Modeling Trend Processes in Parametric Mortality Models

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- joint work with Matthias Börger

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Agenda

Motivation

New Stochastic Trend Process

Parameter Estimation

Stochastic Start Trend

Comparison with other Trend Processes

Summary

Motivation

Uncertainty about the evolution of mortality

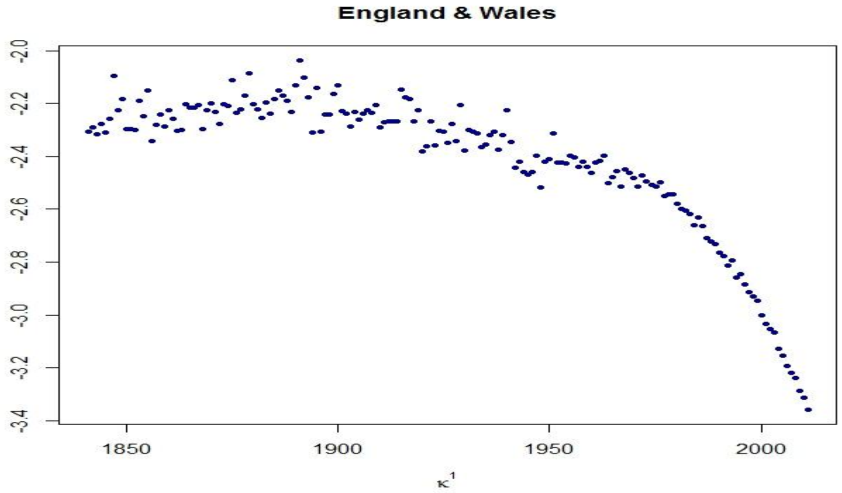
- increasing attention on longevity risk
- measure longevity risk in pension or annuity portfolios with stochastic mortality models

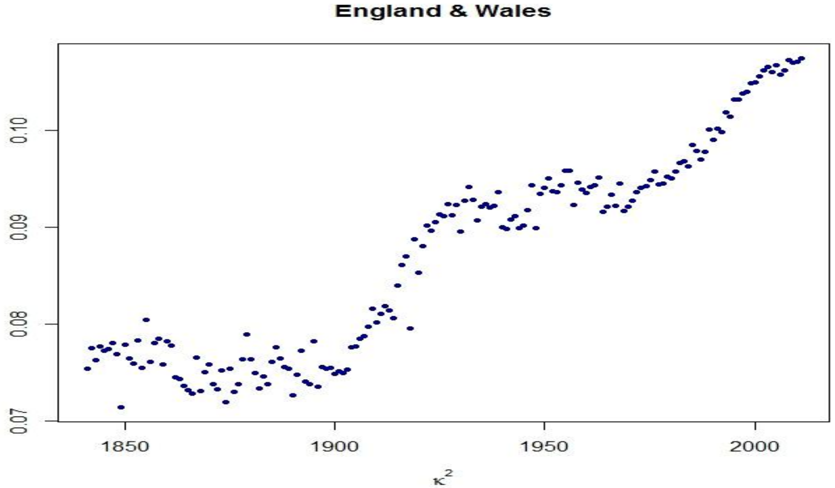
- parametric mortality models
- one or more time dependent parameter
- projection of these parameter processes with stochastic processes
  - stochastic forecasts of future mortality
- focus here: A new stochastic trend process to model the parameter processes in parametric mortality models

Motivation

Stochastic processes:

- popular choice: (multivariate) random walk with drift (RWD)
  - one constant drift
  - simple estimation
- curves are unlikely to be generated by a RWD (Börger et al. 2014)
- new trend process
  - continuous and piecewise linear trends
  - random changes in the trend's slope
  - random fluctuations around the trend





New Stochastic Trend Process

Trend process with multiple breakpoints:

- modeling the trend with random noise:

  $\kappa_t = \tilde{\kappa}_t + \epsilon_t$

- extrapolation with most recent trend:

  $\tilde{\kappa}_t = \tilde{\kappa}_{t-1} + d_t$

- inclusion of a possible trend change with a trend change probability p

  $d_t = d_{t-1} + \lambda_t$

  and with no trend change with a probability $1 - p$

  $d_t = d_{t-1}$

- trend change intensity:

