Modeling Mortality Trend under Modern Solvency Regimes

Matthias Börger
Daniel Fleischer
Nikita Kuksin

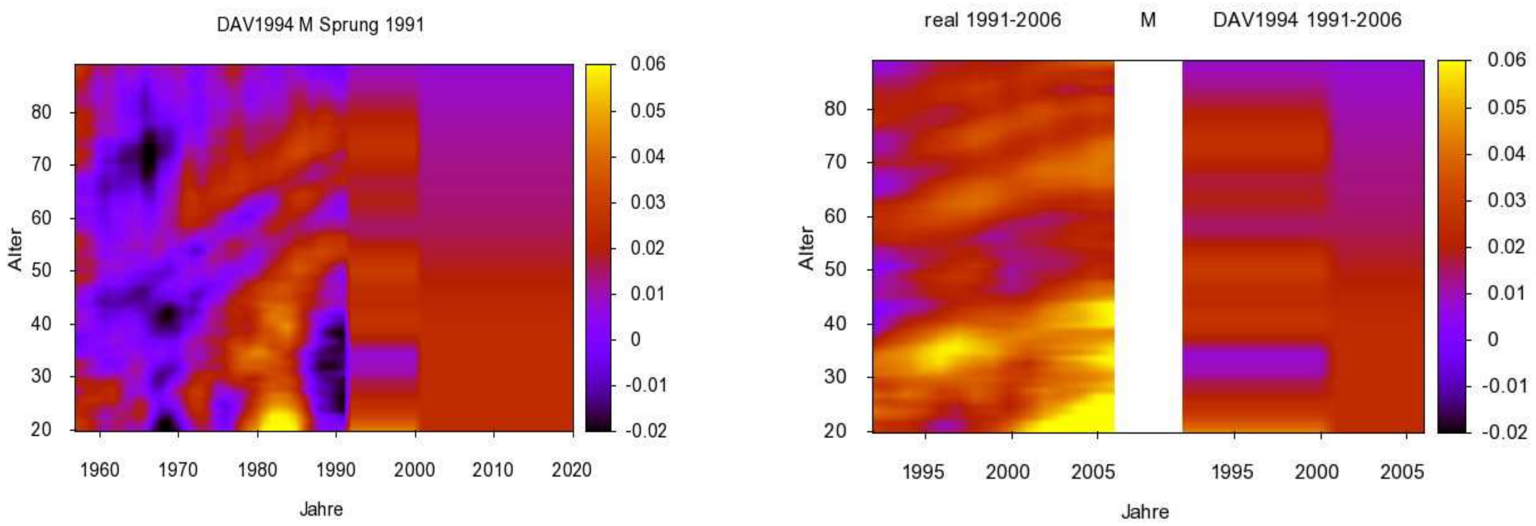
February 2012

Helmholtzstraße 22
D-89081 Ulm
phone +49 (0) 731/50-31230
fax +49 (0) 731/50-31239
email ifa@ifa-ulm.de

ifa
Institut für Finanz- und Aktuarwissenschaften

Introduction

- **What is longevity risk?**



→ **Longevity risk is the risk of underestimating future mortality improvements**

- Trend risk
- Mortality risk has a trend risk and a catastrophe risk component
- Systematic and non-hedgeable risk
  → Explicitly accounted for under Solvency II and the Swiss Solvency Test (SST)

ifa Institut für Finanz- und Aktuarwissenschaften

Capital Requirements under Solvency II

- **General concept for Solvency Capital Requirement (SCR) under Solvency II**
  - SCR = 99.5% Value-at-Risk (VaR) of Available Capital over 1 year
  - „Capital necessary to cover losses over next year with at least 99.5% probability“
  - Overall risk is typically split into several modules, individual SCRs are finally aggregated
- **Stochastic mortality model is required for mortality/longevity trend risk under Solvency II**
- **In a 1-year setting, longevity/mortality trend risk consists of two components:**
  - Low/high realized mortality in the one year
  - Decrease/increase in expected future mortality, i.e. changes in the long-term mortality trend

ifa Institut für Finanz- und Aktuarwissenschaften

Mortality Trend Model Requirements

- **Goal: Specification and calibration of a mortality model with the following properties**
  - Simultaneous modeling of mortality and longevity risk
    - Exploiting of diversification effects
  - Full age range
    - 20 to 105 in our case
  - Consideration of several populations at the same time
    - Males and females in the same country
    - Populations from different countries
  - Quantification of risk over limited time horizons
    - One-year view of Solvency II and the SST particularly relevant
  - Plausible tail scenarios
    - 99.5% VaR
  - Conservative calibration

