Multi-year non-life insurance risk
A case study

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BELTIOS P&C GmbH

About the speakers (1/2)

Institut für Finanz- und Aktuarwissenschaften (ifa)

- ifa is an independent actuarial consulting firm.
- Our consulting services in all lines of insurance business include:
  - typical actuarial tasks and actuarial modelling
  - insurance product development
  - risk management, Solvency II, asset liability management
  - data analytics
  - market entries (cross-border business, setup of new insurance companies, Fintechs)
  - professional education
  - academic research on actuarial topics of practical relevance
- located in Ulm, Germany
- currently about 30 consultants
- academic cooperation with the University of Ulm (offering the largest actuarial program in Germany)

Lukas J. Hahn
Consultant

- joined ifa in 2013
- main working areas: data analytics, actuarial modelling, Solvency II
- Master of Science (Mathematics and Management, University of Ulm, 2015)
- Master of Mathematics (Statistics, University of Waterloo, 2014)
- Ph.D. candidate (University of Ulm)
- completed qualification to become certified actuary (German Association of Actuaries DAV)

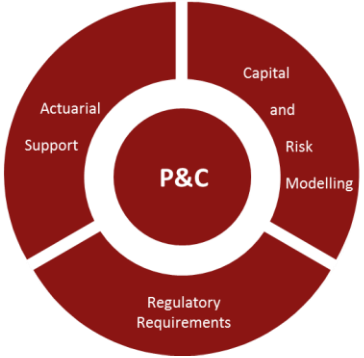
About the speakers (2/2)

Marc Linde
Senior Manager

BELTIOS P&C GmbH

- BELTIOS P&C is an independent actuarial consulting firm founded in 2014 by a well-established team
- Broad range of actuarial services for non-life-insurers in the German-speaking area and the European area
  - Actuarial consultancy in loss reserving and pricing
  - Capital and risk modelling
  - Implementation of regulatory requirements
- Loss Reserving Tool "RESTA"
- Located in Cologne and Munich, Germany

BELTIOS



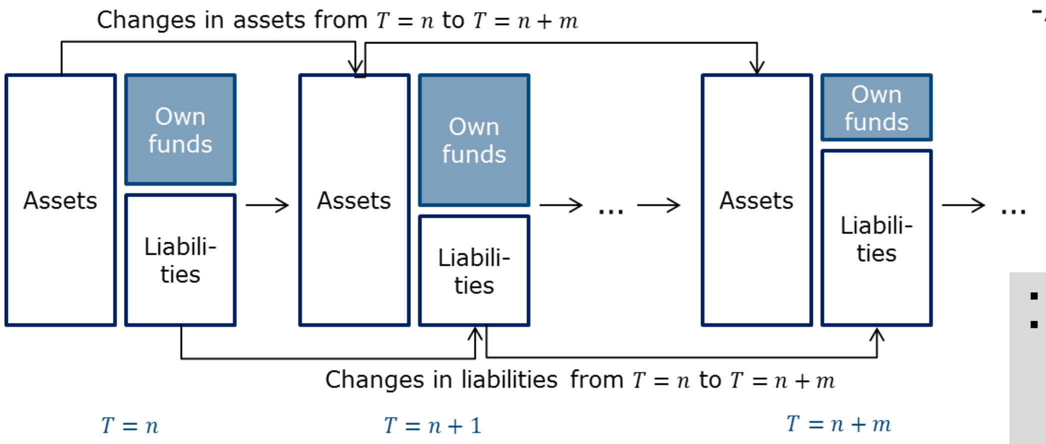
- More than 10 years working experience in the insurance sector
- Main working areas: Loss Reserving, Capital Modelling, Solvency II
- joined BELTIOS P&C in 2014
- Diploma in Mathematics (University of Duisburg-Essen, 2007)
- Certified Non-Life actuary (German Association of Actuaries DAV)
- Member of the working group „Internal models in non-life insurance" within the DAV
- Teacher for the final DAV exam for non-life insurance on „Internal models"

