# A New Methodology for Measuring Actual to Expected Performance


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- This talk is based on joint work with Nan Zhu, Illinois State University
- October 2012

**Despite its importance, the details of credible A to E studies is a topic that only an actuary could get excited about.**

(Russel Dorsett, CLU, Director, Veris Settlement Partners in Fasano e-Newsletter September 2012)

**So, let's get excited!**

(The Pointer Sisters, 1982)



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# Agenda

**Previous Approaches to measuring Actual to Expected Performance**

Counting Based Approaches

Actual to Expected Ratios

**A new Approach: Difference in Curtate Life Expectancy (DCLE)**

**Testing for Dispersion and Shape of the Mortality Curve**

**Asymmetric Information: Who knows the insured best? The insured!**

**Summary and Outlook**



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# Previous Approaches to measuring Actual to Expected Performance

## Counting based approaches

- In early years, a counting based approach was used to assess the quality of LE-providers.
  - Count how many people have died before their LE and how many have died after their LE.
- This method yields meaningful results only after all individuals have either died or outlived their LE.
- Before that time, it makes the provider look more conservative, because at the beginning, only early deaths are possible.
- So even if the provider is too aggressive, you start counting a few early deaths and conclude that the provider is conservative.
- It will be years until the late deaths occur and you realize that the provider is too aggressive.

- **Counting based approaches are statistically biased: LE-providers look more conservative than they are.**
- **This effect is the larger, the more recently the analyzed portfolio has been underwritten.**



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# Previous Approaches to measuring Actual to Expected Performance

## Actual to Expected Ratios

- As a solution, Actual to Expected Ratios were introduced.
- Compare expected number of deaths with actual number of deaths.
  - Expected number of deaths is based on mortality probabilities.
  - E.g. if 100 individuals with an LE of 5 years have been underwritten exactly one year ago and the probability of dying within the first year is 7% (for each individual), then the expected number of deaths is 7 (even if no one has reached their LE).
- This results in an A/E-ratio: actual number of deaths divided by expected number of deaths
- An A/E-ratio of 100% means that between time of underwriting and today, exactly as many individuals have died as was predicted by the LE-provider.
- An A/E-ratio close to 100% is therefore seen as an indication for good underwriting.
  - However, this will not always be true!

# Previous Approaches to measuring Actual to Expected Performance

## Actual to Expected Ratios

- Currently, market participants observe a significant difference between LEs given by different LE-providers while at the same time, all of them claim to have an A/E-ratio close to 100%.
- **The A/E-puzzle: How can different LE-providers give significantly different LEs and still all be right?**
  - The primary reason has to do with the comparison of so-called restated LEs in some analyses with actual LEs in others.
  - However, we believe that the methodology of calculating A/E-ratios can add to this apparent disconnect.
- **The very methodology of calculating an A/E-ratio contains a statistical bias that "pulls" results towards 100% over time. This effect is the larger, the longer ago the analyzed portfolio has been underwritten.**
  - This will be explained in the following slides.

- **Over time, A/E-ratios are artificially 'pulled' towards a level of 100%. Therefore, under certain circumstances, A/E-ratios may be only of limited explanatory value!**

# Previous Approaches to measuring Actual to Expected Performance

## Actual to Expected Ratios

- **In what follows, we use a simple hypothetical example to explain the statistical bias of current A/E analyses.**
  - Two hypothetical (!) LE-providers
  - We assumed that the individuals die according to some **given LE**. Actual mortality is **red** in all charts.
  - **Provider 1** is too aggressive by 2 years (Provider 1 is **blue** in all charts).
  - **Provider 2** is too conservative by 2 years (Provider 1 is **green** in all charts).
- All slides that are based on this simple example are marked with the following label:

Hypothetical Example

- We will also present results based on actual Fasano Associates data. Those are marked with the following label:

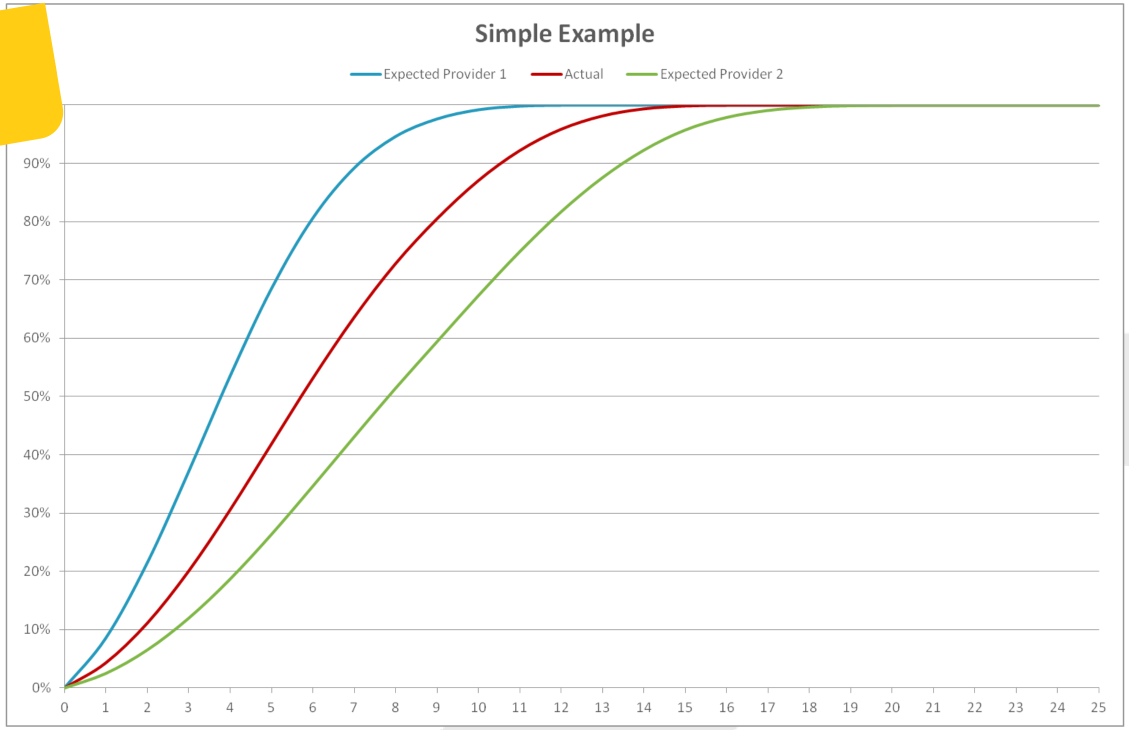
Fasano Data

# Previous Approaches to measuring Actual to Expected Performance

## Actual to Expected Ratios

The chart shows the evolution of expected mortality for the two hypothetical providers (blue and green) as well as actual mortality (red).