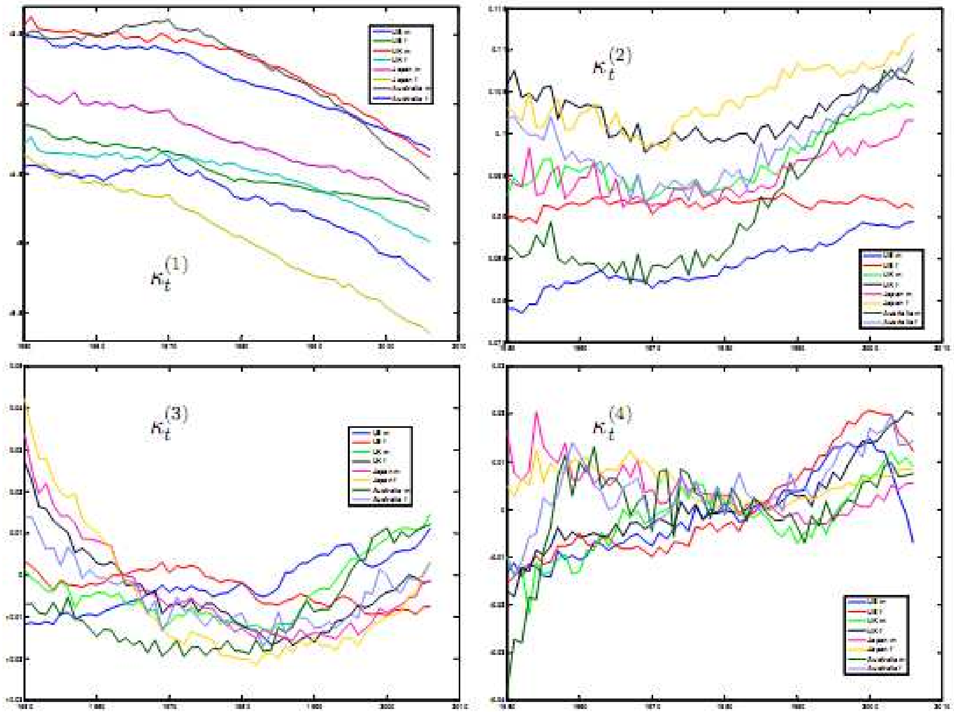
Model Specification and Estimation

- **We model the logit of mortality rates**

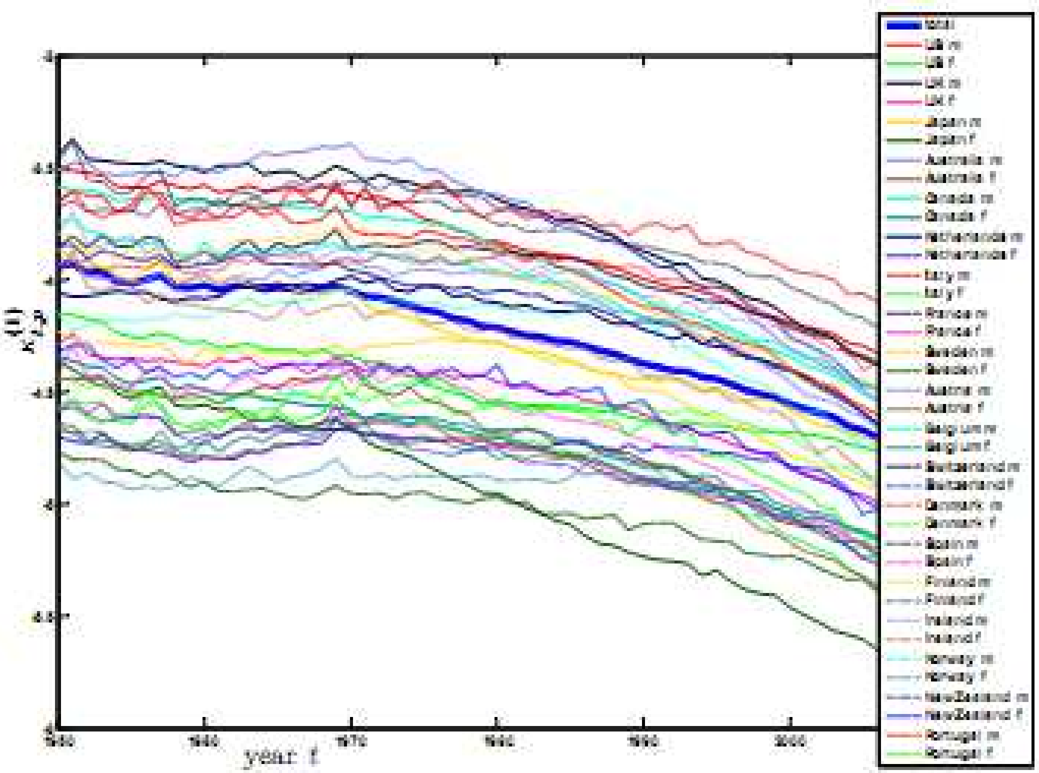
  $$\text{logit}(q_{x,t}) = \alpha_x + \kappa_t^{(1)} + \kappa_t^{(2)}(x - x_{center}) + \kappa_t^{(3)}(x_{young} - x)^+ + \kappa_t^{(4)}(x - x_{old})^+ + \gamma_{t-x}$$

