Institut für Finanz- und Aktuarwissenschaften ifa

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Multi-year non-life insurance risk of dependent lines of business

The multivariate additive loss reserving model

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Multi-year non-life insurance risk for dependent portfolios

The multivariate additive loss reserving model

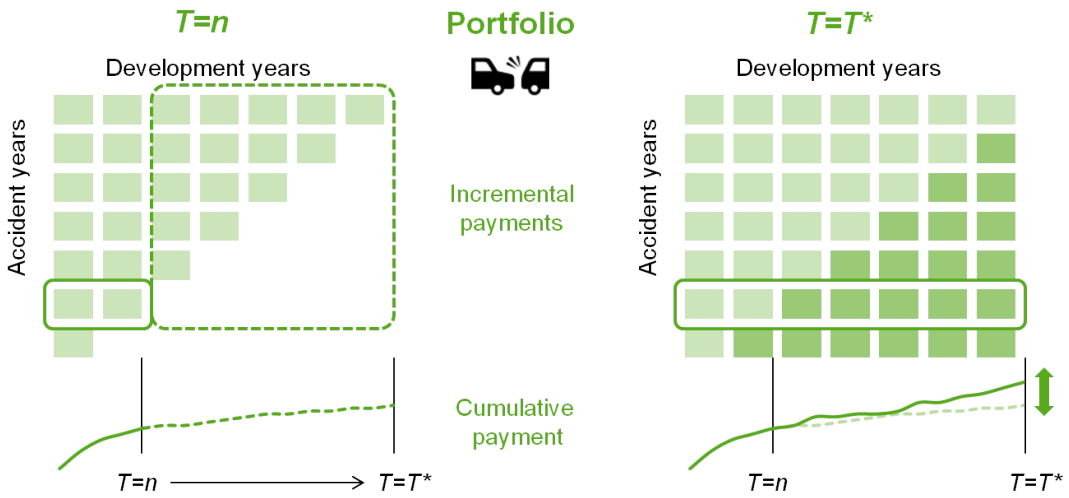
Analytical risk estimators

Case study

Conclusion

Risk horizon

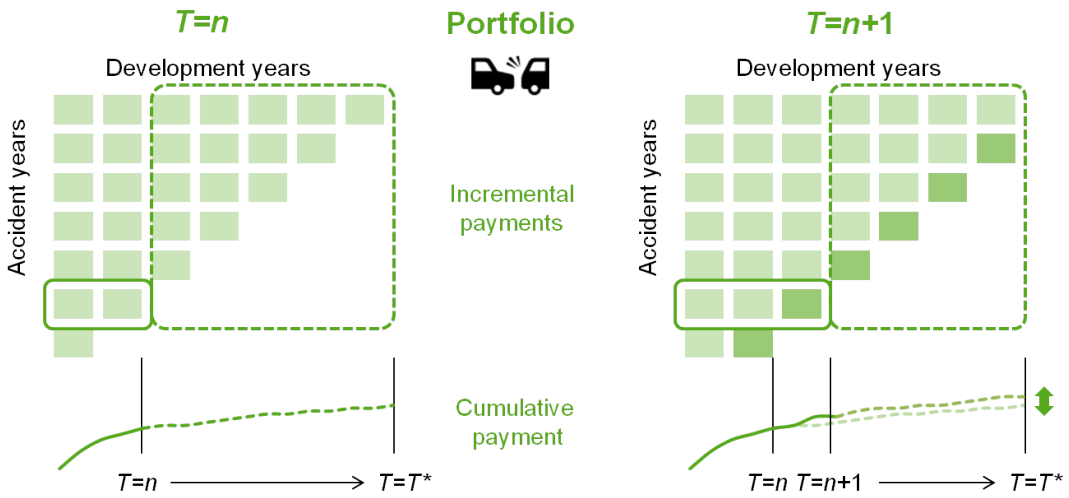
- **Risk horizon** for non-life insurance risk



- Traditional ultimo view measures uncertainty stemming from all future occurrences and payments until final settlement at $T = T^*$