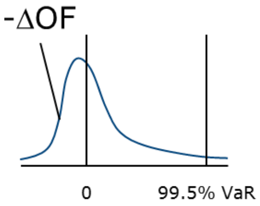
Agenda

- Motivation
  - Multi-year non-life insurance risk
  - Quantification of multi-year non-life insurance risk
- Bootstrap approach
- Case Study
  - Data and Setup
  - Results
- Conclusion

Multi-year non-life insurance risk (1/3)

- Balance sheet approach for risks in a **multi-year view**: change in own funds (OF) over *m* future years



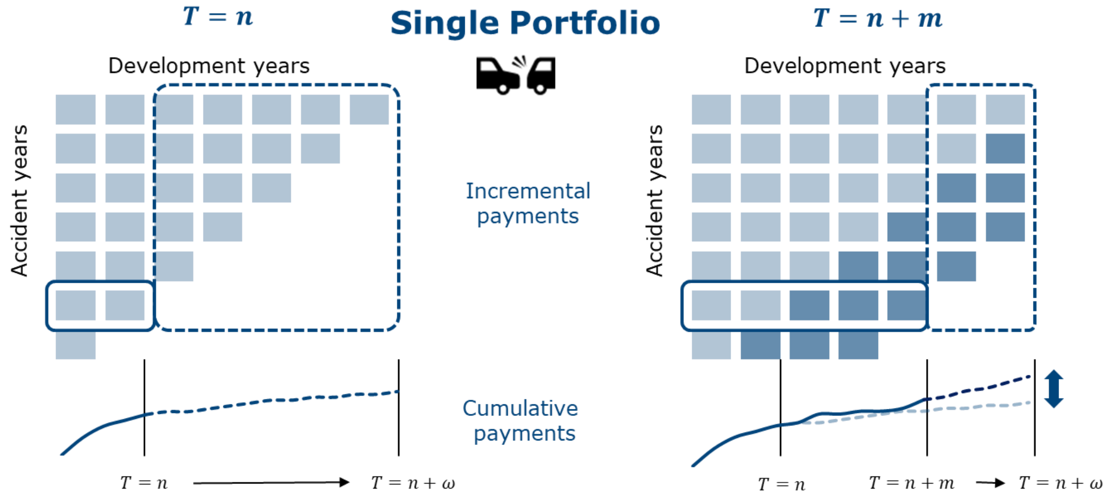


- **Solvency Capital Requirement (SCR)**
- **Own Risk and Solvency Assessment (ORSA)**:
  - overall solvency needs
  - ***undertaking-specific*** risk profile, tolerance, and business plans
  - ***medium or long-term perspective*** (usually horizon of 3-5 years)
- Derivation of risk margins and risk loadings

- Focus on reserve and premium risks

Multi-year non-life insurance risk (2/3)

- **Claims development result** (CDR): difference in best estimate ultimates over time