  - $x_{center} = 60$, $x_{young} = 55$, $x_{old} = 85$
  - $\kappa_t^{(1)}$ describes the general level of mortality, $\kappa_t^{(2)}$ is the slope of the mortality curve, $\kappa_t^{(3)}$ and $\kappa_t^{(4)}$ describe additional effects in young and old age mortality, respectively

- **Model estimation via Generalized Linear Model Theory**
  - Logit is canonical link function for Binomial distribution
  - Number of deaths is binomially distributed given initial exposures
  - Cohort parameters are fitted to residuals separately



Multi-Population Setting



- There is clearly a common trend in $\kappa_t^{(1)}$
- A model for several populations must account for that
- We apply cointegration and an error correction model for deviations from the common trend

ifa
Institut für Finanz- und Aktuarwissenschaften

Model Simulation

Projection of $\kappa_{t,total}^{(1)}$ for the total population

- **Linear trends with breaks in the historical data**
  - Commonly used random walk with drift does not allow for such trend breaks
  - Trend breaks are particularly important under one-year view (change of best estimate trend)

- **Idea: Each year, fit regression line to historical data and forecast future best estimate mortality as**

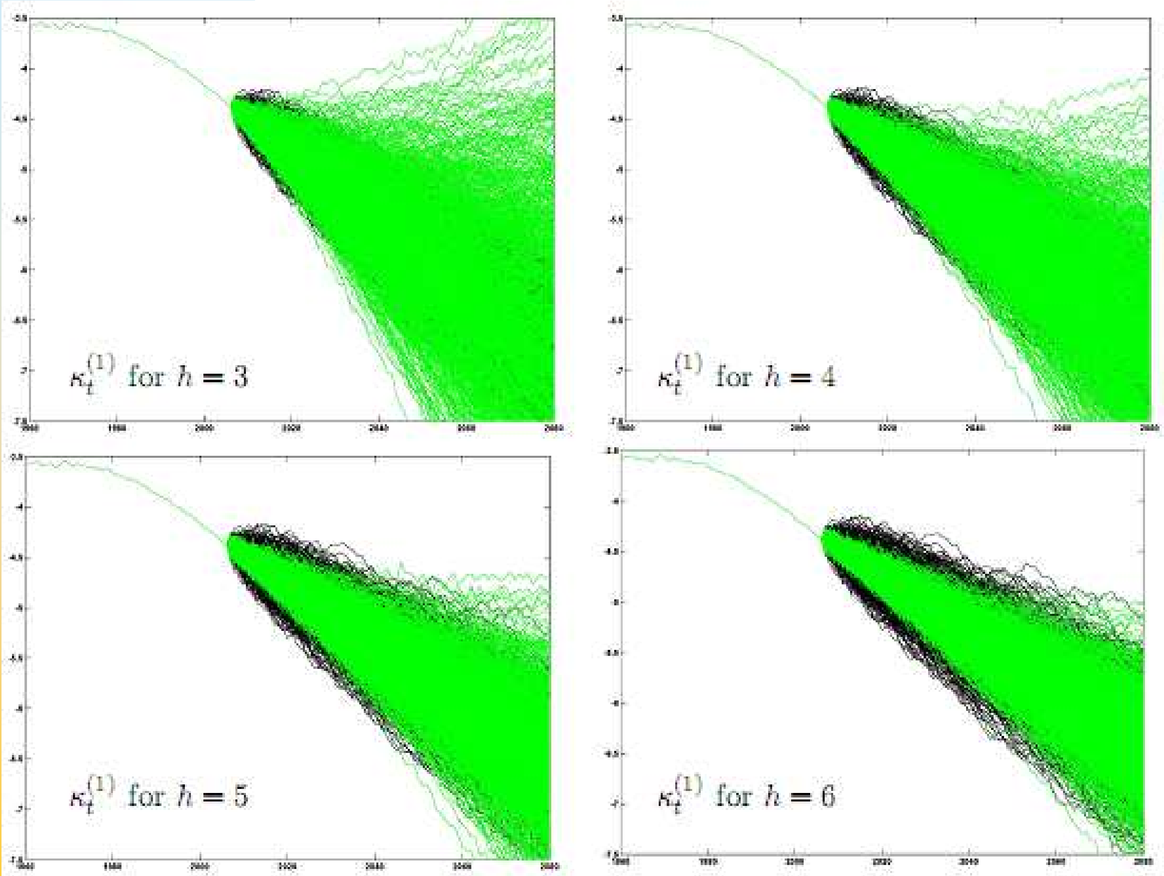
$$\kappa_{t+1,total}^{(1)} = l_t(t+1) + \varepsilon_{t+1}^{(1)}(\sigma^{(1)} + \overline{\sigma}^{(1)})$$

