It Takes Two:
Why Mortality Trend Modeling is more than Modeling one Mortality Trend

Matthias Börger (m.boerger@ifa-ulm.de)
Jochen Russ (j.russ@ifa-ulm.de)

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Introduction

- Around the world, life expectancy increases and mortality rates decrease
- The decrease in log mortality rates often appears linear:

![Log q_60 England & Wales males](image)
Introduction

• What if we look further into the past?
Agenda

• Why two mortality trends?
  – Actual mortality trend (AMT)
  – Expected mortality trend (EMT)
  – Some examples for applications

• A combined model for both trends
  – AMT component
  – Stochastic start trend
  – Comparison with other AMT approaches
  – EMT component

• Conclusion
Actual Mortality Trend

- The first trend is the actual mortality trend (AMT)
  - The AMT describes realized future mortality and is the core of most existing mortality models
  - Goal: plausible extrapolation of historically observed mortality
  - Frequency and magnitude of changes in the AMT plus random fluctuations around the AMT need to be modeled
- Today’s AMT is not (fully) observable!
- We know historical mortality
  - Random fluctuations can be filtered out
  - Historical trend changes and slopes of piecewise linear trends can be estimated
Actual Mortality Trend

- But we do not know the current slope
  - There might be a trend change this year
  - There might have been a trend change over the last years which is misinterpreted as random fluctuations or vice versa
Estimated Mortality Trend

- The **second trend** is the estimated mortality trend (EMT)
- The EMT is an observer’s estimate of the AMT at some given point in time
  - Estimate of the unknown AMT based on available data
Estimated Mortality Trend

• The EMT is based on the most recent historical mortality evolution and updated as soon as new data becomes available
• The EMT is the basis for mortality projections and (generational) mortality tables, e.g., for reserving
Why Two Mortality Trends?

- Which mortality should be considered depends on the application in view, examples:
  - Capital for a portfolio run-off → AMT over the run-off
  - Reserves for the portfolio after 10 years → EMT after 10 years and AMT over the 10 years
    • AMT for the next 10 years is required to be able to compute EMT in 10 years time
  - Payout of a mortality derivative which reduces GAO risk
    → EMT at maturity and AMT up to maturity
  - Analysis of hedge effectiveness of the derivative
    → EMT at maturity of the derivative, AMT also beyond
  - Solvency Capital Requirement: combined 99.5th percentile of actual payments over the next year and changes in liabilities → AMT for actual payments and EMT for change in liabilities
Combined AMT/EMT Model – AMT Component

- Implementation of both trends in the Cairns-Blake-Dowd model

\[ \logit(q_{x,t}) := \log\left(\frac{q_{x,t}}{1 - q_{x,t}}\right) = \kappa_1(t) + \kappa_2(t) \cdot (x - x), \]

- Model parameters for English and Welsh males aged 60-89:

\begin{figure}
\centering
\includegraphics[width=\textwidth]{kappa_1}
\caption{kappa_1}
\end{figure}

\begin{figure}
\centering
\includegraphics[width=\textwidth]{kappa_2}
\caption{kappa_2}
\end{figure}
Combined AMT/EMT Model – AMT Component

- 7 trend changes for both kappa processes ➔ trend change probability $p = \frac{7}{169}$
- Trend change intensity (different from Sweeting (2011)):
  - sign of trend change: Bernoulli distributed with values 1 and -1 and probability $\frac{1}{2}$
  - absolute magnitude of trend change, normally distributed with parameters according to sample mean and sample variance
Combined AMT/EMT Model – Stochastic Start Trend

- The AMT at the start of a simulation is not observable
- What if another trend change occurred after the last significant one?
  - Additional uncertainty
- This is more than parameter uncertainty in estimating a linear trend
- In our model, this uncertainty is accounted for by a stochastic start trend
  - In the paper, we explain in detail how a stochastic start trend can be implemented
Combined AMT/EMT Model – Comparison of AMT Approaches

- Remaining period life expectancy for a 60-year old (with 10th and 90th percentiles)

- All models: Similar, plausible medians
- Random walk with drift: first widest and then narrowest confidence bands, 3.1 years in 2050 seem unrealistically small
- Sweeting’s approach: implausibly wide confidence bands (23.3 years in 2050).
- In our opinion due to his modeling of the trend change intensity (not consistent with parameter estimation)
- Our model: confidence bands look plausible; Stochastic start trend widens range in 2050 from 7.7 years to 8.5 years
Combined AMT/EMT Model – The EMT Component

• The EMT is the best estimate of the AMT at any point in time
• In principle, every estimation procedure is feasible for the EMT
• „Obvious“ choice for the EMT at time t in our setting: Mean of the distribution of the stochastic start trend at time t
  – Not feasible within a simulation → Simpler methods required for the EMT in simulations.
Combined AMT/EMT Model – The EMT Component

- We propose to compute the EMT by weighted regression on observed trend parameters kappa1 and kappa2 in CBD model
  - Extrapolation of linear trend in most recent data points
  - Crucial question: How many data points?
    - Too many data points: Delayed reaction to change in the AMT
    - Too little data points: EMT is exposed to random noise in the AMT
  - Weights decrease exponentially going backwards in time
  - “Optimal” weighting parameters can be derived by minimizing the MSE between AMT and EMT
Conclusion

• Two trends need to be distinguished and modeled:
  – The actual mortality trend (AMT) which is unobservable
  – The estimated mortality trend (EMT) which is an observer’s estimate of the AMT

• Which trend(s) to consider depends on the question in view

• The AMT can be modeled as a piecewise linear function with random changes in the slope
  – Commonly used random walk with drift underestimates long-term longevity risk systematically

• Since the AMT at the start of a simulation is unknown a stochastic start trend should be considered

• The choice of the EMT approach is important in practice
  – A weighted regression approach seems reasonable. We show how optimal weights can be derived
References
