It Takes Two:
Why Mortality Trend Modeling is more than Modeling one Mortality Trend

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Introduction

- Around the world, life expectancy increases and mortality rates decrease
- The decrease in log mortality rates often appears linear:

![Graph showing log mortality rates from 1980 to 2005 for England & Wales males]

- Log mortality is usually projected as random walk with drift
- Drift coincides with historically observed slope
**Introduction**

- **What if we look further into the past?**

- Trend in log mortality appears only piecewise linear; Slope of the mortality trend changes
- Random walk with drift does not account for trend changes
  - Finding is not new, see e.g. Sweeting (2011) or Li et al. (2011)

  ➔ Additional uncertainty and the need for modeling mortality trend changes
  ➔ However, we propose a model with two different mortality trends.
Agenda

- Why two mortality trends?
  - Actual mortality trend (AMT)
  - Expected mortality trend (EMT)
  - Some examples for applications

- A combined model for both trends
  - AMT component
  - Stochastic start trend
  - Comparison with other AMT approaches
  - EMT component

- Conclusion
Actual Mortality Trend

- The first trend is the actual mortality trend (AMT)
  - The AMT describes realized future mortality and is the core of most existing mortality models
  - Goal: plausible extrapolation of historically observed mortality
  - Frequency and magnitude of changes in the AMT plus random fluctuations around the AMT need to be modeled
- Today’s AMT is not (fully) observable!
- We know historical mortality
  - Random fluctuations can be filtered out
  - Historical trend changes and slopes of piecewise linear trends can be estimated
- But we do not know the current slope
  - There might be a trend change this year
  - There might have been a trend change over the last years which is misinterpreted as random fluctuations or vice versa
Estimated Mortality Trend

- The second trend is the estimated mortality trend (EMT)
- The EMT is an observer’s estimate of the AMT at some given point in time
  - Estimate of the unknown AMT based on available data

- The EMT is based on the most recent historical mortality evolution and updated as soon as new data becomes available
- The EMT is the basis for mortality projections and (generational) mortality tables, e.g., for reserving
Why Two Mortality Trends?

The mortality trend to consider depends on the application in view, examples:

- Capital for a portfolio run-off \( \rightarrow \) AMT over the run-off
- Reserves for the portfolio after 10 years
  \( \rightarrow \) EMT after 10 years and AMT over the 10 years
  AMT for the next 10 years is required to be able to compute EMT in 10 years time
- Payout of a mortality derivative which reduces GAO risk
  \( \rightarrow \) EMT at maturity and AMT up to maturity
- Analysis of hedge effectiveness of the derivative
  \( \rightarrow \) EMT at maturity of the derivative, AMT also beyond
- Solvency Capital Requirement: combined 99.5th percentile of actual payments over the next year and changes in the liabilities
  \( \rightarrow \) AMT for actual payments and EMT for change in liabilities
Combined AMT/EMT Model – AMT Component

- For the AMT model component, we use the model of Sweeting (2011):

\[
\text{logit}(q_{x,t}) := \log \left( \frac{q_{x,t}}{1 - q_{x,t}} \right) = \kappa_1(t) + \kappa_2(t) \cdot (x - \bar{x}),
\]

- Model parameters for English and Welsh males aged 60-89:

- 7 trend changes for both kappa processes $\Rightarrow$ trend change probability $p = 7/169$

- Trend change intensity (different from Sweeting (2011)):
  - sign of trend change: Bernoulli distributed with values 1 and -1 and probability 1/2
  - absolute magnitude of trend change, normally distributed with parameters according to sample mean and sample variance
Combined AMT/EMT Model – Stochastic Start Trend

- The AMT at the start of a simulation is not observable
- What if another trend change occurred after the last significant one?
  - Additional uncertainty
- This is more than parameter uncertainty in estimating a linear trend
- In our model, this uncertainty accounted for by stochastic start trend
  - In the paper, we explain in detail how a stochastic start trend can be implemented
**Combined AMT/EMT Model – Comparison of AMT Approaches**

- **Remaining period life expectancy for a 60-year old (with 10th and 90th percentiles)**

  All models: Similar, plausible medians

  Random walk with drift: first widest and then narrowest confidence bands, 3.1 years in 2050 seem unrealistically small

  Sweeting's approach: implausibly wide confidence bands (23.3 years in 2050). In our opinion due to his modeling of the trend change intensity (not consistent with parameter estimation)

  Our model: confidence bands look plausible; Stochastic start trend widens range in 2050 from 7.7 years to 8.5 years
The EMT is the best estimate of the AMT at any point in time

In principle, every estimation procedure is feasible for the EMT

„Obvious“ choice for the EMT at time $t$ in our setting: Mean of the distribution of the stochastic start trend at time $t$

- Not feasible within a simulation. → Simpler methods required for the EMT in simulations.

We propose to compute the EMT by weighted regression

- Extrapolation of linear trend in most recent data points
- Crucial question: How many data points?
  - Too many data points: Delayed reaction to change in the AMT
  - Too little data points: EMT is exposed to random noise in the AMT
- Weights decrease exponentially going backwards in time
- “Optimal” weighting parameters can be derived by minimizing the MSE between AMT and EMT
Conclusion

Two trends need to be distinguished and modeled:
- The actual mortality trend (AMT) which is unobservable
- The estimated mortality trend (EMT) which is an observer’s estimate of the AMT

Which trend(s) to consider depends on the question in view

The AMT can be modeled as a piecewise linear function with random changes in the slope
- The commonly used random walk with drift underestimates long-term longevity risk systematically
- The distribution for the trend change intensity needs to be consistent with the derivation of the trend change probability

Since the AMT at the start of a simulation is unknown a stochastic start trend should be considered

The choice of the EMT approach is important in practice
- A weighted regression approach seems reasonable. We show how optimal weights can be derived
References

