It Takes Two: Why Mortality Trend Modeling is more than Modeling one Mortality Trend

Matthias Börger

Jochen Russ

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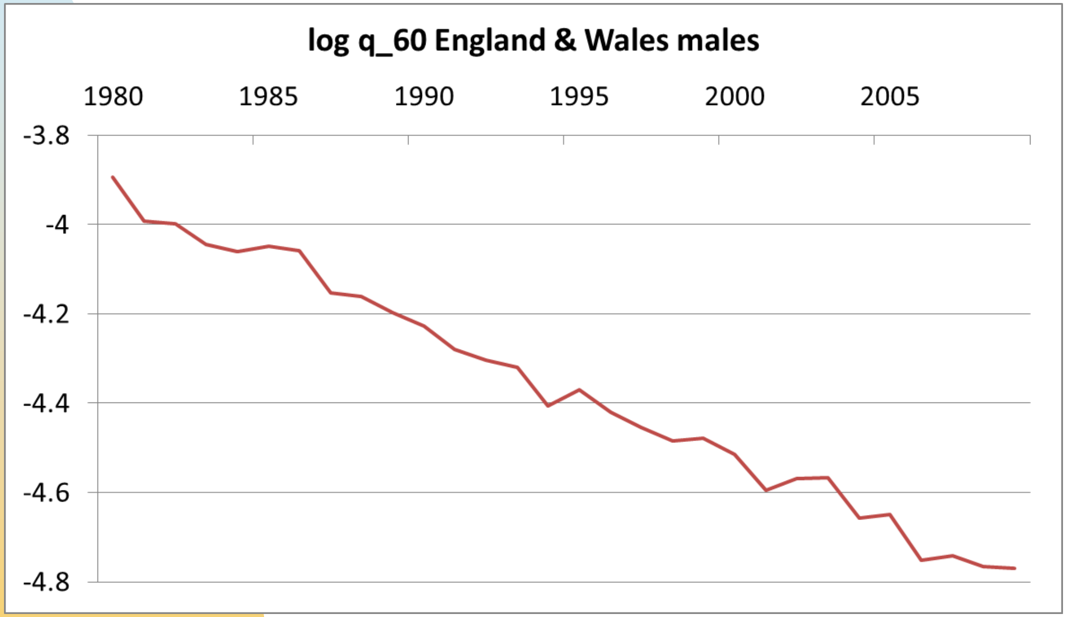
Helmholtzstraße 22
D-89081 Ulm
phone +49 (0) 731/50-31230
fax +49 (0) 731/50-31239
email ifa@ifa-ulm.de

ifa Institut für Finanz- und Aktuarwissenschaften

Introduction

- Around the world, life expectancy increases and mortality rates decrease
- The decrease in log mortality rates often appears linear:



- Log mortality is usually projected as random walk with drift
- Drift coincides with historically observed slope

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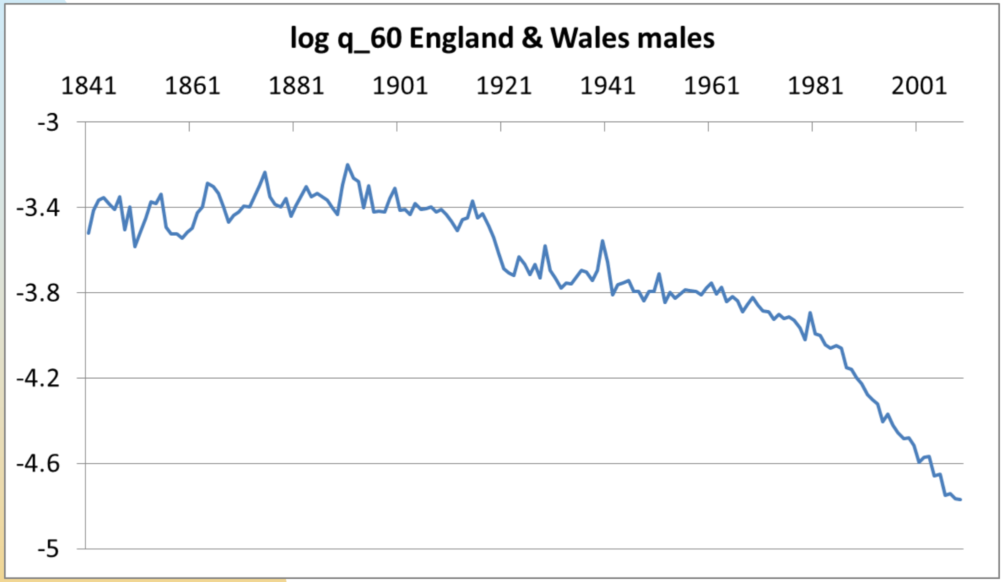
Introduction