  - $\overline{\sigma}^{(1)}$ is a volatility add-on
  - volatility $\sigma^{(1)}$ may be weighted to stress most recent past
  - Implicit „re-calibration“ of the model with respect to the long-term trend
  - To stress most recent mortality experience, the regression line is fitted with weights

$$w_s = \left(1 + \frac{1}{h}\right)^{s-t}$$

Model Simulation (ctd.)

Iterative application of the trend forecasting:



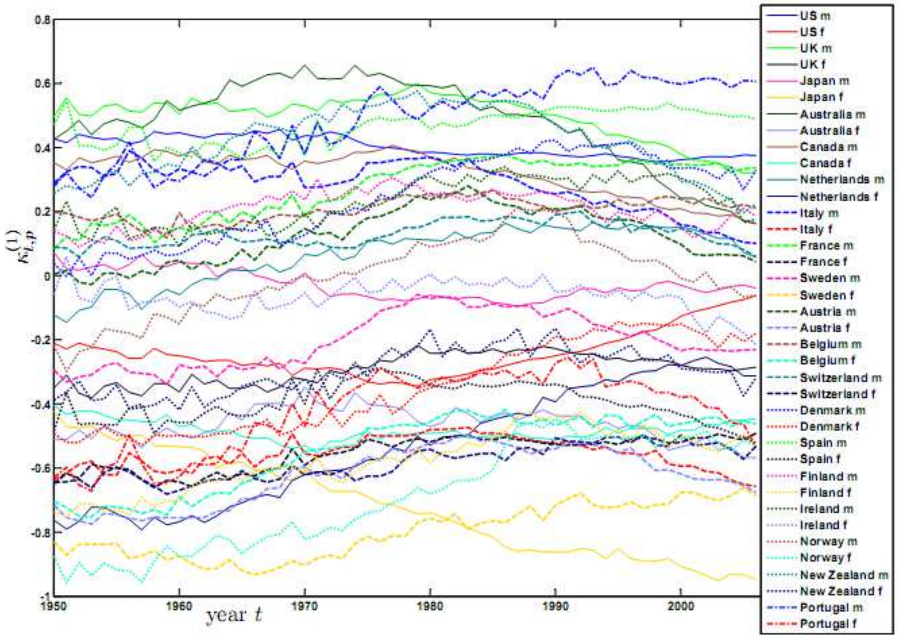
- **Weighting parameter h has massive impact**
- **Plausible one-year and run-off scenarios**
- **Each run-off scenario is a combination of one-year scenarios**
- **Disentangling of one-year noise and long-term trend uncertainty**
- **Possibly more plausible confidence bounds than for a random walk with drift**

Model Simulation (ctd.)