Risk horizon

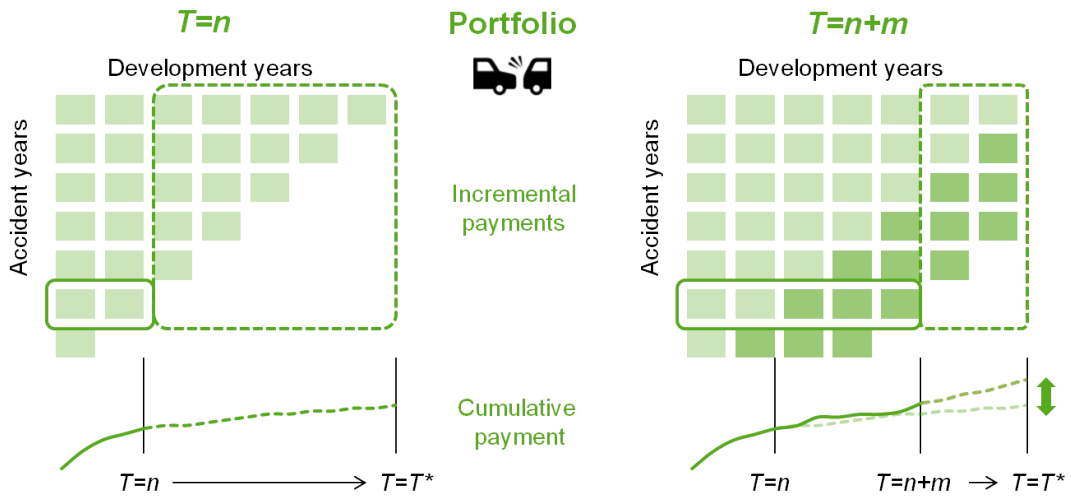
- **Risk horizon** for non-life insurance risk



- Modern regulation requires a one-year view, i.e. uncertainty in solely next year's claims settlement and in updating reserves for outstanding payments

Risk horizon

- **Risk horizon** for non-life insurance risk

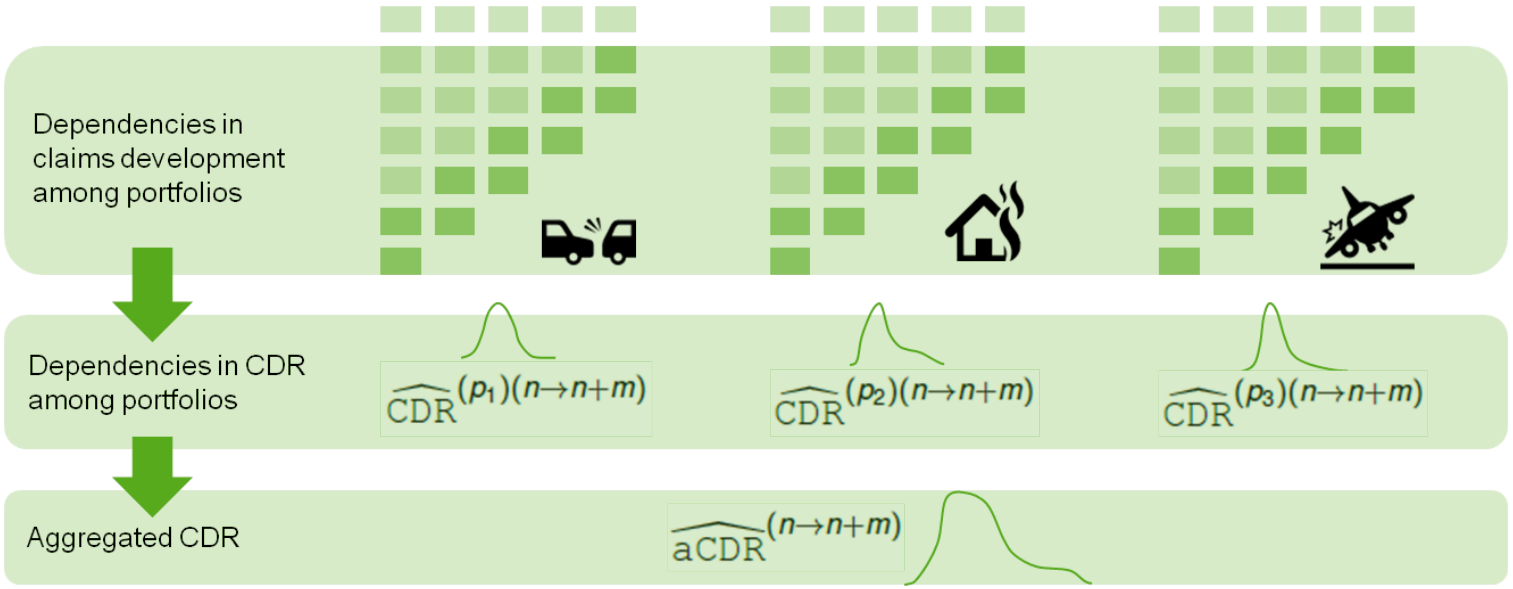


- A general multi-year view builds a bridge between both horizons: uncertainty in settling $m = 1, \ldots, T^* - n$ years and in updating remaining reserves
  - Forward-looking risk management
  - Projection of SCR as part of ORSA and for risk margin calculation

