Modeling Trend Processes in Parametric Mortality Models

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Agenda

Motivation

New Stochastic Trend Process

Parameter Estimation

Stochastic Start Trend

Comparison with other Trend Processes

Summary
Motivation

Uncertainty about the evolution of mortality

- increasing attention on longevity risk
- measure longevity risk in pension or annuity portfolios with stochastic mortality models
- parametric mortality models
- one or more time dependent parameter
- projection of these parameter processes with stochastic processes
  - stochastic forecasts of future mortality
- focus here: A new stochastic trend process to model the parameter processes in parametric mortality models
Motivation

Stochastic processes:
- popular choice: (multivariate) random walk with drift (RWD)
  - one constant drift
  - simple estimation
- curves are unlikely to be generated by a RWD (Börger et al. 2014)
- new trend process
  - continuous and piecewise linear trends
  - random changes in the trend’s slope
  - random fluctuations around the trend
**New Stochastic Trend Process**

**Trend process with multiple breakpoints:**

- modeling the trend with random noise:
  \[ \kappa_t = \tilde{\kappa}_t + \varepsilon_t \]

- extrapolation with most recent trend:
  \[ \tilde{\kappa}_t = \tilde{\kappa}_{t-1} + d_t \]

- inclusion of a possible trend change with a trend change probability \( p \)
  \[ d_t = d_{t-1} + \lambda_t \]
  and with no trend change with a probability \( 1 - p \)
  \[ d_t = d_{t-1} \]

- trend change intensity:
  \[ \lambda_t = S_t \cdot M_t \]

  - absolute magnitude of trend change \( M_t \)
  - sign of the trend change \( S_t \)
Parameter Estimation

Parameters to be estimated:

- trend change probability $p$
- decomposed trend change intensity $\lambda_t = S_t \cdot M_t$
  - $S_t$ Bernoulli distributed with values -1 and 1 each with probability ½.
  - $M_t$ Lognormally distributed with parameters $\mu, \sigma^2$
  
  Other approaches use a normal distribution or a Pareto distribution

- values of $r_t$ and $d_t$ for the starting value and trend
- variance of the error terms $\epsilon_t \sim N(0, \sigma^2)$
Parameter Estimation

Idea: Estimation based on historical trend changes

- Comparison and selection of models with information criteria, e.g.:
  - Akaike Information Criterion (AIC)
  - Bayesian Information Criterion (BIC)
  - Modified Bayesian Information Criterion (MBIC)
- Consider changing variance in $\epsilon_t$ with a CUSUM test.
Parameter Estimation

Historic trend processes for females in England & Wales and Sweden
Parameter Estimation

Estimate parameters based on historical trend changes:

- trend change probability \( p = \frac{\text{#breaks}}{\text{data length}} \)
- trend change intensity:
  - use historic absolute trend changes to estimate the parameters of the lognormal distribution \( \mu, \sigma^2 \)
  - optional: sampling of absolute trend changes for different countries
- values of \( \bar{k}_t \) and \( d_t \) for the starting value and trend based on historic trends
- variance of the error term with CUSUM test
Stochastic Start Trend

Uncertainty about the most recent trend and the starting point of simulation

- breakpoint which was not detected yet?
- possible different actual trend?
- possible different starting value?

Idea: include a stochastic distribution for the most recent trend $d_n$ and for the starting point of simulation $\bar{\kappa}_2$.

- Include another last breakpoint after the last detected breakpoint in the historic trend.

- Use relative likelihood to estimate a discrete distribution for the models with an additional last breakpoint.
Stochastic Start Trend

Uncertainty about the most recent trend
- stochastic start trend considers latest evolution
- extrapolation with the most recent trend which is set to be stochastic
- here: upward movement of confidence interval and increase of uncertainty
Comparison with other Trend Processes

Random walk with drift (RWD) and models of Sweeting and Li et al.

- **RWD:**
  - bivariate random walk with one constant drift
  - preselection of data history; here: data since last breakpoint

- **Sweeting (2011):**
  - identification of trend model with Chow-test
  - magnitude of changes normally distributed with mean 0

- **Li et al. (2014):**
  - VARIMA process
  - extrapolation of trends and errors
  - fitted to total data history
Comparison with other Trend Processes

RWD and models of Sweeting and Li
- rather small confidence intervals for RWD and VARIMA; location seems questionable
  - preselection on data history
  - large uncertainty in nearby future and moderate increases in more distant future
- trend process indicates larger uncertainty in the future
- extremely wide confidence intervals with the approach of Sweeting
Comparison with other Trend Processes

RWD and models of Sweeting and Li

- same result for the period life expectancy at age 60
- rather small upside potential for RWD and VARIMA
- trend process includes uncertainties in the future, i.e.:
  - possible continuation of latest improvements
  - extremely wide confidence intervals with the approach of Sweeting
Summary

- Standard random walk with drift does not always extrapolate historic evolution.
  - one constant drift only for limited period of time
- new trend model with:
  - continuous and piecewise linear trends
  - random changes in the trend’s slope
  - random fluctuations around the trend
- parameter estimation based on historic trends
- reasonable trend models for all countries with long data histories provided by the HMD
- Uncertainty in current trend can be accounted for by a stochastic start trend distribution.
- comparison with other approaches including trends
  - reasonable results with the new trend process
  - adequate extrapolation of historic evolution
  - large uncertainty in the more distant future
Literature

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