Projection of $\kappa_{t,p}^{(1)}$ for individual populations

- **For each individual population we project as**
  - $\kappa_{t,p}^{(1)} = \kappa_{t,total}^{(1)} + a_p + b_p(\kappa_{t-1,p}^{(1)} - \kappa_{t-1,total}^{(1)}) + \varepsilon_{t,p}$
  - b_p denotes the „mean reversion speed“ (absolute value should be smaller than 1)
  - $a_p/(1-b_p)$ is the long-term difference between the total population and population p

- **Different approaches of calibrating the long-term difference**
  - Fitting of an AR(1) process to historical differences
  - Weighted/unweighted average of historical differences
  - …



ifa Institut für Finanz- und Aktuarwissenschaften

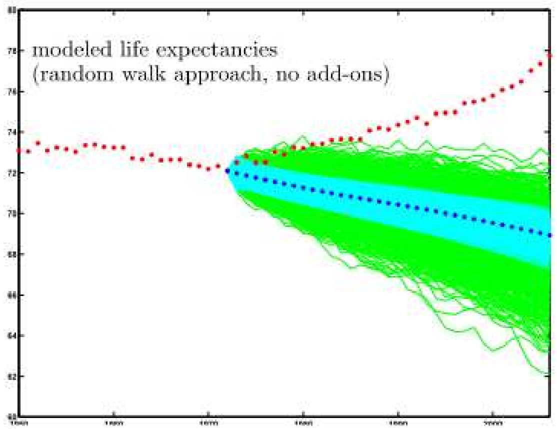
Model Simulation (ctd.)

- **Projection of $\kappa_t^{(2)}, \kappa_t^{(3)}$, and $\kappa_t^{(4)}$ for the individual populations**
  - No substantial trend obvious in the historical data
  - Forecast as correlated 3-dimensional random walk
  - No substantial correlation with $\kappa_t^{(1)}$
  - Volatility add-on $\overline{\sigma}^{(2)}$ for $\kappa_t^{(2)}$
    - The larger the changes in the slope of the mortality, the smaller the correlation between young and old ages
    - Thus the add-on affects diversification between mortality and longevity risk
  - Between populations, increments of $\kappa_t^{(1)}$ and $\kappa_t^{(2)}$ are correlated
    - This also implies slight correlation between the $\kappa_t^{(3)}$ and $\kappa_t^{(4)}$
    - Historical correlations should be checked carefully though and possibly adjusted

- **Projection of γ_{t-x}**
  - Cohort parameters should stay around zero
  - Forecast as Gaussian noise
  - Cohort parameters are rather irrelevant for short-term simulations as under Solvency II

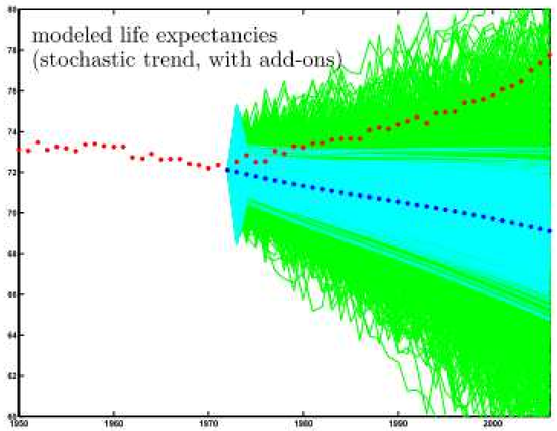
Weighting Parameters and Volatility Add-ons

- **Parameters $h, \overline{\sigma}^{(1)}$, and $\overline{\sigma}^{(2)}$ have a massive impact on simulation outcomes and thus SCRs**
- **Add-on $\overline{\sigma}^{(1)}$ determines possible severity of short-term events**
- **Weighting parameter h determines trend changes over one year and width of confidence bounds**



- **Calibration is difficult but should be conservative**

  → Fitting to most severe events/evolutions in the past
  - Example: Rapid increase in Dutch life expectancy gains starting from about 1970
  - Question: At which percentile should such extreme evolutions be observed?