- **What if we look further into the past?**



- **Trend in log mortality appears only piecewise linear; Slope of the mortality trend changes**
- **Random walk with drift does not account for trend changes**
  - Finding is not new, see e.g. Sweeting (2011) or Li et al. (2011)

→ **Additional uncertainty and the need for modeling mortality trend changes**

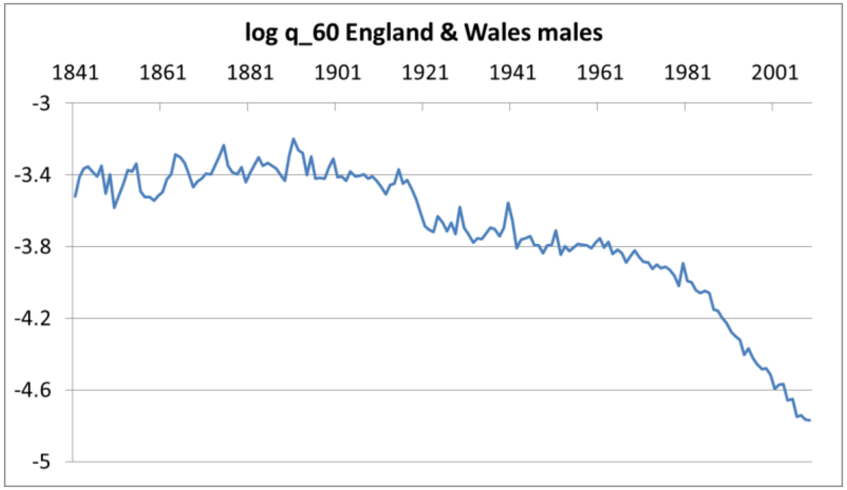
→ **However, we propose a model with two different mortality trends.**

Agenda

- **Why two mortality trends?**
  - Actual mortality trend (AMT)
  - Expected mortality trend (EMT)
  - Some examples for applications
- **A combined model for both trends**
  - AMT component
  - Stochastic start trend
  - Comparison with other AMT approaches
  - EMT component
- **Conclusion**

Actual Mortality Trend

- **The first trend is the actual mortality trend (AMT)**
  - The AMT describes realized future mortality and is the core of most existing mortality models
  - Goal: plausible extrapolation of historically observed mortality
  - Frequency and magnitude of changes in the AMT plus random fluctuations around the AMT need to be modeled
- **Today's AMT is not (fully) observable!**
- **We know historical mortality**
  - Random fluctuations can be filtered out
  - Historical trend changes and slopes of piecewise linear trends can be estimated
- **But we do not know the current slope**
  - There might be a trend change this year
  - There might have been a trend change over the last years which is misinterpreted as random fluctuations or vice versa