Hypothetical Example

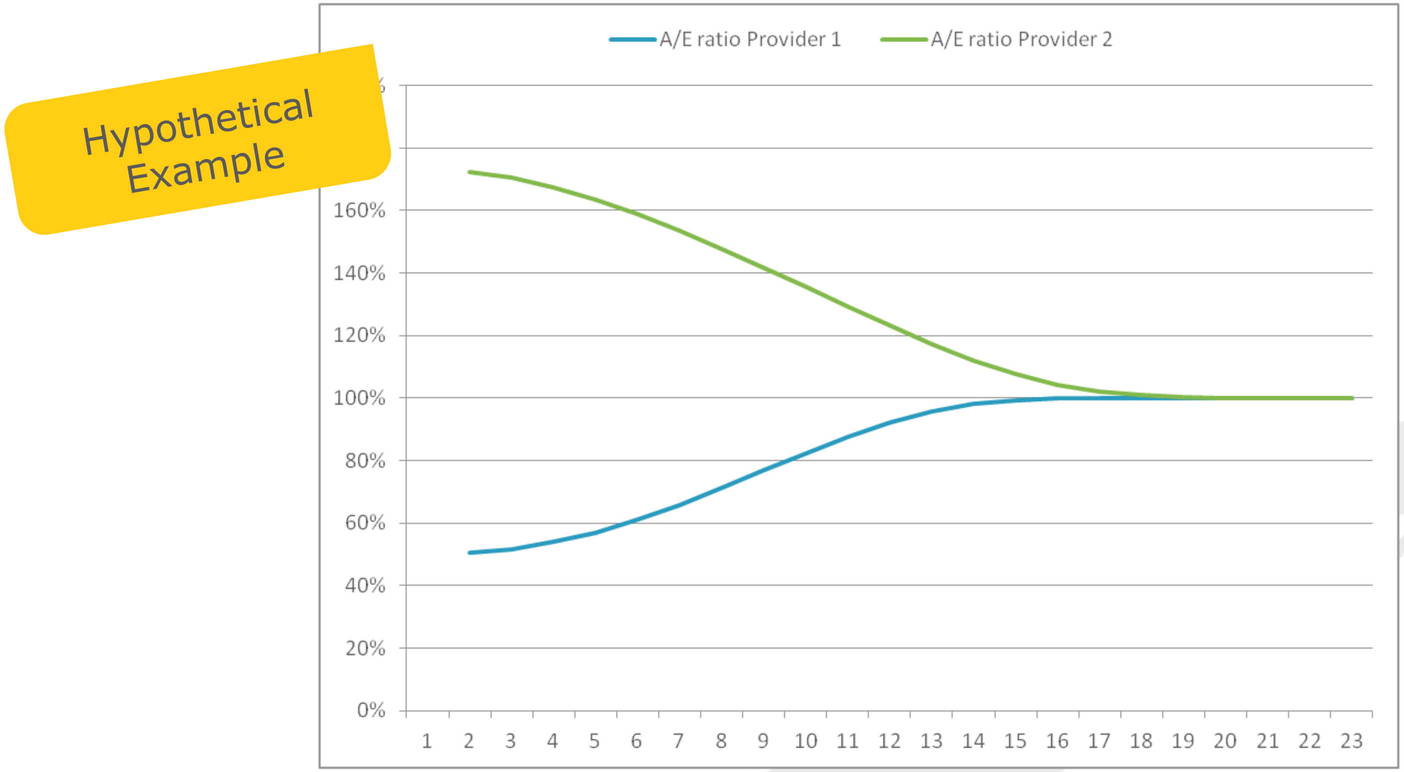



- **Provider 1 is too aggressive**: Expected mortality is higher than actual.
- **Provider 2 is too conservative**: Expected mortality is lower than actual.

# Previous Approaches to measuring Actual to Expected Performance

## Actual to Expected Ratios

The chart shows the evolution of A/E ratios for both providers in the same example.

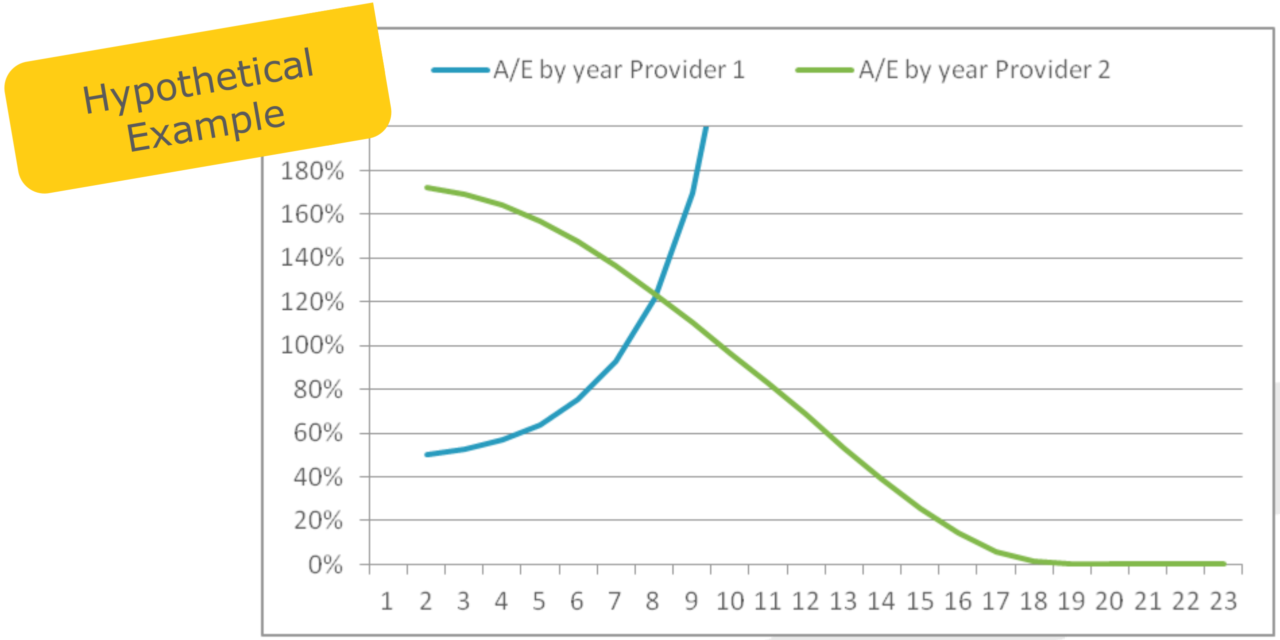


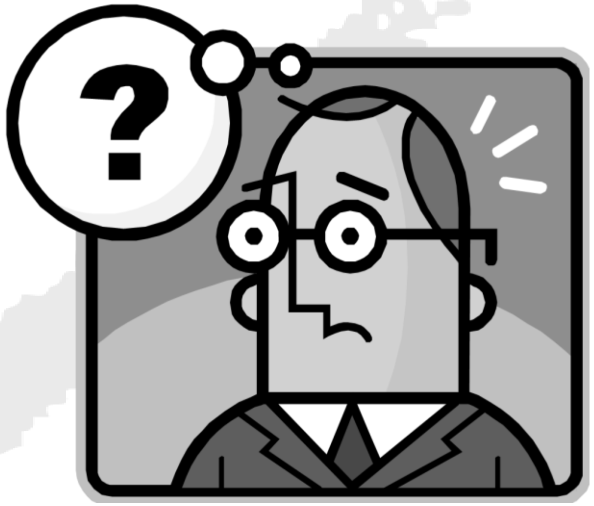

- As time goes by, both A/E ratios approach 100%.
- **Looking at A/E ratios conceals differences between LE-providers in later years.**

