It Takes Two:
Why Mortality Trend Modeling is more than Modeling one Mortality Trend

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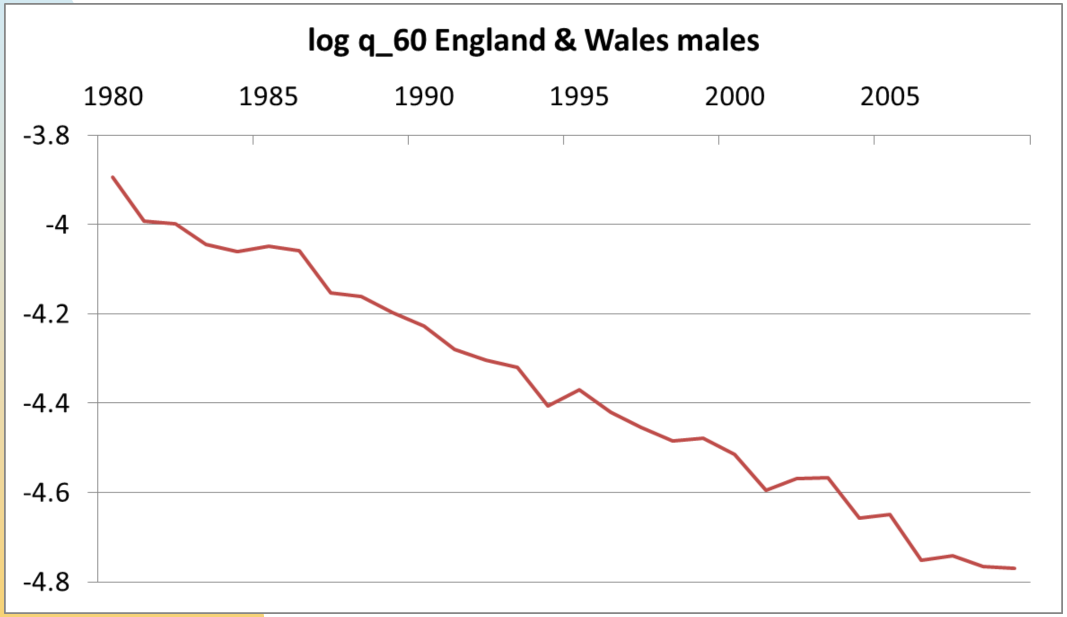
Introduction

- In almost every country, life expectancies increase and mortality rates decrease
- The decrease in log mortality rates often appears linear:



- Log mortality is usually projected as random walk with drift
- Drift coincides with historically observed slope

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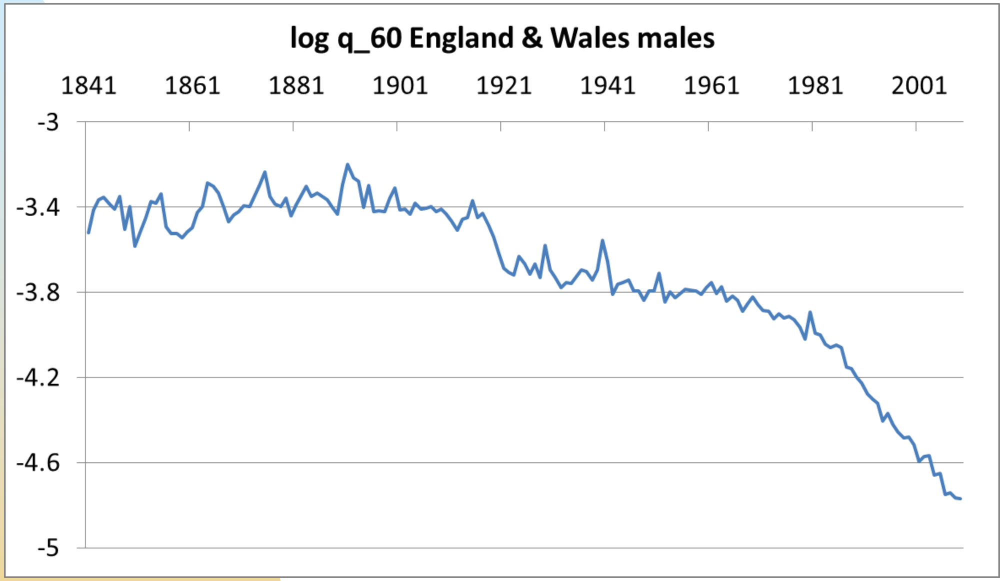
Introduction

