 20 data points	11.682	3.418

- **Practical implication: derivative with payout equal to this life expectancy**
  - The payout is computed based on the EMT in 2050
  - Underestimation of life expectancy critical from hedger‘s point of view
  - Probabilities of underestimating the life expectancy:

EMT estimation method	> 5 years	> 10 years
Optimal weighting	3%	0.4%
Unweighted regression on 20 data points	7.7%	2.1%

➔ EMT approach has a crucial impact on the payout and the hedge effectiveness

Conclusion

- **Two trends need to be distinguished and modeled:**
  - The actual mortality trend (AMT) which is unobservable
  - The estimated mortality trend (EMT) which is an observer‘s estimate of the AMT
- **The trend to consider depends on the question in view**
- **The AMT should be modeled as a piecewise linear function with random changes in the slope**
  - The commonly used random walk with drift underestimates longevity risk systematically
- **Since the AMT at the start of a simulation is unknown a stochastic start trend should be considered**
- **The choice of the EMT approach is crucial in practice**
  - A weighted regression approach seems most reasonable
  - We show how optimal weights can be derived

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References

- Sweeting, P., 2011. A Trend-Change Extension of the Cairns-Blake-Dowd Model. *Annals of Actuarial Science*, Volume 5, pp. 143-162.

- Li, J., Chan, W. S. & Cheung, S. H., 2011. Structural Changes in the Lee-Carter Mortality Indexes: Detection and Implications. *North American Actuarial Journal*, Volume 15, pp. 13-31.

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