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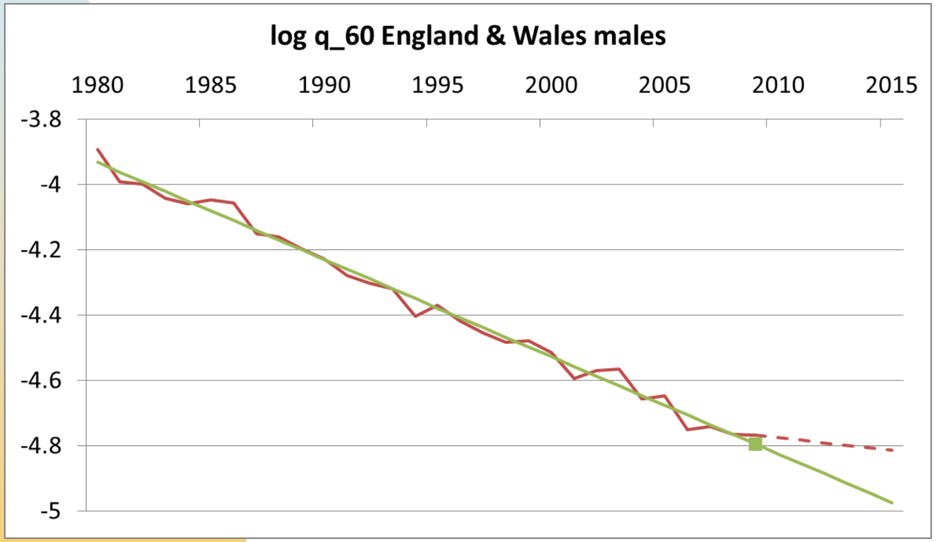
Estimated Mortality Trend

- **The second trend is the estimated mortality trend (EMT)**
- **The EMT is an observer's estimate of the AMT at some given point in time**
  - Estimate of the unknown AMT based on available data



- **The EMT is based on the most recent historical mortality evolution and updated as soon as new data becomes available**
- **The EMT is the basis for mortality projections and (generational) mortality tables, e.g., for reserving**

Why Two Mortality Trends?

- **The mortality trend to consider depends on the application in view, examples:**
  - Capital for a portfolio run-off ➔ AMT over the run-off
  - Reserves for the portfolio after 10 years
    ➔ EMT after 10 years and AMT over the 10 years
    - AMT for the next 10 years is required to be able to compute EMT in 10 years time
  - Payout of a mortality derivative which reduces GAO risk
    ➔ EMT at maturity and AMT up to maturity
  - Analysis of hedge effectiveness of the derivative
    ➔ EMT at maturity of the derivative, AMT also beyond
  - Solvency Capital Requirement: combined 99.5th percentile of actual payments over the next year and changes in the liabilities
    ➔ AMT for actual payments and EMT for change in liabilities

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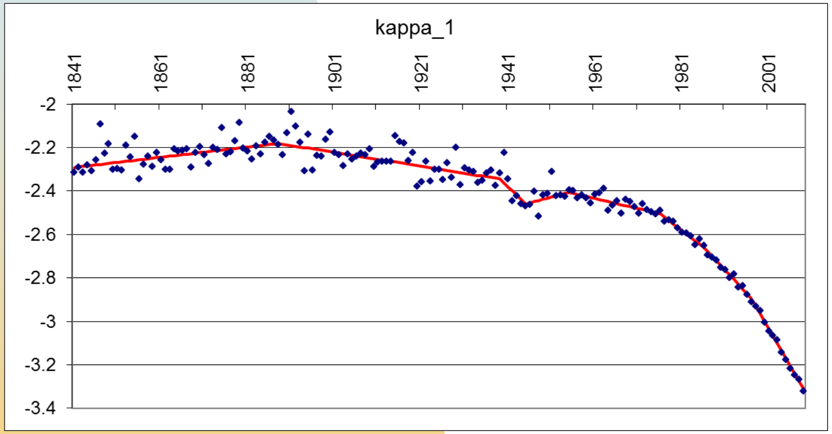
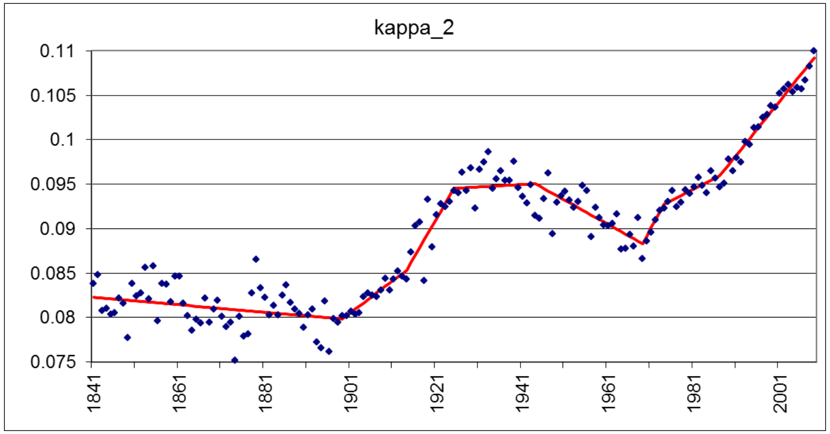
Combined AMT/EMT Model – AMT Component