- **What if we look further into the past?**



- **Trend in log mortality appears only piecewise linear**
- **Slope of the mortality trend changes**
- **Random walk with drift does not account for trend changes**
  - Finding is not new, see e.g. Sweeting (2011) or Li et al. (2011)

➔ **Additional uncertainty and the need for modeling mortality trend changes**

Agenda

- **Why two mortality trends?**
  - Actual mortality trend (AMT)
  - Expected mortality trend (EMT)
  - Some examples for applications

- **A combined model for both trends**
  - AMT component
  - Stochastic start trend
  - Comparison with other AMT approaches
  - EMT component
  - Comparison with other EMT approaches

- **Conclusion**

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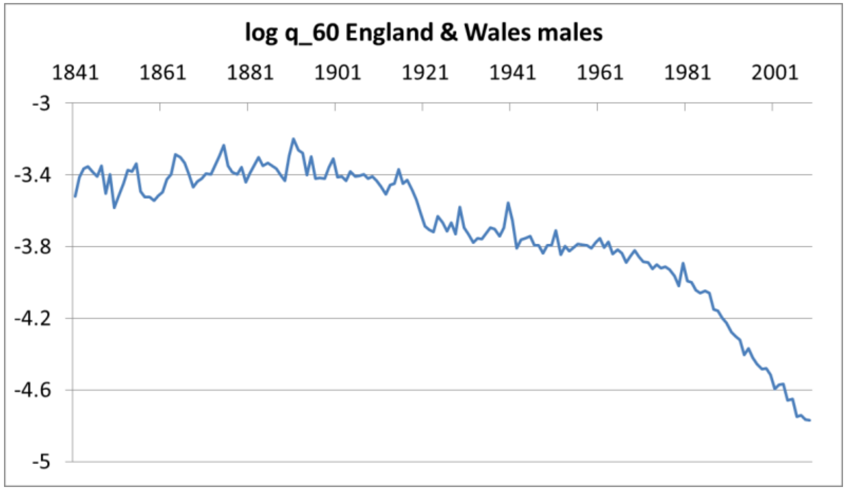
Actual Mortality Trend

- **One trend is the actual mortality trend (AMT)**
  - The AMT describes realized future mortality and is the core of most existing mortality models
  - Goal: plausible extrapolation of historically observed mortality
  - Time and magnitude of changes in the AMT plus random fluctuations around the AMT need to be modeled



- **The AMT is not (fully) observable!**
- **We „know" the historical AMT**
  - Random fluctuations can be filtered out
  - Historical trend changes and slopes of piecewise linear trends are rather obvious
- **We have an idea of the current value of the AMT**
- **But we do not know the current slope**
  - There might be a trend change this year
  - There might have been a trend change over the last years which is covered by random fluctuations

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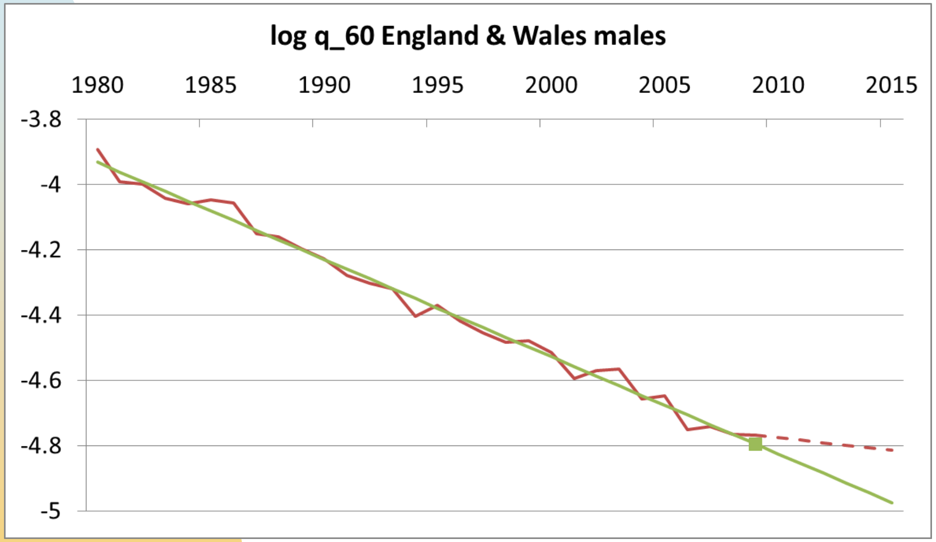
Expected Mortality Trend