  $\lambda_t = S_t \cdot M_t$

  - absolute magnitude of trend change M_t
  - sign of the trend change S_t

Parameter Estimation

Parameters to be estimated:

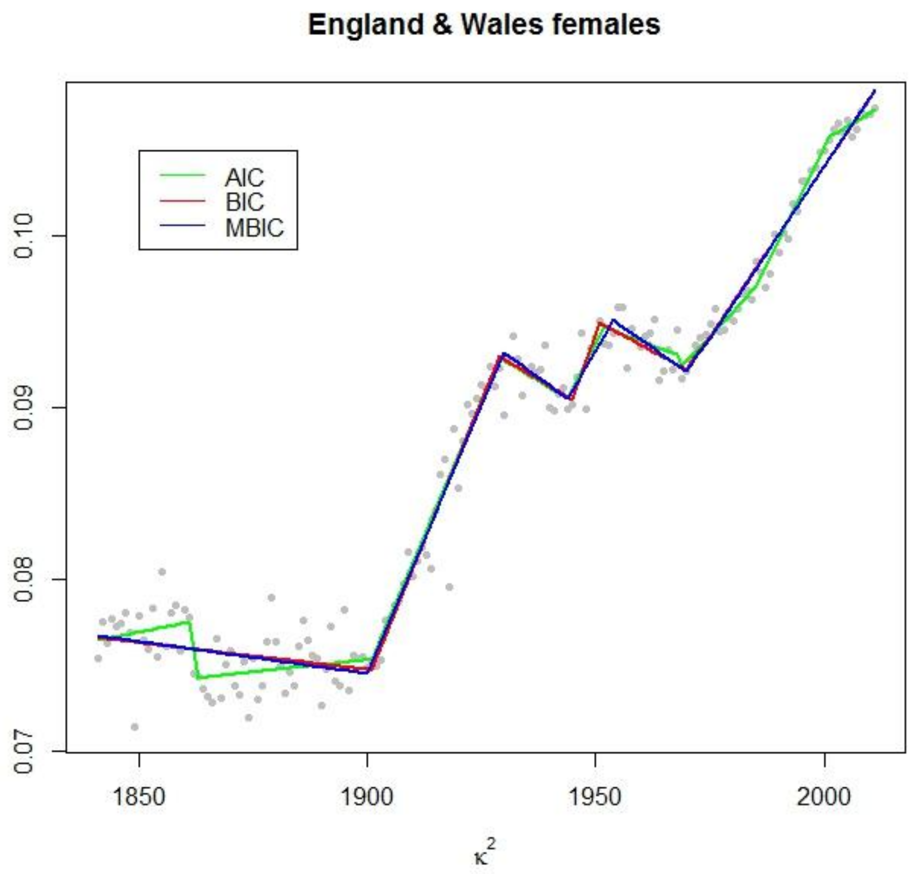
- trend change probability p
- decomposed trend change intensity $\lambda_t = S_t \cdot M_t$
  - S_t bernoulli distributed with values -1 and 1 each with probability ½.
  - M_t lognormally distributed with parameters μ, σ^2

    Other approaches use a normal distribution or a Pareto distribution
- values of $\tilde{\kappa}_t$ and d_t for the starting value and trend
- variance of the error terms $\epsilon_t \sim \mathcal{N}(0, \sigma_t^2)$