Single portfolio

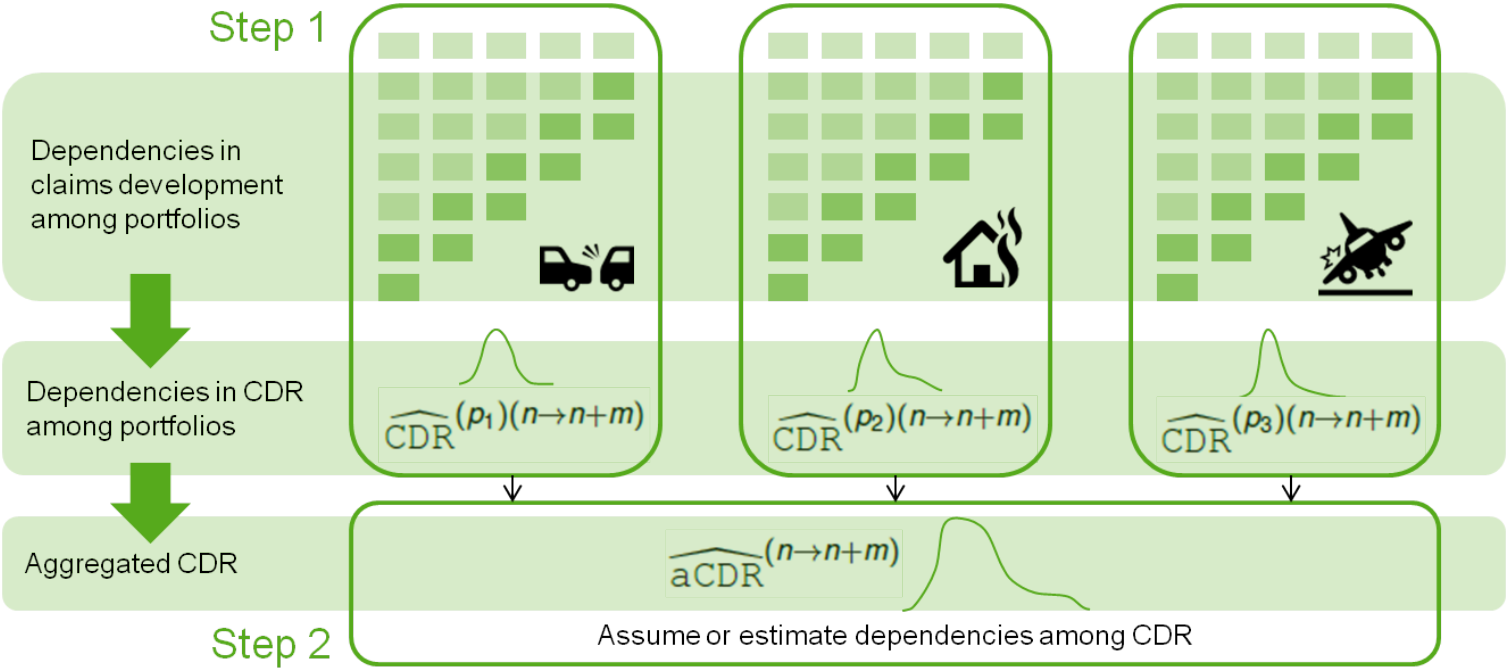
- Uncertainty in claims development result $\widehat{\mathtt{CDR}}^{(n \to n+m)}$ of a single portfolio
  - **Analytical approaches**: Closed-form estimators for the mean squared error of prediction (msep) of $\widehat{\mathtt{CDR}}^{(n \to n+m)}$ in distribution-free stochastic reserving models
    - additive loss reserving model (Diers and Linde, 2013)
    - Mack chain ladder (CL) model (Diers et al., 2016)
  - **Bootstrap methods**: Stochastic m-year re-reserving ("actuary in a box") in Diers et al. (2013) to estimate a full predictive distribution of $\widehat{\mathtt{CDR}}^{(n \to n+m)}$
    - generalizing the one-year bootstrap by Ohlsson and Lauzeningks (2009)