Reserve risk:
Prior accident years

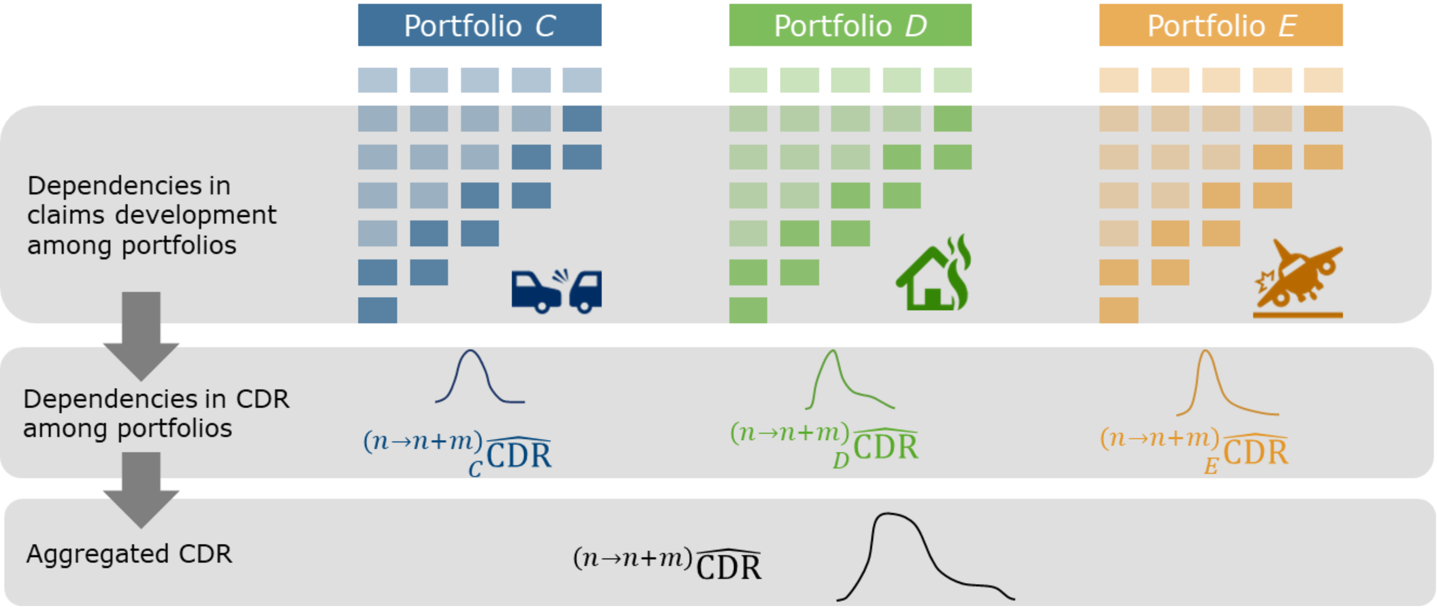
Premium risk:
Future accident years

Non-life insurance risk:
All prior and future accident years

$$^{(n\to n+m)}\widehat{\mathrm{CDR}}$$

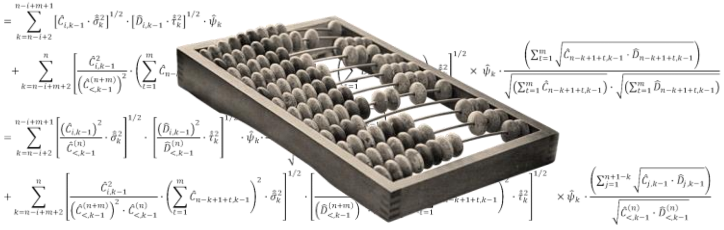
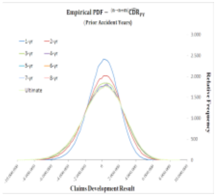
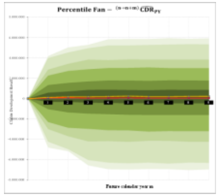
Multi-year non-life insurance risk (3/3)

Multivariate view



Quantification of multi-year non-life insurance risk

Quantification of the aggregated multi-year non-life insurance risk for multiple lines of business

Analytical Approaches	Bootstrap Approaches
	  

Analytical Approaches

- Closed-form analytical formulae
- Only provides moments up to 2nd order, i.e. prediction error (= standard deviation of the predictive distribution)
- Fast computation
- Available for most common reserving models
- Consistent with well-known one-year and ultimo formulae

Bootstrap Approaches

- Simulation-based
- Leads to a full predictive distribution (provides also higher moments and risk measures like VaR and TVaR)
- More time-consuming, subject to simulation error
- Applicable to most common reserving models
- Consistent with well-known bootstrap approaches

Bootstrap Approach (1/10)

- **Model Calibration**
- Bootstrap Process
- Simulation of Future Payments
- Stochastic Re-reserving
- Portfolio Aggregation

- The **reserving methods** used for best-estimate prediction determine the **stochastic model**.
- Focus on two of the most common reserving methods:
  - **C**hain-**L**adder method (CL) $\Longrightarrow$ Mack chain-ladder model
  - **I**ncremental **L**oss **R**atio method (ILR) $\Longrightarrow$ Additive loss reserving model
- Estimate model parameters:
  - CL: Chain-ladder factors, Mack volatility parameters
  - ILR: Incremental loss ratios, volatility parameters
- Predict best estimates for ultimates per accident year.