# Previous Approaches to measuring Actual to Expected Performance

## Actual to Expected Ratios

- This problem cannot be easily solved with current A/E methodology unless you are willing to look at several numbers, e.g. A/E-ratios by year since underwriting, etc.
  - Even then, you need to be careful when interpreting the data.



- **If you are looking for <u>one</u> meaningful number that measures the performance of an LE-Provider, A/E-ratios are not suitable if a significant portion of the considered portfolio has been underwritten a rather long time ago (e.g., longer than the average portfolio LE)!**

# Agenda

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Counting Based Approaches

Actual to Expected Ratios

**A new Approach: Difference in Curtate Life Expectancy (DCLE)**

**Testing for Dispersion and Shape of the Mortality Curve**

**Asymmetric Information: Who knows the insured best? The insured!**

**Summary and Outlook**

# A new Approach: Difference in Curtate Life Expectancy (DCLE)

## What are we trying to measure?

- **Before designing a statistical test, we need to decide what we are trying to measure.**

| | |
|---|---|
| Level | • Are the given life expectancy estimates right on average?<br>• If half of them are too long and half of them are too short, the effects would 'cancel out'<br>• That's what current A/E analyses try to measure! |
| Level + Dispersion | • Are all given life expectancy estimates right?<br>• This is significantly more than what current A/E analyses measure.<br>• It might be asking too much! |
| Level + Dispersion + Shape | • Are all given mortality probabilities (and not only the resulting life expectancy estimates) correct?<br>• This is clearly asking too much. |



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# A new Approach: Difference in Curtate Life Expectancy (DCLE)

## What are we trying to measure?

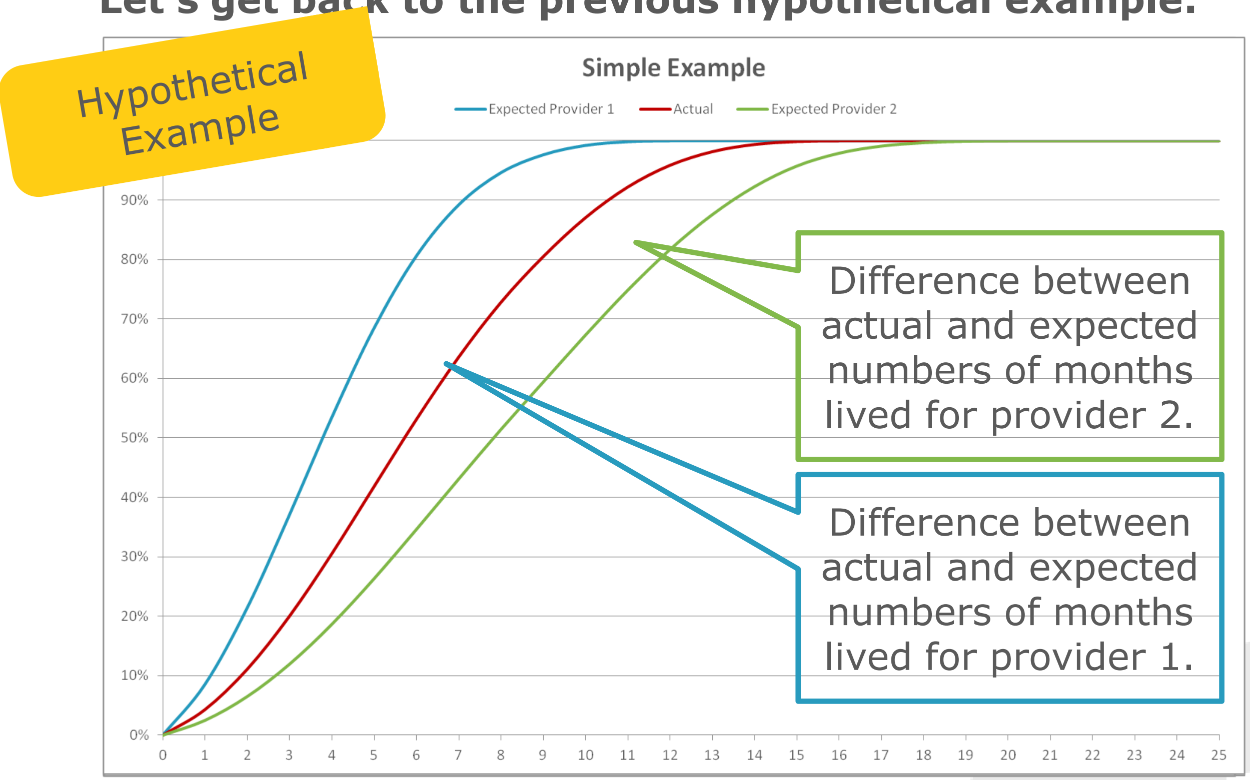
- **We have designed statistically meaningful tests for all three of them.**