Dependent portfolios



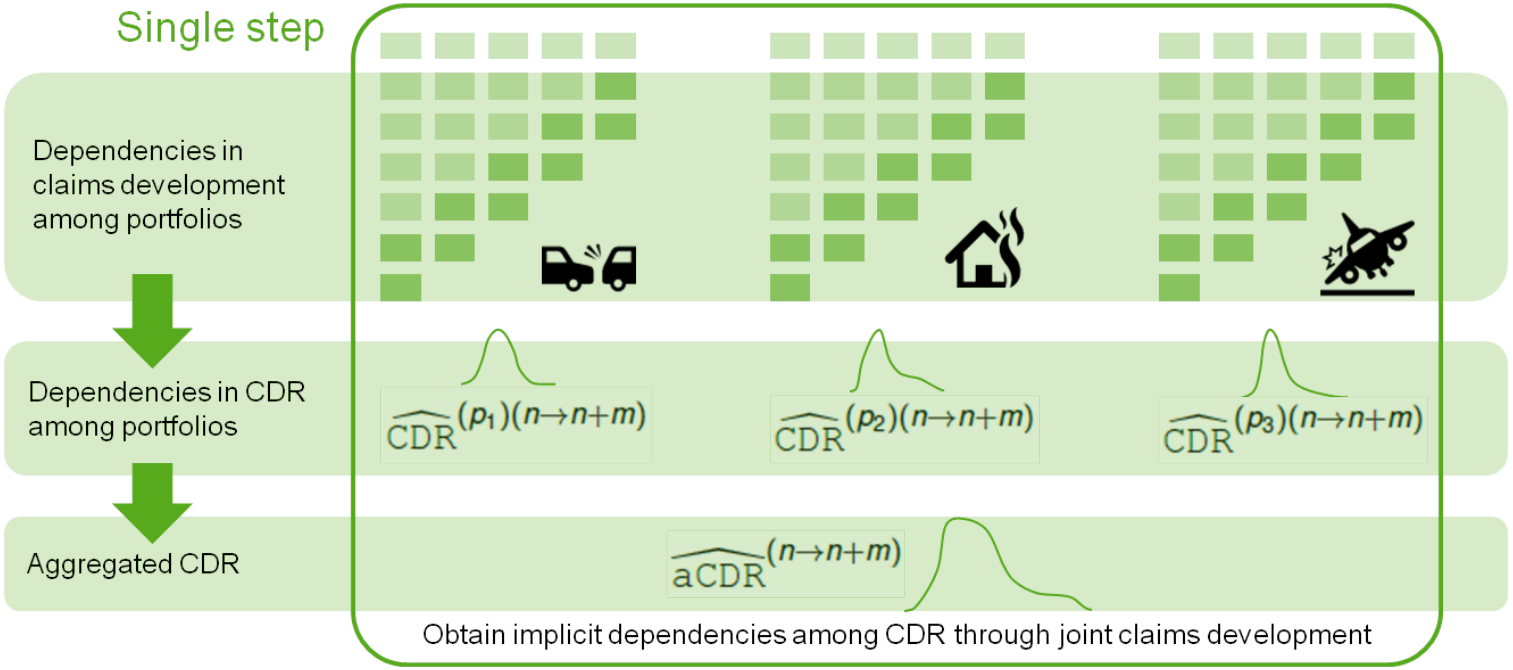
- **Goal**
  - Estimate joint risk subject to dependencies among CDR

Dependent portfolios



- **Two-step approach**
  - Analyze risk in each portfolio's CDR individually
  - Analyze dependencies among individual CDR in a subsequent step

Dependent portfolios



- **Single-step approach**
  - Analyze risk in aggregated CDR subject to dependencies among claims developments of individual portfolios (i.e. no aggregated portfolio!)

Agenda

Multivariate extended additive loss reserving model

Definition

The **multivariate extended additive loss reserving model** is defined through

- $s_{i,k}$, $i = 1, \ldots, n+m$, $k = 1, \ldots, n$ are independent
- For each $k = 1, \ldots, n$ there exists $\mu_k = \left(\mu_k^{(1)}, \ldots, \mu_k^{(q)}\right)'$ such that $\mathbb{E}(s_{i,k}) = V_i \mu_k$.
- For each $k = 1, \ldots, n$ there exists a symmetric positive definite $q \times q$ matrix Σ_k such that $\mathbb{V}(s_{i,k}) = V_i^{1/2} \Sigma_k V_i^{1/2}$.

- Features compared to Hess et al. (2006)
  - We assume global independence
  - We integrate future accident years

Multi-year framework

Definition

Given $\Delta_n \to \Delta_{n+m}$, the m-year (observable) **claims development result** for a single accident year $i \in \{1, \ldots, n+m\}$ is

$$\widehat{\mathrm{CDR}}_i^{(n \to n+m)} = {}^{(n)}\widehat{u}_i - {}^{(n+m)}\widehat{u}_i.$$

- In the multivariate extended additive model,

$$\widehat{\mathrm{CDR}}_i^{(n \to n+m)} = V_i \sum_{k=\max\{1, n-i+2\}}^{\min\{n+m-i+1, n\}} \left({}^{(n)}\widehat{\mu}_k - m_{i,k} \right) + V_i \sum_{k=n+m-i+2}^{n} \left({}^{(n)}\widehat{\mu}_k - {}^{(n+m)}\widehat{\mu}_k \right)$$

- Risk measure: msep

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Estimator for risk in a single accident year