Bootstrap Approach (2/10)

Bivariate chain-ladder model

Model Calibration

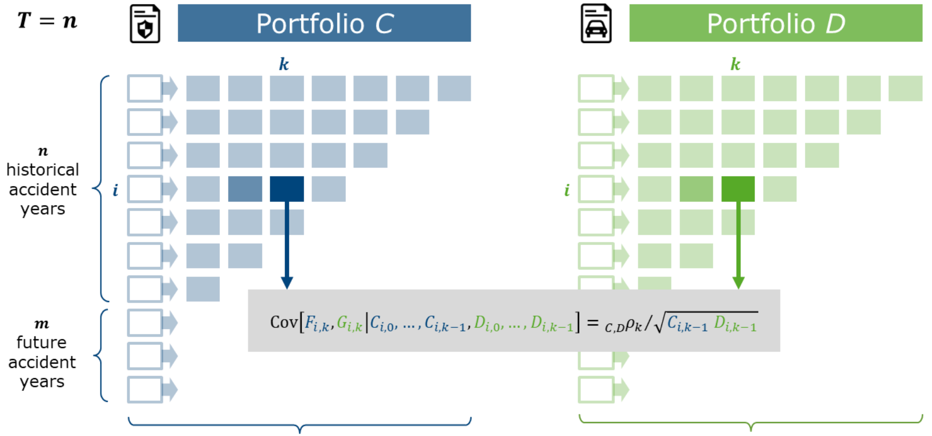
Bootstrap Process

Simulation of Future Payments

Stochastic Re-reserving

Portfolio Aggregation

$T = n$



$$\text{Cov}\left[F_{i,k}, G_{i,k} \middle| C_{i,0}, \dots, C_{i,k-1}, D_{i,0}, \dots, D_{i,k-1}\right] = {}_{C,D}\rho_k / \sqrt{C_{i,k-1}\, D_{i,k-1}}$$

Marginal Mack chain-ladder model
- Independence of accident years
- $\mathbb{E}\left[F_{i,k} \middle| C_{i,0}, \dots, C_{i,k-1}\right] = f_k$
- $\mathbb{V}\left[F_{i,k} \middle| C_{i,0}, \dots, C_{i,k-1}\right] = \sigma_k^2 / C_{i,k-1}$

Marginal Mack chain-ladder model
- Independence of accident years
- $\mathbb{E}\left[G_{i,k} \middle| D_{i,0}, \dots, D_{i,k-1}\right] = g_k$
- $\mathbb{V}\left[G_{i,k} \middle| D_{i,0}, \dots, D_{i,k-1}\right] = \tau_k^2 / D_{i,k-1}$

Bootstrap Approach (3/10)