- **For the AMT model component, we use the model of Sweeting (2011):**

$$logit(q_{x,t}) := \log\left(\frac{q_{x,t}}{1-q_{x,t}}\right) = \kappa_1(t) + \kappa_2(t)\cdot(x-\bar{x}),$$

- **Model parameters for English and Welsh males aged 60-89:**





- **7 trend changes for both kappa processes ➔ trend change probability p = 7/169**
- **Trend change intensity (different from Sweeting (2011)):**
  - sign of trend change: Bernoulli distributed with values 1 and -1 and probability 1/2
  - absolute magnitude of trend change, normally distributed with parameters according to sample mean and sample variance

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Combined AMT/EMT Model – Stochastic Start Trend

- **The AMT at the start of a simulation is not observable**
- **What if another trend change occurred after the last significant one?**

  ➔ Additional uncertainty
- **This is more than parameter uncertainty in estimating a linear trend**
- **In our model, this uncertainty accounted for by stochastic start trend**
  - In the paper, we explain in detail how a stochastic start trend can be implemented

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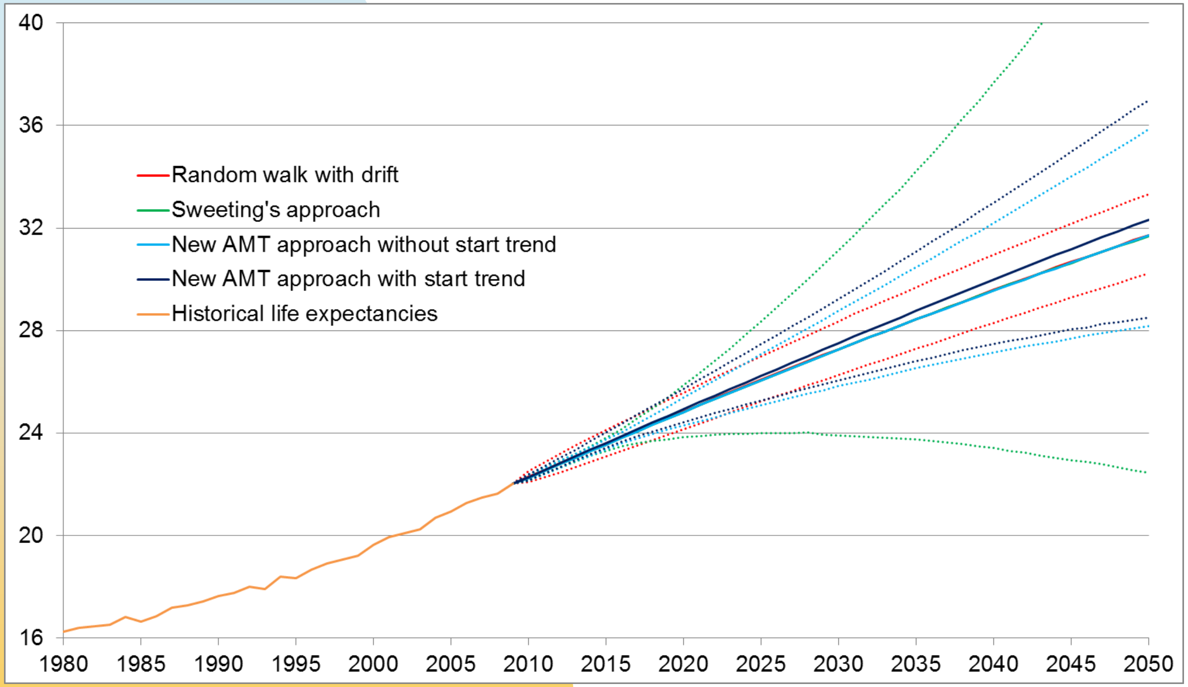
Combined AMT/EMT Model – Comparison of AMT Approaches