Theorem

The unconditional covariance matrix of the m-year CDR for one accident year $i \in \{2, \ldots, n+m\}$ is given by

$$\mathbb{V}\left(\widehat{\mathrm{CDR}}_i^{(n \to n+m)}\right)$$
$$= V_i \left[\sum_{k=\max\{1,n-i+2\}}^{\min\{n+m-i+1,n\}} \left({}^{(n)}V\Sigma_{\leq k}^{-1} + V_i^{-1/2}\Sigma_k V_i^{-1/2} \right) + \sum_{k=n+m-i+2}^{n} \left({}^{(n)}V\Sigma_{\leq k}^{-1} - {}^{(n+m)}V\Sigma_{\leq k}^{-1} \right) \right] V_i$$

- If Σ_k is unknown, substitute by suitable estimator $\widehat{\Sigma}_k$

Estimator for non-life insurance risk

- More generally, we obtain estimators for
  - non-life insurance risk: $\mathbb{V}\left(\widehat{\mathtt{CDR}}^{(n\to n+m)}\right)$
  - reserve risk: $\mathbb{V}\left(\widehat{\mathtt{CDR}}_{\mathsf{PY}}^{(n\to n+m)}\right)$
  - premium risk: $\mathbb{V}\left(\widehat{\mathtt{CDR}}_{\mathsf{NY}}^{(n\to n+m)}\right)$
- Special cases for
  - one-year view: $m = 1$
  - ultimo view: $m = i - 1$ (single accident year i) or $m = n - 1$ (reserve risk)
- Aggregated risk
  - Use, e.g., $\mathbb{V}\left(\widehat{\mathtt{aCDR}}_i^{(n\to n+m)}\right) = \mathbf{1}'\mathbb{V}\left(\widehat{\mathtt{CDR}}_i^{(n\to n+m)}\right)\mathbf{1}$
  - For aggregated reserve risk in ultimo view, we obtain the same result as in Merz and Wüthrich (2009)
- Single portfolio
  - Use $q = 1$

Estimator for non-life insurance risk

- Effective correlation between reserve and premium risk
  - Estimator for aggregated business

$$\mathrm{Corr}\left(\widehat{\mathtt{aCDR}}_{\mathsf{PY}}^{(n\to n+m)}, \widehat{\mathtt{aCDR}}_{\mathsf{NY}}^{(n\to n+m)}\right)$$
$$= \frac{\mathbb{V}\left(\widehat{\mathtt{aCDR}}^{(n\to n+m)}\right) - \mathbb{V}\left(\widehat{\mathtt{aCDR}}_{\mathsf{PY}}^{(n\to n+m)}\right) - \mathbb{V}\left(\widehat{\mathtt{aCDR}}_{\mathsf{NY}}^{(n\to n+m)}\right)}{2\sqrt{\mathbb{V}\left(\widehat{\mathtt{aCDR}}_{\mathsf{PY}}^{(n\to n+m)}\right)\mathbb{V}\left(\widehat{\mathtt{aCDR}}_{\mathsf{NY}}^{(n\to n+m)}\right)}}$$

  - Analogous estimator for marginal effective correlation
  - Similar logic for effective correlation of fixed risk component between marginal portfolios
- Risk in future one-year view $\mathbb{V}\left(\widehat{\mathtt{CDR}}^{(n+t\to n+t+1)}\right)$
  - Closed-form estimators available
  - $\mathbb{V}\left(\widehat{\mathtt{CDR}}^{(n\to n+t+1)}\right) =$
    $\mathbb{V}\left(\widehat{\mathtt{CDR}}^{(n\to n+t)}\right) + \mathbb{V}\left(\widehat{\mathtt{CDR}}^{(n+t\to n+t+1)}\right)$

Agenda

Data

- **Data** used in Braun (2004), Merz and Wüthrich (2009), Ludwig and Schmidt (2010), Diers and Linde (2013)
  - $q = 2$ portfolios
    - general third-party liability
    - motor vehicle third-party liability
  - $n = 14$ historic accident years with volumes
- Include volumes for $m = 5$ future accident years
- Gauss-Markov parameter estimation as in Merz and Wüthrich (2009)