- **The second trend is the estimated mortality trend (EMT)**
- **The EMT is the actuary‘s/demographer‘s estimate of the AMT**
  - Current value and current slope of the AMT



- **The EMT is based on the most recent historical mortality evolution and updated as soon as new data becomes available**
- **The EMT is the basis for mortality projections and (generational) mortality tables, e.g., for reserving**

Why Two Mortality Trends?

- **The mortality trend to consider depends on the application in view, examples:**
- **Capital for a portfolio run-off ➔ AMT over the run-off**
- **Reserves for the portfolio after 10 years**
  ➔ EMT after 10 years and AMT over the 10 years
  - AMT for the next 10 years is required to be able to compute EMT in 10 years time
- **Payout of a mortality derivative which reduces GAO risk**
  ➔ EMT at maturity and AMT up to maturity
- **Analysis of hedge effectiveness of the derivative**
  ➔ EMT at maturity, AMT also beyond
- **Solvency Capital Requirement: combined 99.5th percentile of actual payments over the next year and changes in the liabilities**
  ➔ AMT for actual payments and EMT for change in liabilities

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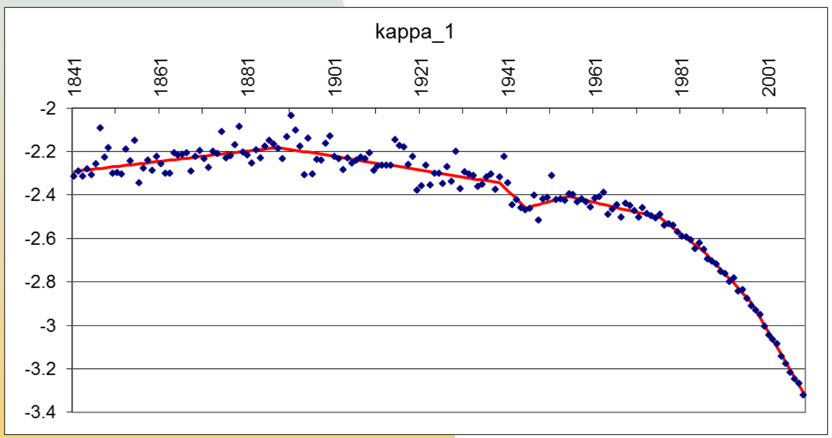
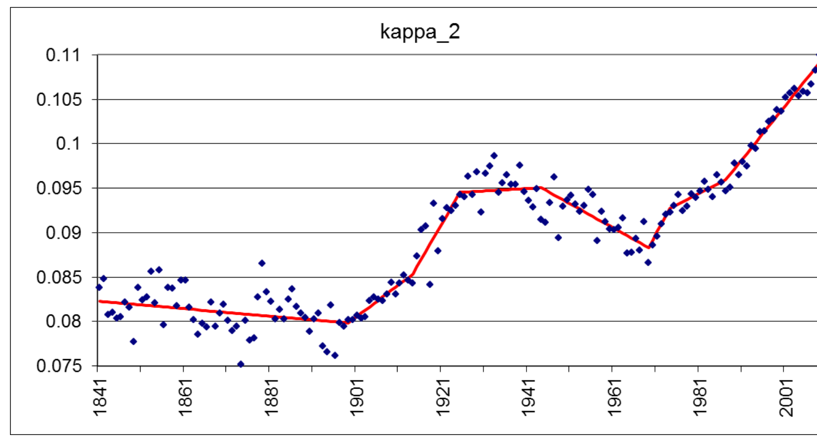
Combined AMT/EMT Model – AMT Component