Bivariate additive reserving model

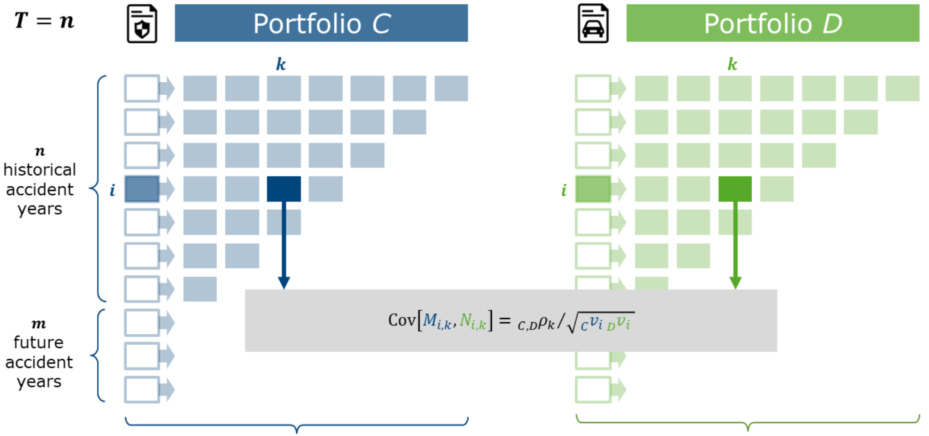
Model Calibration

Bootstrap Process

Simulation of Future Payments

Stochastic Re-reserving

Portfolio Aggregation



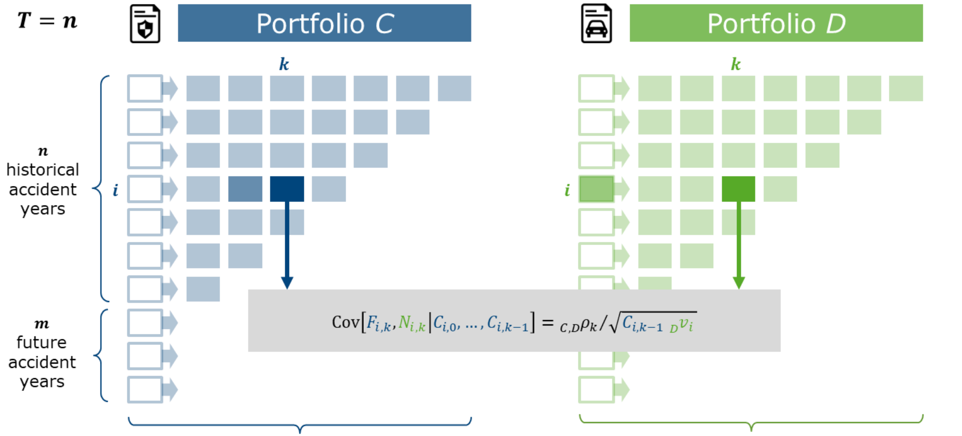
Marginal additive reserving model
- Independence of incremental payments
- $\mathbb{E}[M_{i,k}] = m_k$
- $\mathbb{V}[M_{i,k}] = s_k^2 / {}_C\nu_i$

Marginal additive reserving model
- Independence of incremental payments
- $\mathbb{E}[N_{i,k}] = n_k$
- $\mathbb{V}[N_{i,k}] = t_k^2 / {}_D\nu_i$

Bootstrap Approach (4/10)

Combination chain-ladder and additive models

- **Model Calibration**
- Bootstrap Process
- Simulation of Future Payments
- Stochastic Re-reserving
- Portfolio Aggregation



$$\mathrm{Cov}\left[F_{i,k}, N_{i,k} \middle| C_{i,0}, \dots, C_{i,k-1}\right] = {}_{C,D}\rho_k / \sqrt{C_{i,k-1}\ {}_{D}v_i}$$

Marginal Mack chain-ladder model
- Independence of accident years
- $\mathbb{E}\left[F_{i,k} \middle| C_{i,0}, \dots, C_{i,k-1}\right] = f_k$
- $\mathbb{V}\left[F_{i,k} \middle| C_{i,0}, \dots, C_{i,k-1}\right] = \sigma_k^2 / C_{i,k-1}$

Marginal additive reserving model
- Independence of incremental payments
- $\mathbb{E}\left[N_{i,k}\right] = n_k$
- $\mathbb{V}\left[N_{i,k}\right] = t_k^2 / {}_{D}v_i$

Bootstrap Approach (5/10)