Parameter Estimation

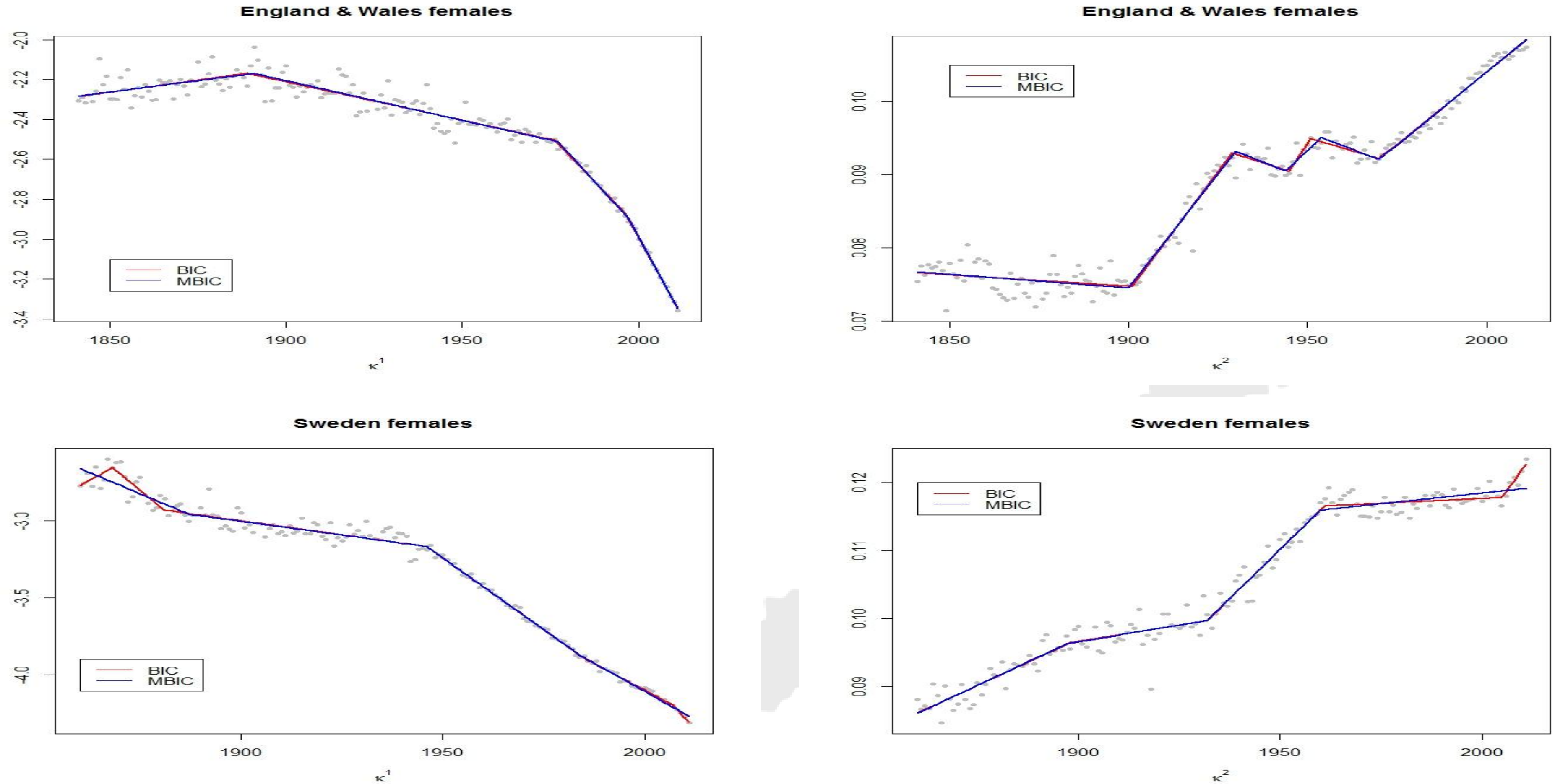
Idea: Estimation based on historical trend changes

- Use an iterative 'segmented' method proposed by Muggeo (2003).
- comparison and selection of models with information criteria, e.g.:
  - Akaike Information Criterion (AIC)
  - Bayesian Information Criterion (BIC)
  - Modified Bayesian Information Criterion (MBIC)
- Consider changing variance in ϵ_t with a CUSUM test.