| | |
|---|---|
| Level | • Relevant for the industry<br>• Natural substitute for current A/E-analyses<br>• Comes without bias; results easy to interpret<br>• Details follow |
| Level + Dispersion | • Probably less relevant for the industry, although consequences may be relevant<br>• Harder to explain, harder to perform, difficult to interpret<br>• Some details follow – more in an academic paper. |
| Level + Dispersion + Shape | • Probably no relevance for the industry. Due to size of portfolio, even deviations that would be considered small by practitioners would be statistically significant.<br>• But it may lead to improved understanding of deviations.<br>• Some details follow – more in an academic paper. |

# A new Approach: Difference in Curtate Life Expectancy (DCLE)

## The basic idea

**Let's get back to the previous hypothetical example:**




- **Methodology is based on Difference in Curtate Life Expectancy (DCLE)**
- At any point in time, the aggressive provider is identified as being too aggressive and the conservative provider is identified as being too conservative!

- In late years number of actual deaths approaches number of expected deaths, i.e. A/E converges to 1
  - Even if all deaths occurred later than expected (provider 1) or earlier than expected (provider 2)
- **Solution**: Compare actual number of months lived until today to expected number of months lived until today (curtate life expectancy)
  - **Actual**: In every month count the number of people alive and sum them up
  - **Expected**: In every month count the expected number of people alive and sum them up
  - The difference is the **area between the actual and expected curve.**



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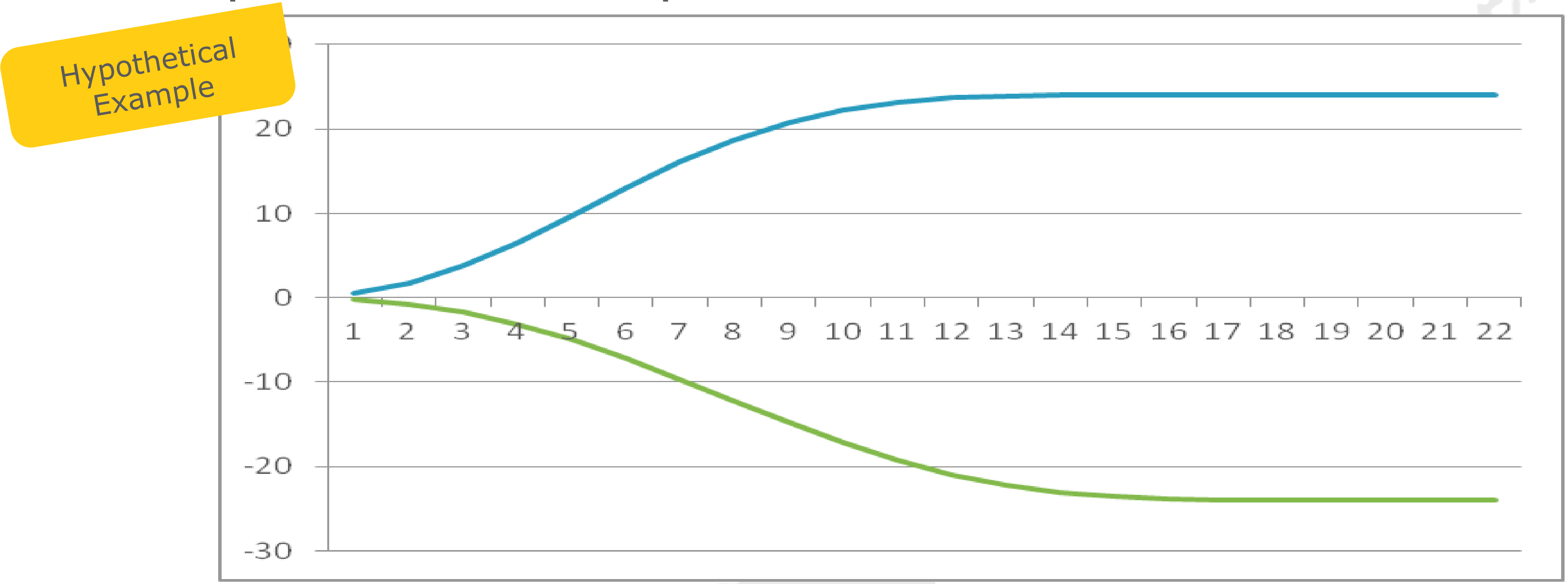
# A new Approach: Difference in Curtate Life Expectancy (DCLE)