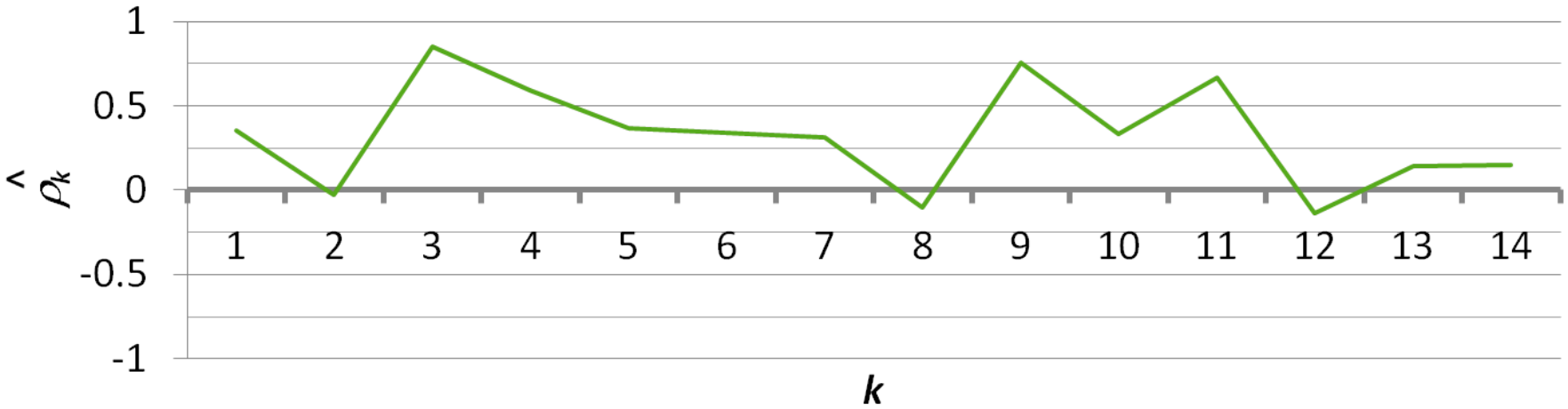
Selected results

▸ Parameter estimates

Parameter estimates in the multivariate additive loss reserving model

Notation: $\widehat{\sigma}_k^{(p)}$ denote estimated standard errors at k for $p = 1, 2$, i.e. $\widehat{\sigma}_k^{(p)} := \sqrt{\widehat{\Sigma}_k^{(p,p)}}$, and $\widehat{\rho}_k^{(1,2)} := \widehat{\Sigma}_k^{(1,2)} \Big/ \left(\widehat{\sigma}_k^{(1)}\widehat{\sigma}_k^{(2)}\right)$ the estimated correlation coefficient.
Estimates are calculated as in Merz and Wüthrich (2009, Table 4). For our extended model, we additionally estimate parameters for $k = 1$.

Estimator	Development years k													
	1	2	3	4	5	6	7	8	9	10	11	12	13	14
$\widehat{\mu}_k^{(1)}$	0.08860	0.19974	0.20640	0.17493	0.12119	0.08452	0.04844	0.02476	0.01441	0.01195	0.00614	0.00428	0.00529	0.00371
$\widehat{\mu}_k^{(2)}$	0.26771	0.32899	0.16172	0.09061	0.05572	0.03170	0.01550	0.00910	0.00017	0.00354	−0.00051	0.00354	−0.00097	−0.00026
$\widehat{\sigma}_k^{(1)}$	22.17661	31.57523	20.03153	14.42171	18.92315	13.63922	13.90884	5.78725	7.16307	12.20596	6.09168	1.83922	0.55530	0.16766
$\widehat{\sigma}_k^{(2)}$	27.95800	27.73945	18.19876	15.17124	15.99662	11.73743	5.17320	4.69929	2.05411	4.96015	1.34770	2.99827	1.34770	0.60578
$\widehat{\rho}_k^{(1,2)}$	0.35146	−0.02654	0.84893	0.59216	0.37111	0.34034	0.31262	−0.10467	0.75529	0.33235	0.66612	−0.13921	0.14399	0.14894



Selected results

- **Best estimate reserves**
  - No material difference between univariate and multivariate models

Best estimate reserves for outstanding and future claims by line of business and aggregated for overall business.
Best estimate reserves are calculated via the multivariate additive loss reserving model ("multivariate") and the univariate additive loss reserving model applied individually to single triangles ("univariate (marginal)") and to the overall triangle when first aggregating both triangles by adding all entries ("univariate (overall)").

	General liability business		Auto liability business		Overall business		
Reserve component	Multivariate	Univariate (individual)	Multivariate	Univariate (individual)	**Multivariate**	Univariate (individual)	Univariate (overall)
Prior years PY $= \{1, \ldots, 14\}$	6,315,254	6,311,503	2,050,866	2,047,680	**8,366,120**	8,359,183	8,123,852
Future years NY $= \{15, \ldots, 19\}$	14,594,119	14,591,110	8,311,027	8,307,522	**22,905,145**	22,898,632	22,698,279
of which $i = n + 1 = 15$	2,601,723	2,601,187	1,512,389	1,511,751	**4,114,112**	4,112,938	4,077,509
All years PY $\cup$ NY $= \{1, \ldots, 19\}$	20,909,372	20,902,613	10,361,893	10,355,203	**31,271,265**	31,257,815	30,822,130
of which PY $\cup \{15\}$	8,916,977	8,912,690	3,563,255	3,559,431	**12,480,232**	12,472,121	12,201,361

Selected results

- **Reserve risk**
  - Considerably higher in multivariate model
  - Stable correlation

Estimated reserve risk by line of business and aggregated for overall business.
Reserve risk is estimated via prediction errors (PE, standard errors of prediction variance) for all prior years and coefficients of variation (CoV, in terms of best estimate reserves) by models as in previous table.
For the multivariate model we also calculate the effective linear correlation between both portfolios (Corr).