Parameter Estimation

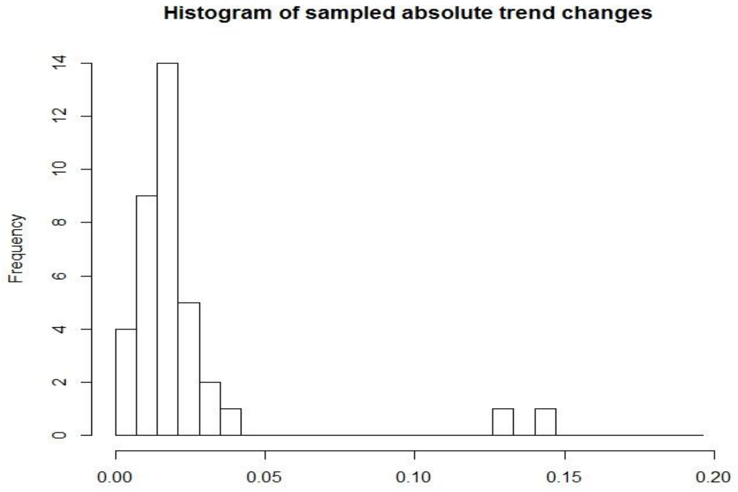
Historic trend processes for females in England & Wales and Sweden

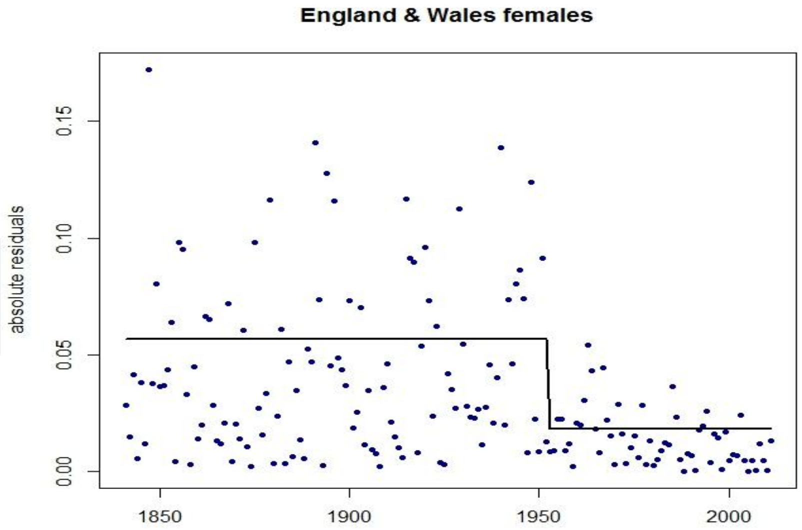


Parameter Estimation

Estimate parameters based on historical trend changes:

- trend change probability $p = \frac{\#breaks}{data\ length}$
- trend change intensity:
  - use historic absolute trend changes to estimate the parameters of the lognormal distribution μ, σ^2
  - optional: sampling of absolute trend changes for different countries
- values of $\tilde{\kappa}_t$ and d_t for the starting value and trend based on historic trends
- variance of the error term with CUSUM test