- **Remaining period life expectancy for a 60-year old (with 10th and 90th percentiles)**



- All models: Similar, plausible medians
- Random walk with drift: first widest and then narrowest confidence bands, 3.1 years in 2050 seem unrealistically small
- Sweeting's approach: implausibly wide confidence bands (23.3 years in 2050). In our opinion due to his modeling of the trend change intensity (not consistent with parameter estimation)
- Our model: confidence bands look plausible; Stochastic start trend widens range in 2050 from 7.7 years to 8.5 years

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Combined AMT/EMT Model – The EMT Component

- **The EMT is the best estimate of the AMT at any point in time**
- **In principle, every estimation procedure is feasible for the EMT**
- **„Obvious“ choice for the EMT at time t in our setting: Mean of the distribution of the stochastic start trend at time t**
  - Not feasible within a simulation. → Simpler methods required for the EMT in simulations.
- **We propose to compute the EMT by weighted regression**
  - Extrapolation of linear trend in most recent data points
  - Crucial question: How many data points?
    - Too many data points: Delayed reaction to change in the AMT
    - Too little data points: EMT is exposed to random noise in the AMT
  - Weights decrease exponentially going backwards in time
  - “Optimal” weighting parameters can be derived by minimizing the MSE between AMT and EMT

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Conclusion

- **Two trends need to be distinguished and modeled:**
  - The actual mortality trend (AMT) which is unobservable
  - The estimated mortality trend (EMT) which is an observer‘s estimate of the AMT
- **Which trend(s) to consider depends on the question in view**
- **The AMT can be modeled as a piecewise linear function with random changes in the slope**
  - The commonly used random walk with drift underestimates long-term longevity risk systematically
  - The distribution for the trend change intensity needs to be consistent with the derivation of the trend change probability
- **Since the AMT at the start of a simulation is unknown a stochastic start trend should be considered**
- **The choice of the EMT approach is important in practice**
  - A weighted regression approach seems reasonable. We show how optimal weights can be derived

References

- Sweeting, P., 2011. A Trend-Change Extension of the Cairns-Blake-Dowd Model. *Annals of Actuarial Science*, Volume 5, pp. 143-162.

- Li, J., Chan, W. S. & Cheung, S. H., 2011. Structural Changes in the Lee-Carter Mortality Indexes: Detection and Implications. *North American Actuarial Journal*, Volume 15, pp. 13-31.

Contact Details

ifa

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phone +49 (0) 731/50-31230

fax +49 (0) 731/50-31239

email ifa@ifa-ulm.de

www.ifa-ulm.de

Matthias Börger

m.boerger@ifa-ulm.de

Jochen Ruß

j.russ@ifa-ulm.de

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