	General liability business				Auto liability business				Overall business						
Multi-year view	Multivariate		Univariate (individual)		Multivariate		Univariate (individual)		**Multivariate**			Univariate (individual)		Univariate (overall)	
m	PE	CoV	PE	CoV	PE	CoV	PE	CoV	PE	CoV	**Corr**	PE	CoV	PE	CoV
1	126,647	2.0%	126,707	2.0%	68,738	3.4%	68,788	3.4%	161,656	1.9%	**30.8%**	144,175	1.7%	162,774	2.0%
2	161,783	2.6%	161,846	2.6%	86,051	4.2%	86,103	4.2%	207,972	2.5%	**34.7%**	183,324	2.2%	209,852	2.6%
3	182,838	2.9%	182,897	2.9%	95,830	4.7%	95,880	4.7%	233,761	2.8%	**34.3%**	206,505	2.5%	235,683	2.9%
4	196,249	3.1%	196,309	3.1%	101,398	4.9%	101,446	5.0%	249,429	3.0%	**33.7%**	220,972	2.6%	250,896	3.1%
5	204,234	3.2%	204,294	3.2%	103,953	5.1%	104,000	5.1%	258,155	3.1%	**33.3%**	229,243	2.7%	259,454	3.2%
6	209,462	3.3%	209,525	3.3%	105,057	5.1%	105,104	5.1%	263,461	3.1%	**32.9%**	234,409	2.8%	264,378	3.3%
7	212,578	3.4%	212,643	3.4%	105,793	5.2%	105,840	5.2%	266,686	3.2%	**32.8%**	237,527	2.8%	267,342	3.3%
8	214,854	3.4%	214,919	3.4%	106,285	5.2%	106,332	5.2%	269,088	3.2%	**32.7%**	239,785	2.9%	269,601	3.3%
9	216,240	3.4%	216,305	3.4%	106,638	5.2%	106,685	5.2%	270,555	3.2%	**32.7%**	241,184	2.9%	270,990	3.3%
10	216,520	3.4%	216,585	3.4%	106,781	5.2%	106,828	5.2%	270,873	3.2%	**32.6%**	241,499	2.9%	271,284	3.3%
11	216,546	3.4%	216,611	3.4%	106,878	5.2%	106,925	5.2%	270,927	3.2%	**32.6%**	241,564	2.9%	271,344	3.3%
12	216,548	3.4%	216,613	3.4%	106,897	5.2%	106,944	5.2%	270,937	3.2%	**32.6%**	241,575	2.9%	271,356	3.3%
13	216,548	3.4%	216,613	3.4%	106,900	5.2%	106,947	5.2%	270,939	3.2%	**32.6%**	241,576	2.9%	271,358	3.3%

Selected results

- **Premium risk** assuming one future year
  - Considerably higher in multivariate model

Estimated premium risk of one future year by line of business and aggregated for overall business.
Premium risk is estimated via prediction errors (PE, standard errors of prediction variance) for the next future year and coefficients of variation (CoV, in terms of best estimate reserves) by models as in previous table.
For the multivariate model we also calculate the effective linear correlation between both portfolios (Corr).