Combination chain-ladder and additive models

Model Calibration

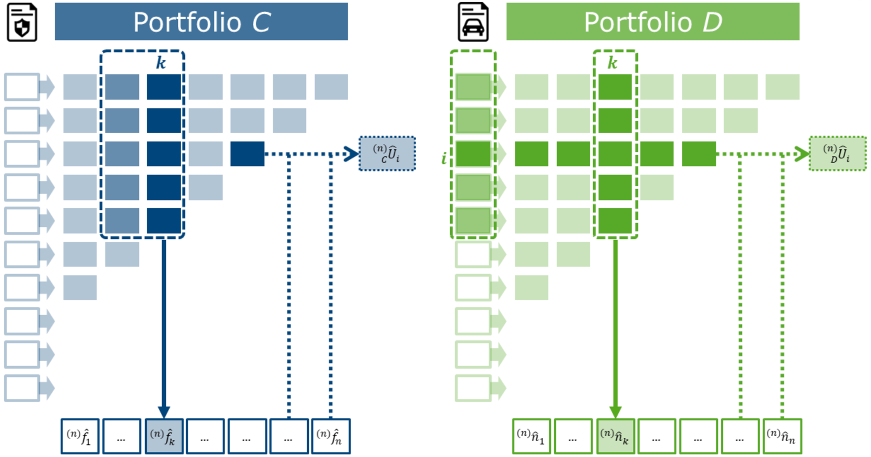
Bootstrap Process

Simulation of Future Payments

Stochastic Re-reserving

Portfolio Aggregation

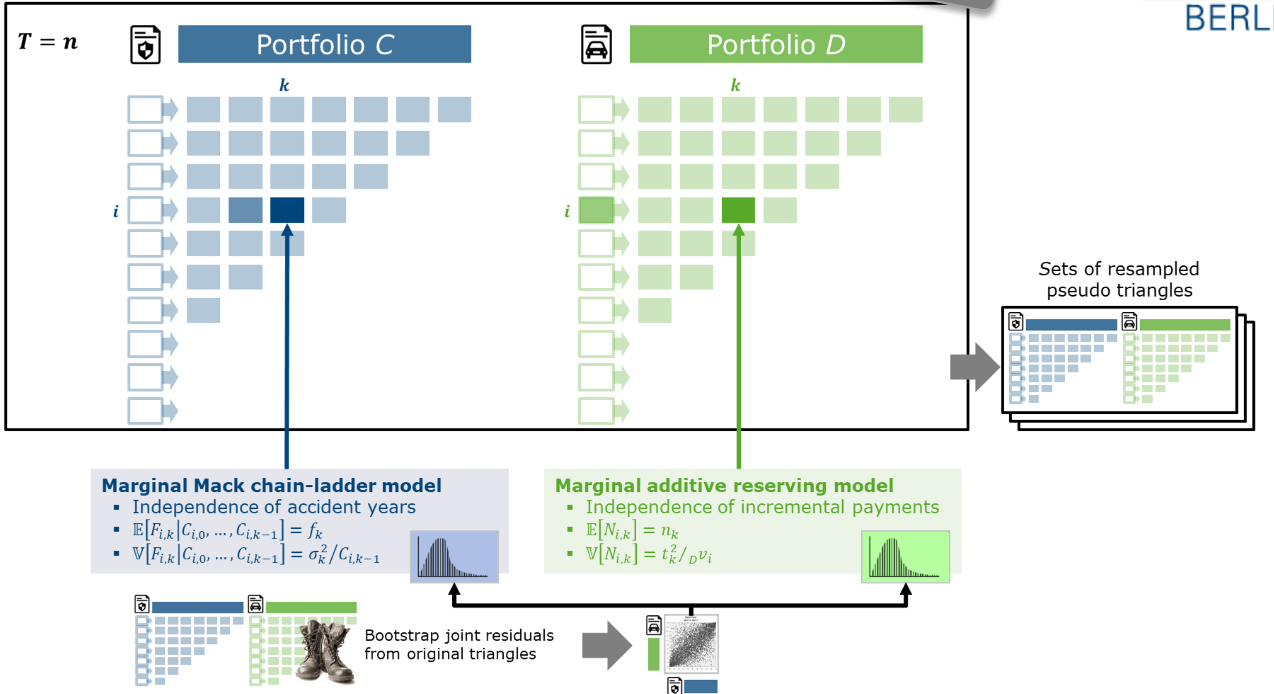
$T = n$



Bootstrap Approach (6/10)

Combination chain-ladder and additive models

- Model Calibration
- **Bootstrap Process**
- Simulation of Future Payments
- Stochastic Re-reserving
- Portfolio Aggregation



Bootstrap Approach (7/10)

