Modeling Mortality Trend under Modern Solvency Regimes

Matthias Börger
Daniel Fleischer
Nikita Kuksin

February 2012
Introduction

What is longevity risk?

- Longevity risk is the risk of underestimating future mortality improvements
  - Trend risk
  - Mortality risk has a trend risk and a catastrophe risk component
  - Systematic and non-hedgeable risk
- Explicitly accounted for under Solvency II and the Swiss Solvency Test (SST)
Capital Requirements under Solvency II

- General concept for Solvency Capital Requirement (SCR) under Solvency II
  - SCR = 99.5% Value-at-Risk (VaR) of Available Capital over 1 year
  - „Capital necessary to cover losses over next year with at least 99.5% probability“
  - Overall risk is typically split into several modules, individual SCRs are finally aggregated

- Stochastic mortality model is required for mortality/longevity trend risk under Solvency II

- In a 1-year setting, longevity/mortality trend risk consists of two components:
  - Low/high realized mortality in the one year
  - Decrease/increase in expected future mortality, i.e. changes in the long-term mortality trend
Mortality Trend Model Requirements

Goal: Specification and calibration of a mortality model with the following properties

- Simultaneous modeling of mortality and longevity risk
  - Exploiting of diversification effects
- Full age range
  - 20 to 105 in our case
- Consideration of several populations at the same time
  - Males and females in the same country
  - Populations from different countries
- Quantification of risk over limited time horizons
  - One-year view of Solvency II and the SST particularly relevant
- Plausible tail scenarios
  - 99.5% VaR
- Conservative calibration
We model the logit of mortality rates

\[
\text{logit}(q_{x,t}) = \alpha_x + \kappa_x^{(1)} + \kappa_x^{(2)}(x - x_{\text{center}}) + \kappa_x^{(3)}(x_{\text{young}} - x)^+ + \kappa_x^{(4)}(x - x_{\text{old}})^+ + \gamma_{t-x}
\]

- \(x_{\text{center}} = 60, x_{\text{young}} = 55, x_{\text{old}} = 85\)
- \(\kappa_x^{(1)}\) describes the general level of mortality, \(\kappa_x^{(2)}\) is the slope of the mortality curve, \(\kappa_x^{(3)}\) and \(\kappa_x^{(4)}\) describe additional effects in young and old age mortality, respectively

**Model estimation via Generalized Linear Model Theory**

- Logit is canonical link function for Binomial distribution
- Number of deaths is binomially distributed given initial exposures
- Cohort parameters are fitted to residuals separately
Multi-Population Setting

- There is clearly a common trend in $\kappa_t^{(1)}$
- A model for several populations must account for that
- We apply cointegration and an error correction model for deviations from the common trend
Projection of $\kappa_{t,total}^{(1)}$ for the total population

- Linear trends with breaks in the historical data
  - Commonly used random walk with drift does not allow for such trend breaks
  - Trend breaks are particularly important under one-year view (change of best estimate trend)

- Idea: Each year, fit regression line to historical data and forecast future best estimate mortality as

$$
\kappa_{t+1,total}^{(1)} = \lambda_t (t + 1) + \epsilon_{t+1}^{(1)} (\sigma^{(1)} + \bar{\sigma}^{(1)})
$$

- $\bar{\sigma}^{(1)}$ is a volatility add-on
- Volatility $\sigma^{(1)}$ may be weighted to stress most recent past
- Implicit „re-calibration“ of the model with respect to the long-term trend
- To stress most recent mortality experience, the regression line is fitted with weights

$$
w_s = \left(1 + \frac{1}{h}\right)^{t-s}
$$
Model Simulation (ctd.)

Iterative application of the trend forecasting:

- Weighting parameter $h$ has massive impact
- Plausible one-year and run-off scenarios
- Each run-off scenario is a combination of one-year scenarios
- Disentangling of one-year noise and long-term trend uncertainty
- Possibly more plausible confidence bounds than for a random walk with drift
Projection of $\kappa_{t,p}^{(1)}$ for individual populations

For each individual population we project as

\[ \kappa_{t,p}^{(1)} = \kappa_{t,total}^{(1)} + a_p + b_p (\kappa_{t-1,p}^{(1)} - \kappa_{t-1,total}^{(1)}) + \epsilon_{t,p} \]

- $b_p$ denotes the "mean reversion speed" (absolute value should be smaller than 1)
- $a_p/(1-b_p)$ is the long-term difference between the total population and population $p$

Different approaches of calibrating the long-term difference

- Fitting of an AR(1) process to historical differences
- Weighted/unweighted average of historical differences
- ...
Model Simulation (ctd.)