	General liability business				Auto liability business				Overall business						
Multi-year view	Multivariate		Univariate (individual)		Multivariate		Univariate (individual)		**Multivariate**			Univariate (individual)		Univariate (overall)	
m	PE	CoV	PE	CoV	PE	CoV	PE	CoV	PE	CoV	**Corr**	PE	CoV	PE	CoV
1	42,704	1.6%	42,718	1.6%	38,550	2.5%	38,557	2.6%	66,540	1.6%	**34.0%**	57,545	1.4%	77,863	1.9%
2	70,964	2.7%	70,978	2.7%	53,950	3.6%	53,959	3.6%	95,104	2.3%	**14.3%**	89,159	2.2%	103,255	2.5%
3	79,973	3.1%	80,000	3.1%	59,457	3.9%	59,471	3.9%	111,834	2.7%	**27.1%**	99,683	2.4%	119,550	2.9%
4	84,440	3.2%	84,465	3.2%	62,984	4.2%	62,999	4.2%	119,586	2.9%	**30.1%**	105,371	2.6%	128,637	3.2%
5	90,964	3.5%	90,987	3.5%	66,585	4.4%	66,600	4.4%	128,242	3.1%	**30.9%**	112,757	2.7%	136,648	3.4%
6	94,232	3.6%	94,255	3.6%	68,438	4.5%	68,453	4.5%	132,519	3.2%	**31.0%**	116,490	2.8%	141,201	3.5%
7	97,411	3.7%	97,434	3.7%	68,808	4.5%	68,822	4.6%	135,389	3.3%	**30.6%**	119,289	2.9%	143,859	3.5%
8	98,033	3.8%	98,056	3.8%	69,108	4.6%	69,123	4.6%	135,958	3.3%	**30.2%**	119,971	2.9%	144,394	3.5%
9	98,881	3.8%	98,902	3.8%	69,173	4.6%	69,188	4.6%	136,799	3.3%	**30.3%**	120,701	2.9%	145,154	3.6%
10	101,130	3.9%	101,151	3.9%	69,494	4.6%	69,508	4.6%	138,931	3.4%	**30.2%**	122,731	3.0%	147,082	3.6%
11	101,680	3.9%	101,701	3.9%	69,520	4.6%	69,535	4.6%	139,436	3.4%	**30.2%**	123,200	3.0%	147,494	3.6%
12	101,730	3.9%	101,751	3.9%	69,636	4.6%	69,650	4.6%	139,517	3.4%	**30.1%**	123,307	3.0%	147,579	3.6%
13	101,734	3.9%	101,756	3.9%	69,659	4.6%	69,674	4.6%	139,534	3.4%	**30.1%**	123,324	3.0%	147,596	3.6%
14	101,735	3.9%	101,756	3.9%	69,664	4.6%	69,678	4.6%	139,537	3.4%	**30.1%**	123,327	3.0%	147,600	3.6%

Selected results

- Further analyses results possible

	Multivariate								
	Correlation: reserve/premium			Proportion of process and estimation variance					
				Overall business					
Multi-year view	General liability business	Auto liability business	Overall business	Reserve risk		Premium risk		Non-life ins. risk	
m				Process	Estimation	Process	Estimation	Process	Estimation
1	43.6%	24.9%	35.2%	83.0%	17.0%	86.9%	13.1%	83.4%	16.6%
2	33.5%	22.2%	31.7%	70.7%	29.3%	84.9%	15.1%	71.1%	28.9%
3	33.6%	22.5%	30.3%	61.4%	38.6%	83.6%	16.4%	61.6%	38.4%
4	34.2%	22.5%	30.2%	54.5%	45.5%	82.6%	17.4%	54.8%	45.2%
5	33.0%	21.8%	29.2%	50.0%	50.0%	81.8%	18.2%	49.8%	50.2%
6	32.7%	21.4%	28.8%	46.8%	53.2%	81.4%	18.6%	46.4%	53.6%
7	32.1%	21.5%	28.6%	44.7%	55.3%	81.0%	19.0%	44.1%	55.9%
8	32.3%	21.5%	28.7%	43.0%	57.0%	80.8%	19.2%	42.5%	57.5%
9	32.2%	21.5%	28.7%	41.9%	58.1%	80.6%	19.4%	41.4%	58.6%
10	31.5%	21.4%	28.3%	41.6%	58.4%	80.5%	19.5%	40.6%	59.4%
11	31.3%	21.5%	28.2%	41.6%	58.4%	80.5%	19.5%	40.5%	59.5%
12	31.3%	21.4%	28.2%	41.5%	58.5%	80.5%	19.5%	40.4%	59.6%
13	31.3%	21.4%	28.2%	41.5%	58.5%	80.5%	19.5%	40.4%	59.6%
14	31.3%	21.4%	28.2%			80.5%	19.5%	40.4%	59.6%

Agenda

Summary

- We derived **closed-form risk estimators** in the multivariate additive loss reserving model that combine
  - evaluation of both reserve and premium risk and their dependencies for all portfolios in one approach,
  - quantification of all risk types for all marginal portfolios as well as the entire business including the risk loading stemming from portfolio dependencies,
  - detailed evolution of all risk components from one-year to ultimo view and extraction of one-year risk at future accounting dates, and
  - decomposition of risk into its sources of process and estimation uncertainty

Advantages and limitations

- **Advantages**
  - Easily computable and interpretable
  - Consistent with distribution-free reserving process
  - Valuable for adequate strategic and regulatory risk modeling
    - Forward looking view in Solvency II
    - Comparative analysis of prescribed and undertaking-specific parameters (e.g. correlation between one-year reserve and premium risk in standard formula)
- **Limitations**
  - No stand-alone risk evaluation
    - No VaR/TVaR or predictive distribution
  - Only valid if model fits the data
    - Uncommon choice for marginal portfolios
    - Restrictive independence assumption
  - Challenging estimation of covariance matrices

Advantages and limitations

- **Future research**
  - Fruitful as starting point for more sophisticated methods
    - Combination with other reserving models
    - Calibration of distributions for CDR
    - Input parameters for simulation-based approaches (stochastic re-reserving)

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