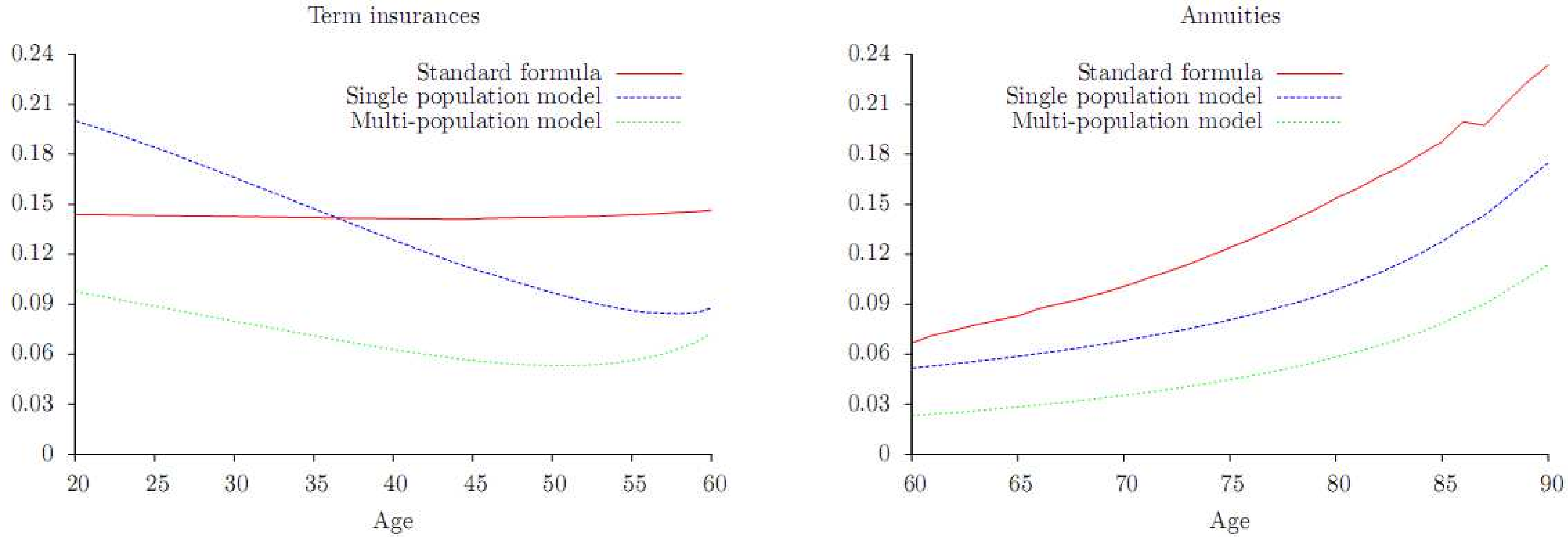


- **Calibration of $\overline{\sigma}^{(2)}$**
  - The larger the add-on the smaller the correlation between young and old ages thus limiting diversification

  → Choose $\overline{\sigma}^{(2)}$ such that correlation between ages at the boundaries of the age range is (close to) zero for most populations

Numerical comparisons

- **Standard formula approach**
  - Reduction/increase of all mortality rates by 20%/15%
  - Change in liabilities is about SCR for longevity/mortality risk
- **SCRs for term insurances (maturity at age 65) and whole life annuities:**



- **Multi-population model demands least capital**
  - Model is worthwhile even for only one population because of reduced trend uncertainty
- **Standard model seems to overstate the risk in general**

Mortality/Longevity Threat Scenarios

- **Available data contains only little information on tail scenarios which we are interested in**
- **Uncertainty remains whether model outcomes are severe enough**
  → Incorporate epidemiological/demographic expert opinion

- **Specification of mortality/longevity threat scenarios**
  - Shock to mortality projection
  - Likely effects of finding of a cure for a certain illness
  - Scenarios which the statistical model cannot generate, e.g., diverging mortality trends between countries/regions
  - …

- **Application of threat scenarios**
  - Check of model calibration: Adjustment of weighting parameter or volatility add-ons if the model outcomes should cover the threat scenarios but do not
  - Inclusion in SCR computations: SCR as a weighted average of model outcomes and threat scenarios

Summary

- **Specification, calibration, and application of a mortality model for solvency purposes**
  - Full age range
  - Variability in simulation outcomes due to 5 stochastic drivers
  - Clear interpretation of the model parameters
  - Non-trivial correlation structure to allow for simultaneous modeling of mortality and longevity risk
  - Stochastic trend modeling spares full re-calibration of the model in each scenario
  - Plausible outcomes in one-year view and run-off view
  - Conservative calibration
  - Inclusion of expert opinion
  - Multi-population model allowing for diversification effects and risk reduction

- **Model can be applied for other purposes as well (with slightly different calibration)**

- **A stochastic trend process with several appealing features**
  - Quantification of trend risk over any desired time horizon
  - Advantages compared to the commonly used random walk with drift (e.g. long-term risk vs. short-term noise)
  - The trend process could easily be incorporated in other mortality models as well, e.g. Lee-Carter or Cairns-Blake-Dowd

Contact Details

Matthias Boerger

Institute of Insurance, Ulm University & Institute for Finance and Actuarial Sciences (ifa), Ulm
Helmholtzstraße 22, 89081 Ulm, Germany
Phone: +49 731 50-31257, Fax: +49 731 50-31239
Email: m.boerger@ifa-ulm.de