## The basic idea

- **Development of DCLE in the example:**




- **DCLE measures the average cumulated deviation per life. It indicates early that the conservative provider is too conservative and the aggressive provider is too aggressive. Once enough information is available, it correctly identifies that each is off by exactly 24 months!**



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# A new Approach: Difference in Curtate Life Expectancy (DCLE)

## Results for real data

Fasano Data

- **Results based on actual Fasano Associates data (no IBNR):**
  - We have analyzed a portfolio of more than 50,000 lives underwritten by Fasano Associates.
  - Whole portfolio minus duplicates (note: different method of removing duplicates might yield different results)
  - In the following table, we give the following statistic:
    - expected number of months lived until today
    - minus actual number of months lived until today
    - divided by number of underwritten lives in the considered portfolio
  - If the resulting number is 0, this means that on average, each person has lived exactly as many months as expected between time of underwriting and today.
  - Significantly positive numbers mean that the LE-provider is too aggressive.
    - E.g., if the resulting number is 10, this means that between time of underwriting and today, on average, each person has lived 10 months longer than expected.
  - Analogously, significantly negative numbers mean that the LE-provider is too conservative.

# A new Approach: Difference in Curtate Life Expectancy (DCLE)

## Results for real data

Fasano Data

- **Results for Fasano Associates actual data:**

| All | LE < 2years |
|---|---|
| 0.76 months | -0.09 months |

| Multiplier below 1.2 | Between 1.2 and 2.5 | Above 2.5 |
|---|---|---|
| 0.57 months | 1.37 months | -0.83 months |

| Underwritten before 12/30/2004 | Underwritten until 03/21/2007 | Underwritten until 04/30/2008 | Underwritten after 04/30/2008 |
|---|---|---|---|
| 4.86 months | 0.69 months | -0.04 months | -0.27 months |

- **Average DCLE in the first period is just under 5 months; in all other periods, it is less than 1 month.**

- **Furthermore, we can determine the limiting distribution under the hypothesis DCLE=0 and develop statistical tests for this hypothesis.**
  - However, most of the tests reject the hypothesis because of the large sample size, although the difference is "practically" negligible.



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# Summary of the main results so far

- **Counting based approaches are biased:** LE-providers look more conservative than they are.
- **A/E analyses as currently used will also become statistically biased:** Over time, A/E-ratios are artificially 'pulled' towards a level of 100%.
  - **Differences between LE providers are concealed.** The differences look the smaller the more information becomes available! Same holds for the difference between projected LE and actual mortality.
  - This problem **cannot be completely solved with current A/E methodology** unless you are willing to look at several numbers that are not very easy to interpret.
- **We propose a new approach based on the Difference of Curtate Life Expectancy (DCLE)**
  - Average difference between actual number of months lived until today and expected number of months lived until today.
  - **No structural statistical bias.** At any point in time, an aggressive provider is identified as being too aggressive and a conservative provider is identified as being too conservative!
  - As more and more information about the difference between projected LE and actual mortality becomes available, the difference is evaluated more and more correctly.
  - Analysis of real data shows rather low difference between projected LE and actual mortality.
- **This methodology could serve as a substitute for current A/E analyses.**

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**Asymmetric Information: Who knows the insured best? The insured!**

**Summary and Outlook**

# Testing for Dispersion and Shape of the Mortality Curve

- **If you want to know more about the quality of the underwriting than 'only' whether the LEs are right on average, the mathematics get a little complex**
- **(1) The "overall quality test": Are level and shape and dispersion correct**
  - Statistical test whether the **complete distribution** of projected mortality (i.e. all mortality probabilities) fits the observed pattern of deaths.
  - In other words: Can the deviation between actual and expected mortality patterns be considered random under the hypothesis that the lifetime distributions implied by the supplied tables and multipliers are correct?
  - Probably no relevance for the industry. Due to size of portfolio, even deviations that would be considered small by practitioners would be statistically significant.
  - However, the methodology is interesting: The test relies on a **time-change transformation** of all the data that "turns the clock" slower for individuals who are expected to decease soon and faster for individuals expected to live a long time. We then can, for example, calculate ratios between observed and expected deaths in each "normalized" time interval and develop a test for statistical significance.