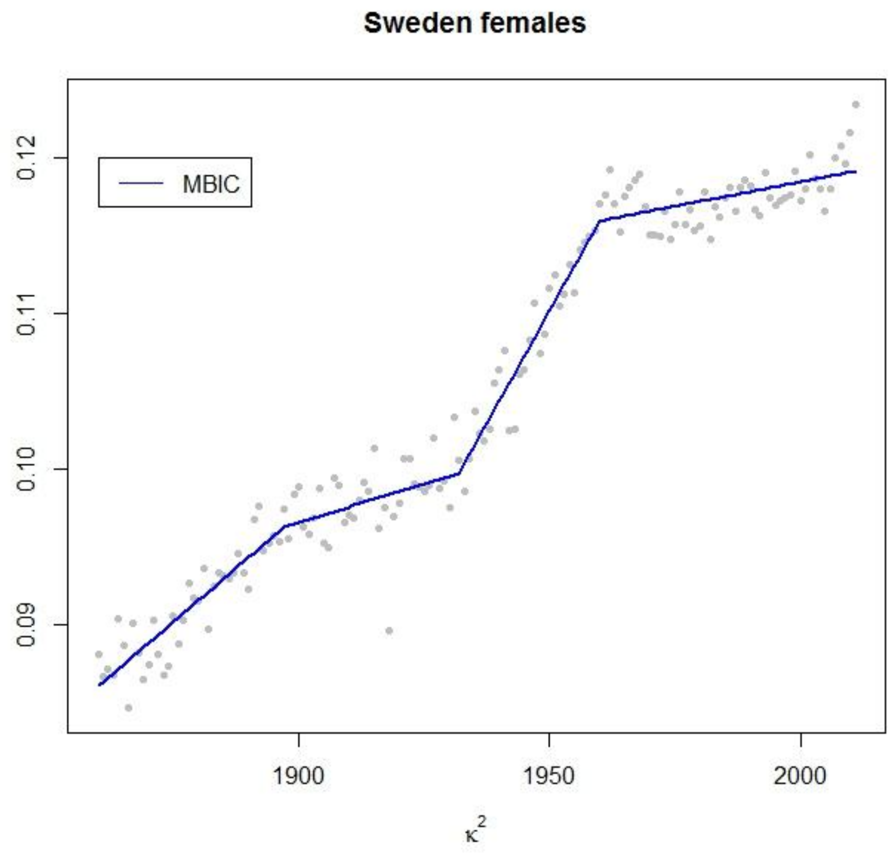




Stochastic Start Trend

Uncertainty about the most recent trend and the starting point of simulation

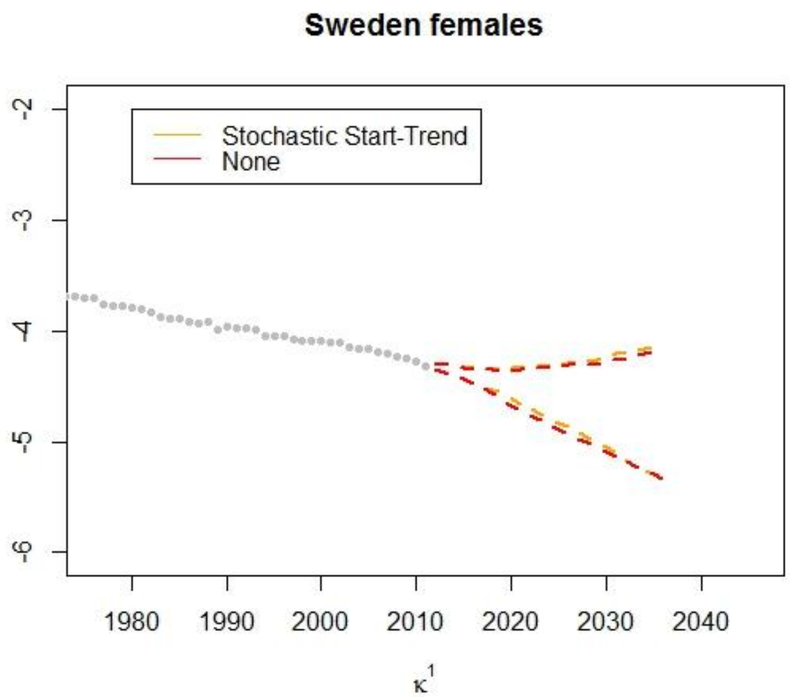
- breakpoint which was not detected yet?
  - possible different actual trend?
  - possible different starting value?
- Idea: include a stochastic distribution for the most recent trend d_t and for the starting point of simulation $\tilde{\kappa}_t$.
- Include another last breakpoint after the last detected breakpoint in the historic trend.
- Use relative likelihood to estimate a discrete distribution for the models with an additional last breakpoint.