- **For the AMT model component, we use the model of Sweeting (2011):**

$$logit(q_{x,t}) := \log\left(\frac{q_{x,t}}{1-q_{x,t}}\right) = \kappa_1(t) + \kappa_2(t)\cdot(x-\bar{x}),$$

- **But in principle, our approach of modeling AMT and EMT could be applied in any model with time process(es)**
- **Model parameters for English and Welsh males aged 60-89:**





- **7 trend changes for both kappa processes ➔ trend change probability p = 7/169**
- **Trend change intensity (different from Sweeting (2011)):** $\lambda_i = S_i \cdot M_i, i = 1{,}2,$
  - $S_i, i = 1{,}2$: sign of trend change, bernoulli distributed with values 1 and -1 and probability 1/2
  - $M_i, i = 1{,}2$: absolute magnitude of trend change, normally distributed with parameters according to sample mean and sample variance

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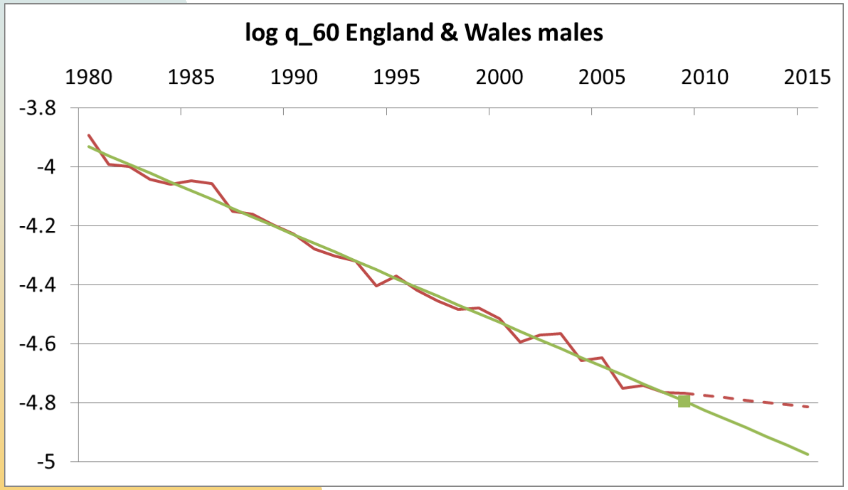
Combined AMT/EMT Model – Stochastic Start Trend

- **The AMT at the start of a simulation is not observable**
- **What if another trend change occured after the last significant one?**
  - **➔ Additional uncertainty**
- **This is more than parameter uncertainty in estimating a linear trend**



- **Uncertainty accounted for by stochastic start trend**
  - We explain how a combined distribution for the current value and the current slope of the AMT can be established

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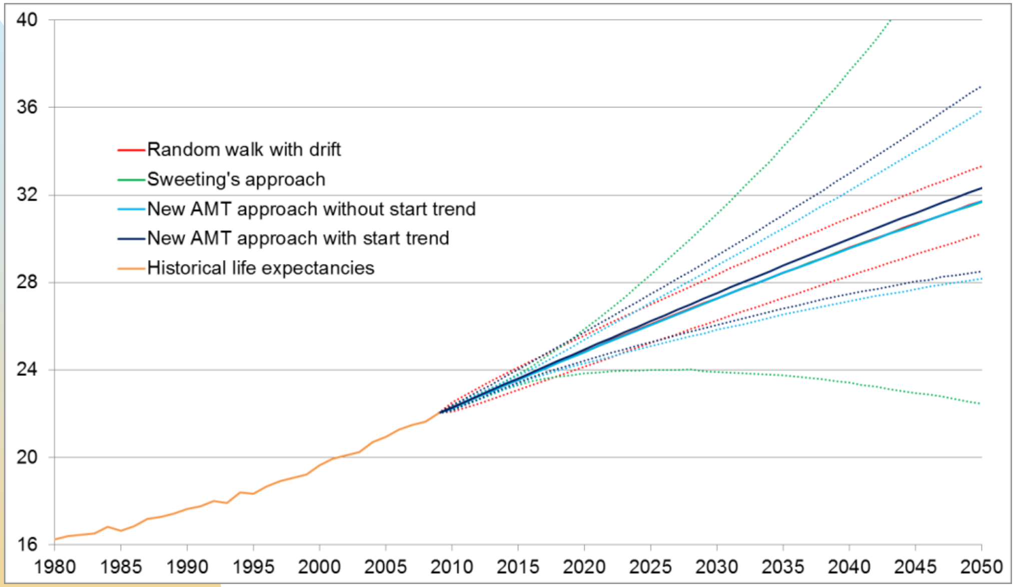
Combined AMT/EMT Model – Comparison of AMT Approaches