# Testing for Dispersion and Shape of the Mortality Curve

- **(2) Testing for shape of the mortality curve.**
  - We derive non-parametric estimates for the time dependent “residual” multiplier in different subgroups of the sample.
  - If the shape were perfectly all right, this residual multiplier should be close to a straight (diagonal) line.
  - Selected results from applying the test to real data:
    - Underwriter has been improving over time.
    - Shape is relatively accurate for lower multipliers (i.e. the base table is about right).
    - Some deviation for higher multipliers (i.e. there is potential to further improve the “shape” of the time dependent frailty factor)
      - But bear in mind that LEs are right on average!
    - The very result of this test might be used to create a time dependent “correction tem” for the current mortality multiplier.
  - Methodology: We use a non-parametric estimator, the so-called Nelson-Aalen estimator, to derive a non-parametric estimate of the multiplicative factor on the estimated force of mortality.

Fasano Data



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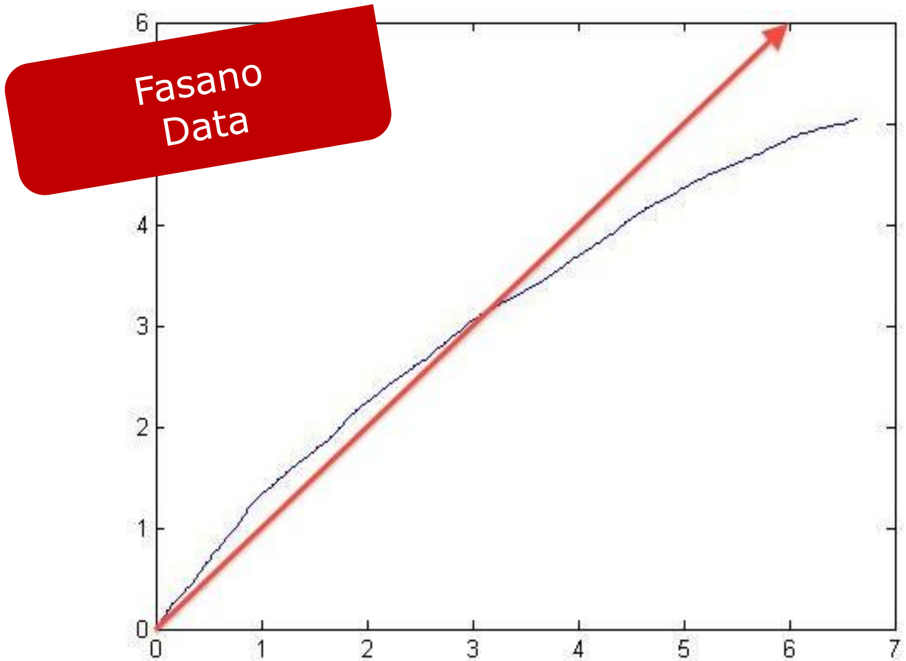
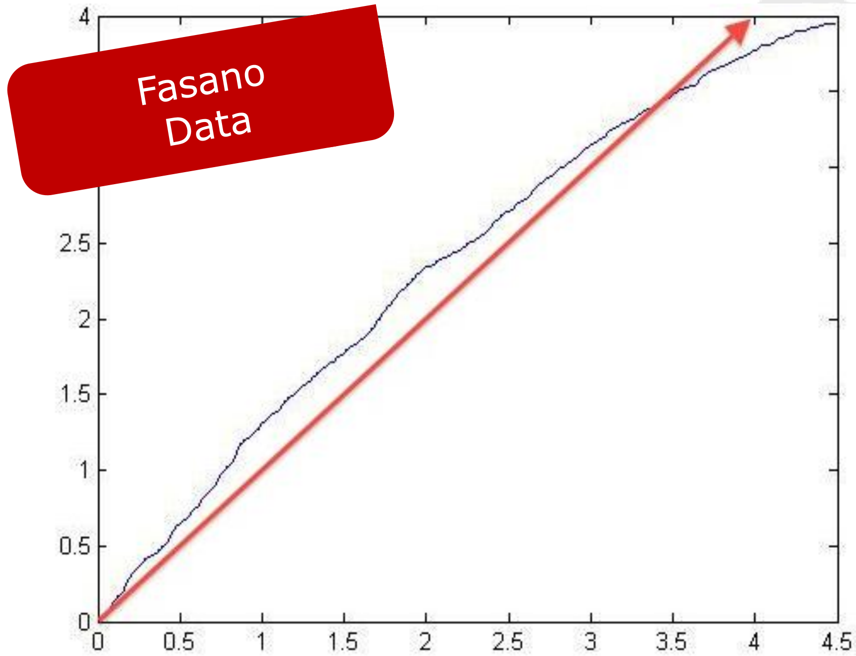
# Testing for Dispersion and Shape of the Mortality Curve

- **(2) Testing for shape of the mortality curve. Example of residual multiplier**
  - Left: cases underwritten rather long time ago – right: cases underwritten rather recently




- This is still work in progress but could yield a significantly deeper understanding how accurate LE projections are in which segment.
- It could also be a starting point for a development of time dependent mortality multipliers.