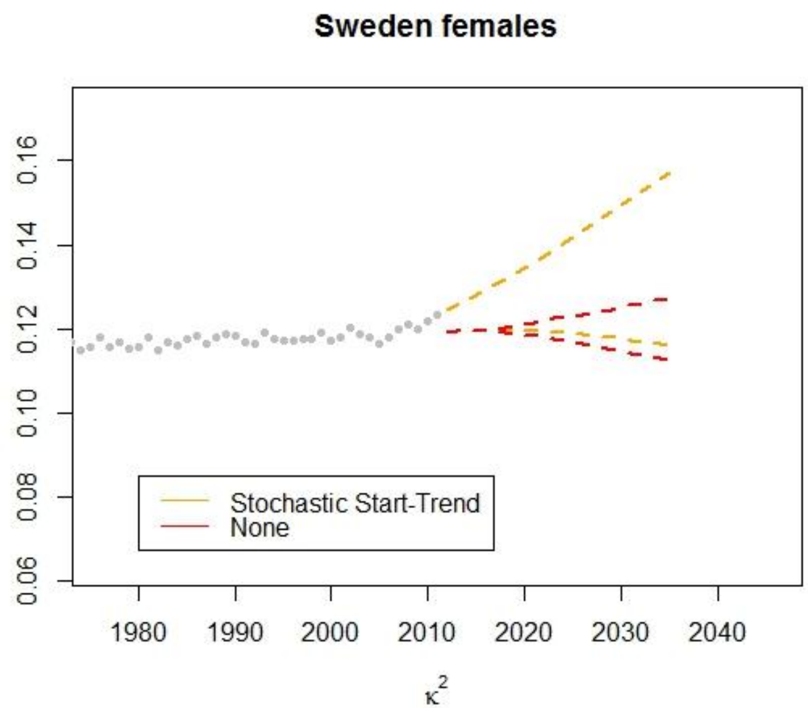


Stochastic Start Trend

Uncertainty about the most recent trend

- stochastic start trend considers latest evolution
- extrapolation with the most recent trend which is set to be stochastic
- here : upward movement of confidence interval and increase of uncertainty





Comparison with other Trend Processes

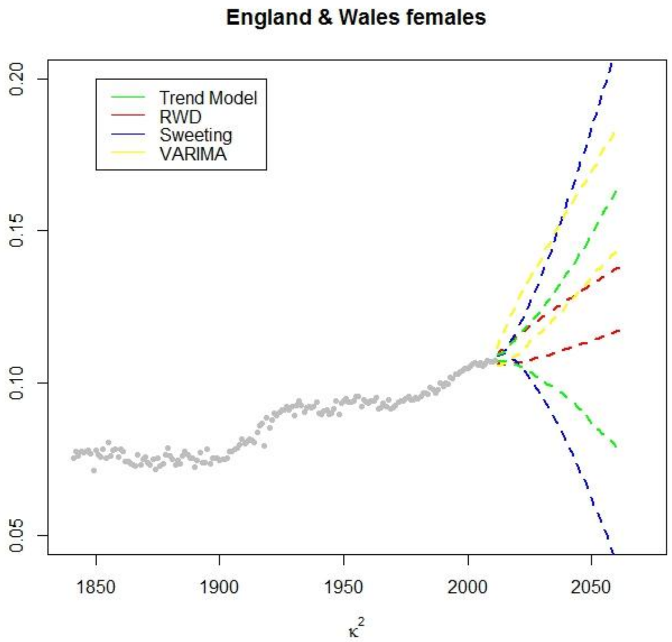
Random walk with drift (RWD) and models of Sweeting and Li et al.

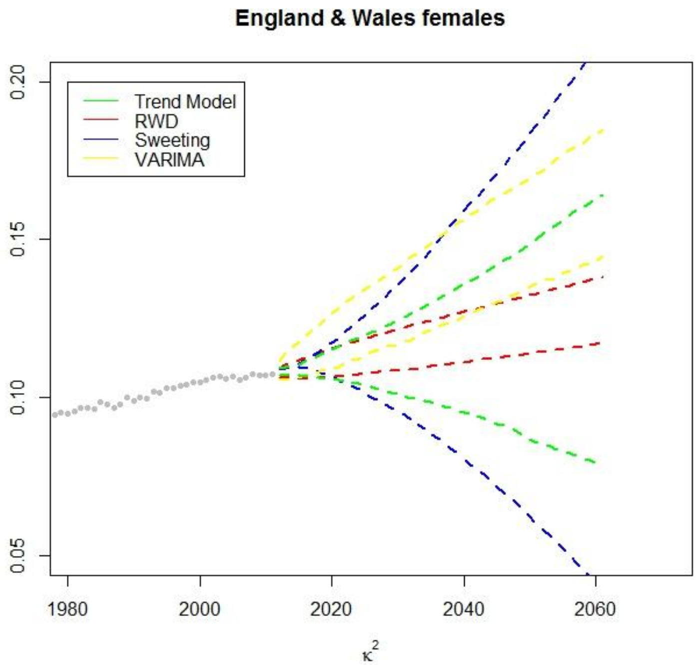
- RWD:
  - bivariate random walk with one constant drift
  - preselection of data history; here: data since last breakpoint
- Sweeting (2011):
  - identification of trend model with Chow-test
  - magnitude of changes normally distributed with mean 0
- Li et al. (2014):
  - VARIMA process
  - extrapolation of trends and errors
  - fitted to total data history