Remaining period life expectancy for a 60-year old (with 10th and 90th percentiles)



- Similar and in every case plausible medians
- Random walk with drift: first widest and then narrowest confidence bounds, 3.1 years in 2050 seem unrealistically small
- Sweeting‘s approach: implausibly wide confidence bounds (23.3 years in 2050)
- New ATM approach: confidence bounds look plausible
- Stochastic start trend widens confidence bound in 2050 from 7.7 years to 8.5 years

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Combined AMT/EMT Model – The EMT Component

- **The EMT is the best estimate of the AMT at any point in time**
- **In principle, every estimation procedure is feasible for the EMT**

- **„Optimal“ EMT in our setting: Mean of start trend distribution**
  - Start trend distribution is too complex to be established whenever the EMT is required
  - Simpler methods required for the EMT in simulations

- **We propose to compute the EMT by weighted regression**
  - Extrapolation of linear trend in most recent data points
  - Crucial question: How many data points?
    - Too many data points: Delayed reaction to change in the AMT
    - Too little data points: EMT is exposed to random noise in the AMT
  - Weights $w_i(t-s) = \frac{1}{\left(1+{}^{1}/_{h_i}\right)^{t-s}}, s \leq t, i = 1{,}2$ decrease exponentially going backwards in time
  - Optimal weighting parameters h_i can be derived as minimizing the MSE between AMT and EMT

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Combined AMT/EMT Model – Comparion of EMT Approaches

- **MSE in estimating the cohort life expectancy for a 60-year old in 2050**

EMT estimation method	Mean squared error	Root mean squared error
Optimal weighting parameters	5.136	2.266
Reduced weighting parameters	6.666	2.582
Increased weighting parameters	5.190	2.278
Average weighting parameters	5.312	2.305
Unweighted regression on 5 data points	10.188	3.192
Unweighted regression on 10 data points	5.801	2.409
Unweighted regression on 20 data points	11.682	3.418

- **Practical implication: derivative with payout equal to this life expectancy**
  - The payout is computed based on the EMT in 2050
  - Underestimation of life expectancy critical from hedger‘s point of view
  - Probabilities of underestimating the life expectancy:

EMT estimation method	> 5 years	> 10 years
Optimal weighting parameters	3%	0.4%
Unweighted regression on 20 data points	7.7%	2.1%

➔ EMT approach has a crucial impact on the payout and the hedge effectiveness

Conclusion

- **Two trends need to be distinguished and modeled:**
  - The actual mortality trend (AMT) which is unobservable
  - The estimated mortality trend (EMT) which is an observer‘s estimate of the AMT
- **The trend to consider depends on the question in view**
- **The AMT should be modeled as a piecewise linear function with random changes in the slope**
  - The commonly used random walk with drift underestimates longevity risk systematically
  - The distribution for the trend change intensity needs to be consistent with the trend change probability
- **Since the AMT at the start of a simulation is unknown a stochastic start trend should be considered**
- **The choice of the EMT approach is crucial in practice**
  - A weighted regression approach seems most reasonable
  - We show how optimal weights can be derived

References

- Sweeting, P., 2011. A Trend-Change Extension of the Cairns-Blake-Dowd Model. *Annals of Actuarial Science*, Volume 5, pp. 143-162.

- Li, J., Chan, W. S. & Cheung, S. H., 2011. Structural Changes in the Lee-Carter Mortality Indexes: Detection and Implications. *North American Actuarial Journal*, Volume 15, pp. 13-31.

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