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**Summary and Outlook**

# Asymmetric Information: Who knows the insured best? The insured!

- **The data we analyzed indicate that asymmetric information might play a role in the life settlement industry.**
- **"Asymmetric information" means that one of the parties entering a risky contractual agreement has better information with regards to the corresponding risk.**
- **In our case: Policyholder has information on his individual life expectancy beyond what can be determined (e.g. by an LE-provider) on the basis of observable factors.**
- **Asymmetric information is a very important topic in (insurance) economics.**
  - The last two Clark Medals awarded by the American Economic Association went to researchers who do work on asymmetric information in (primary) insurance markets (J. Levin (2011), A. Finkelstein (2012)).
  - But google search on "asymmetric information in secondary insurance markets" produces no direct hits.
  - Of course this is important for specifics of the LS market, but understanding how good individuals are at assessing their own LE has even broader repercussions.

# Asymmetric Information: What does it mean and why should you care?

- **Suppose there are three groups of individuals at equal proportions in a portfolio. They have the same LEs based on observables but different "individual" LEs**

| | Group 1 | Group 2 | Group 3 | Average in Portf. |
|---|---|---|---|---|
| **LEs based on observables** | 6 years | 6 years | 6 years | 6 years |
| **Individual LEs** | 4 years | 6 years | 8 years | 6 years |

Hypothetical Example

- **This is what we find in the data – i.e. the LEs are correct on average**

  ➔ If an acquired portfolio consists of three groups at equal proportions, there are no issues.

- **But what if the individual LEs affect the decision to settle? Then the individuals with a shorter LE may choose not to settle whereas the individuals with a longer LE will settle.**

| | Group 1 | Group 2 | Group 3 | Average in Portf. |
|---|---|---|---|---|
| **LEs based on observables** | 6 years | 6 years | 6 years | 6 years |
| **Individual LEs** | Don't settle | 6 years | 8 years | 7 years |

Hypothetical Example

- **Economics literature predicts positive relationship between coverage and risk in the presence of asymmetric information!**



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# Asymmetric Information: Who knows the insured best? The insured!

- **Indication of asymmetric information in the data:**
  - Out of the 50,000+ underwritten lives, we know a (small) subset that closed.

Fasano Data

| DCLE of Subset that we know closed | DCLE of the rest |
|---|---|
| 3.54 months | 0.76 months |

- **Consequences for the industry:**
  - You should expect A/E results (in whatever way they are performed) to be lower in a portfolio of policies that have actually closed than in an LE-provider's portfolio (out of which some cases have not closed).
  - Once the effect is quantified, this can be considered in pricing (cf. Zhu & Bauer, 2012).
  - Idea: Adjust the price you charge for the "decision" to settle.
- **Broader consequences:**
  - It has been established, that individuals have a poor idea regarding their absolute life expectancy (cf. Hurd (2009), Payne et al. (2012)).
  - However, individuals might do pretty well when judging their relative life expectancy given certain impairments – what are the economic implications?

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**Summary and Outlook**

# Summary and Outlook

- **Current approaches for evaluating the accuracy of LE projections are problematic.**
- **We propose a new approach based on the Difference of Curtate Life Expectancy (DCLE).**
  - Average difference between actual number of months lived until today and expected number of months lived until today.
  - **No structural statistical bias.** As more and more information about the difference between projected LE and actual mortality becomes available, the difference is evaluated more and more correctly.
  - Analysis of Fasano data shows rather low difference between projected LE and actual mortality.
- **We also develop tests that evaluate LEs with regards to deviations in the shape of the life table and the dispersion.**
  - **Shape:** Interesting for academic purposes but immediate relevance to LE practice limited. Could be a starting point for further improvement of underwriting methodology.
  - **Dispersion:** We find (preliminary) evidence for the existence of asymmetric information in LEs, i.e. it appears that **policyholders have private information** regarding their expected lifetimes and that they **make use of it** in their settlement decisions.
  - This would have important consequences for the LS market (e.g. for pricing) but also for our understanding of individuals' lifetime assessments more generally. Necessary to examine robustness of this result.

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