Comparison with other Trend Processes

RWD and models of Sweeting and Li

- rather small confidence intervals for RWD and VARIMA; location seems questionable
  - preselection on data history
  - large uncertainty in nearby future and moderate increases in more distant future
- trend process indicates larger uncertainty in the future
- extremely wide confidence intervals with the approach of Sweeting

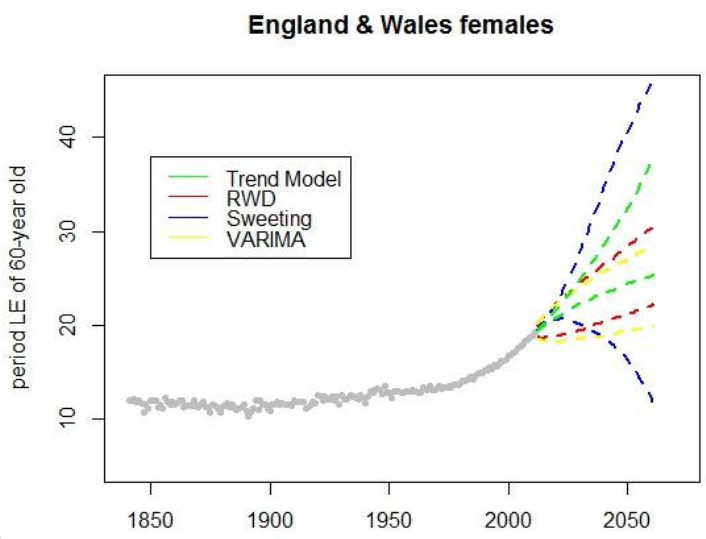


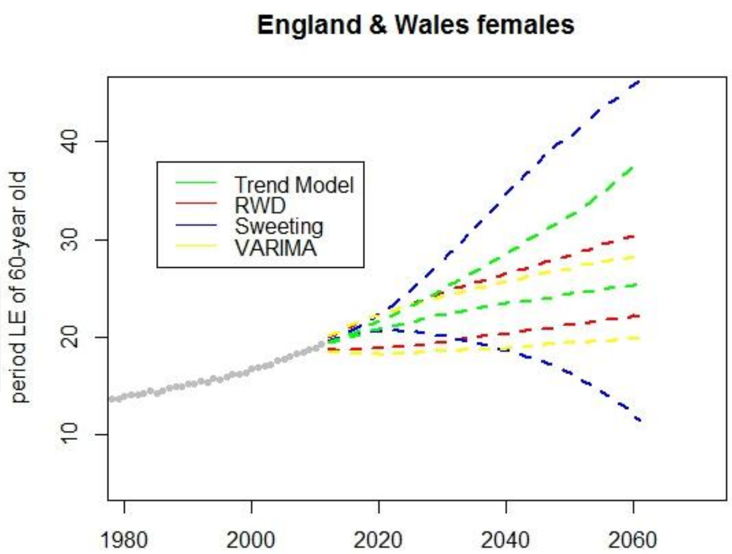


Comparison with other Trend Processes

RWD and models of Sweeting and Li

- same result for the period life expectancy at age 60
- rather small upside potential for RWD and VARIMA
- trend process includes uncertainties in the future, i.e.:
  - possible continuation of latest improvements
- extremely wide confidence intervals with the approach of Sweeting





Summary

- Standard random walk with drift does not always extrapolate historic evolution.
  - one constant drift only for limited period of time
- new trend model with:
  - continuous and piecewise linear trends
  - random changes in the trend’s slope
  - random fluctuations around the trend
- parameter estimation based on historic trends
- reasonable trend models for all countries with long data histories provided by the HMD
- Uncertainty in current trend can be accounted for by a stochastic start trend distribution.
- comparison with other approaches including trends
  - reasonable results with the new trend process
  - adequate extrapolation of historic evolution
  - large uncertainty in the more distant future

Literature

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