- **Projection of** $\kappa_t^{(2)}$, $\kappa_t^{(3)}$, and $\kappa_t^{(4)}$ **for the individual populations**
  - No substantial trend obvious in the historical data
  - Forecast as correlated 3-dimensional random walk
  - No substantial correlation with $\kappa_t^{(1)}$
  - Volatility add-on $\sigma^{(2)}$ for $\kappa_t^{(2)}$
    - The larger the changes in the slope of the mortality, the smaller the correlation between young and old ages
    - Thus the add-on affects diversification between mortality and longevity risk
  - Between populations, increments of $\kappa_t^{(1)}$ and $\kappa_t^{(2)}$ are correlated
    - This also implies slight correlation between the $\kappa_t^{(3)}$ and $\kappa_t^{(4)}$
    - Historical correlations should be checked carefully though and possibly adjusted

- **Projection of** $\gamma_t^{1-x}$
  - Cohort parameters should stay around zero
  - Forecast as Gaussian noise
  - Cohort parameters are rather irrelevant for short-term simulations as under Solvency II
Weighting Parameters and Volatility Add-ons

- Parameters \( h, \sigma^{(1)}, \) and \( \sigma^{(2)} \) have a massive impact on simulation outcomes and thus SCRs
- Add-on \( \sigma^{(1)} \) determines possible severity of short-term events
- Weighting parameter \( h \) determines trend changes over one year and width of confidence bounds

- Calibration is difficult but should be conservative
  - Fitting to most severe events/evolutions in the past
    - Example: Rapid increase in Dutch life expectancy gains starting from about 1970
    - Question: At which percentile should such extreme evolutions be observed?

- Calibration of \( \sigma^{(2)} \)
  - The larger the add-on the smaller the correlation between young and old ages thus limiting diversification
  - Choose \( \sigma^{(2)} \) such that correlation between ages at the boundaries of the age range is (close to) zero for most populations
Numerical comparisons

- **Standard formula approach**
  - Reduction/increase of all mortality rates by 20%/15%
  - Change in liabilities is about SCR for longevity/mortality risk

- **SCRs for term insurances (maturity at age 65) and whole life annuities:**

  ![Graph showing differences in SCR for term insurance and annuities between standard formula, single population model, and multi-population model.](image)

  - **Multi-population model demands least capital**
    - Model is worthwhile even for only one population because of reduced trend uncertainty
  - **Standard model seems to overstate the risk in general**
Mortality/Longevity Threat Scenarios

- Available data contains only little information on tail scenarios which we are interested in.
- Uncertainty remains whether model outcomes are severe enough.
  → Incorporate epidemiological/demographic expert opinion.

**Specification of mortality/longevity threat scenarios**
- Shock to mortality projection
- Likely effects of finding of a cure for a certain illness
- Scenarios which the statistical model cannot generate, e.g., diverging mortality trends between countries/regions
- ...

**Application of threat scenarios**
- Check of model calibration: Adjustment of weighting parameter or volatility add-ons if the model outcomes should cover the threat scenarios but do not.
- Inclusion in SCR computations: SCR as a weighted average of model outcomes and threat scenarios.
Summary

- Specification, calibration, and application of a mortality model for solvency purposes
  - Full age range
  - Variability in simulation outcomes due to 5 stochastic drivers
  - Clear interpretation of the model parameters
  - Non-trivial correlation structure to allow for simultaneous modeling of mortality and longevity risk
  - Stochastic trend modeling spares full re-calibration of the model in each scenario
  - Plausible outcomes in one-year view and run-off view
  - Conservative calibration
  - Inclusion of expert opinion
  - Multi-population model allowing for diversification effects and risk reduction

- Model can be applied for other purposes as well (with slightly different calibration)

- A stochastic trend process with several appealing features
  - Quantification of trend risk over any desired time horizon
  - Advantages compared to the commonly used random walk with drift (e.g. long-term risk vs. short-term noise)
  - The trend process could easily be incorporated in other mortality models as well, e.g. Lee-Carter or Cairns-Blake-Dowd
Matthias Boerger

Institute of Insurance, Ulm University & Institute for Finance and Actuarial Sciences (ifa), Ulm
Helmholtzstraße 22, 89081 Ulm, Germany
Phone: +49 731 50-31257, Fax: +49 731 50-31239
Email: m.boerger@ifa-ulm.de