Combination chain-ladder and additive models

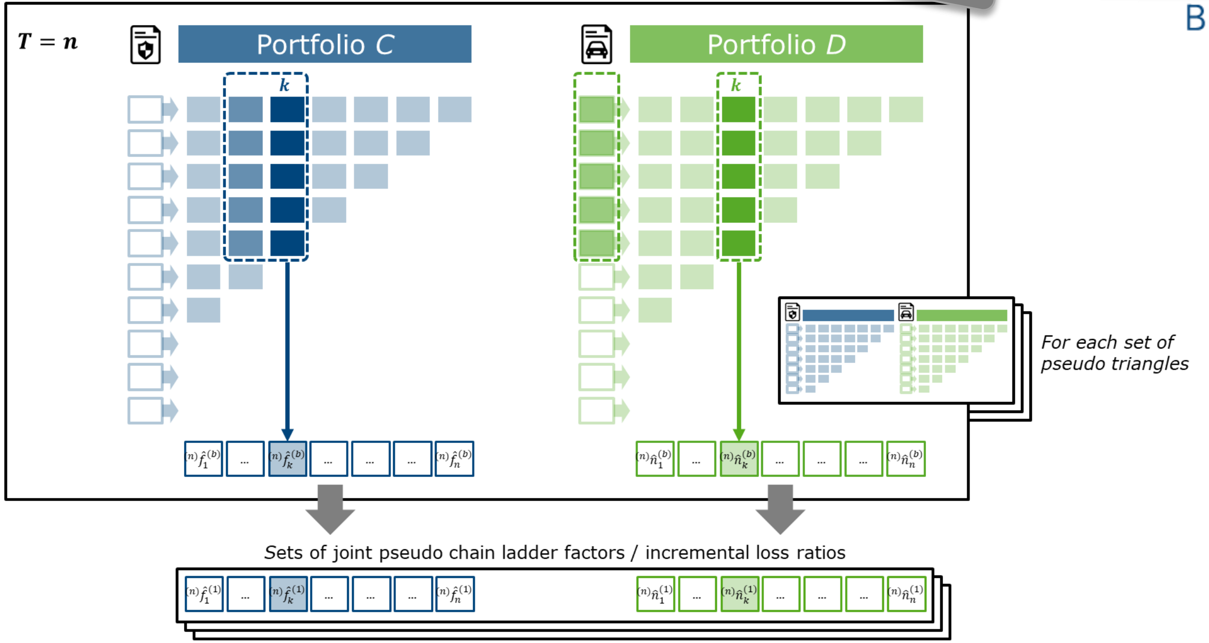
Model Calibration

Bootstrap Process

Simulation of Future Payments

Stochastic Re-reserving

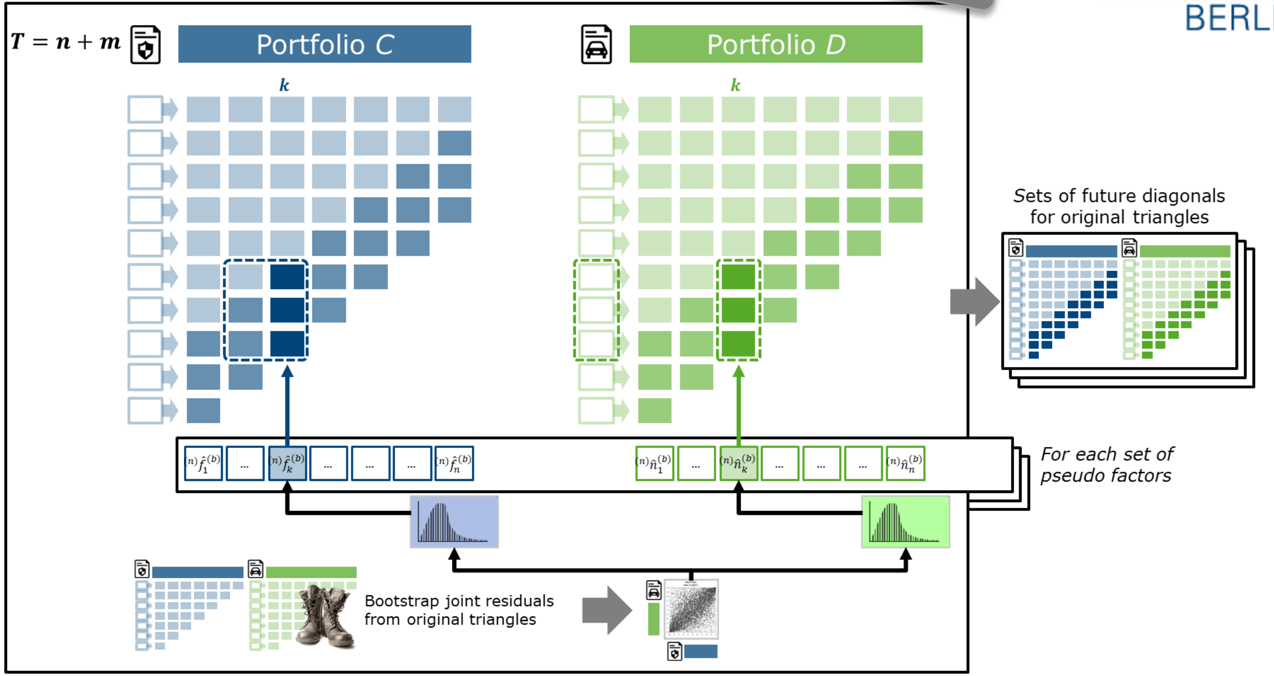
Portfolio Aggregation



Bootstrap Approach (8/10)

Combination chain-ladder and additive models

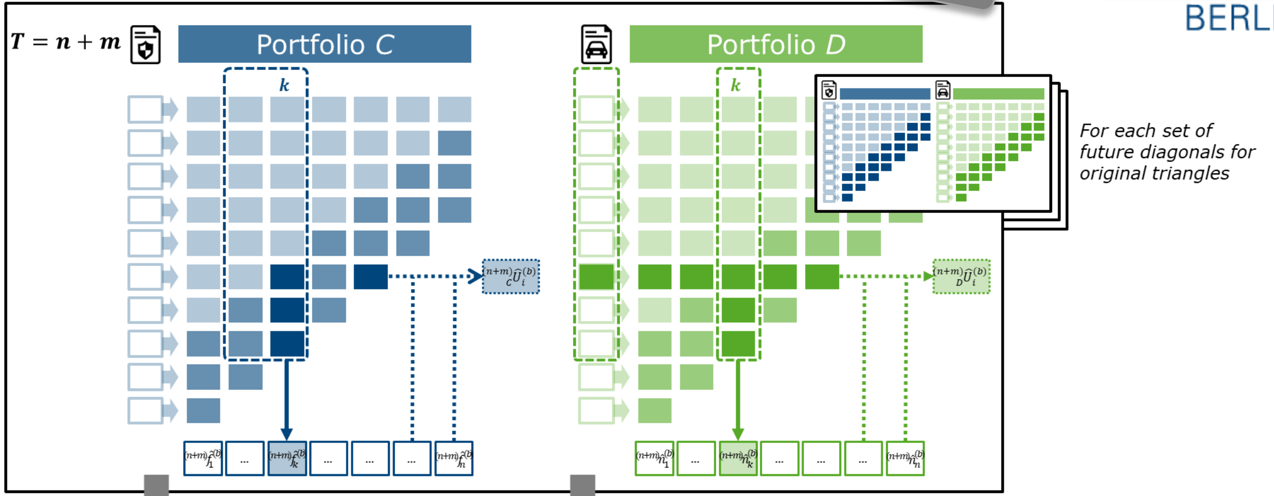
- Model Calibration
- Bootstrap Process
- **Simulation of Future Payments**
- Stochastic Re-reserving
- Portfolio Aggregation



Bootstrap Approach (9/10)

Combination chain-ladder and additive models

- Model Calibration
- Bootstrap Process
- Simulation of Future Payments
- **Stochastic Re-reserving**
- Portfolio Aggregation



Sets of joint best estimate ultimates at $T = n + m$

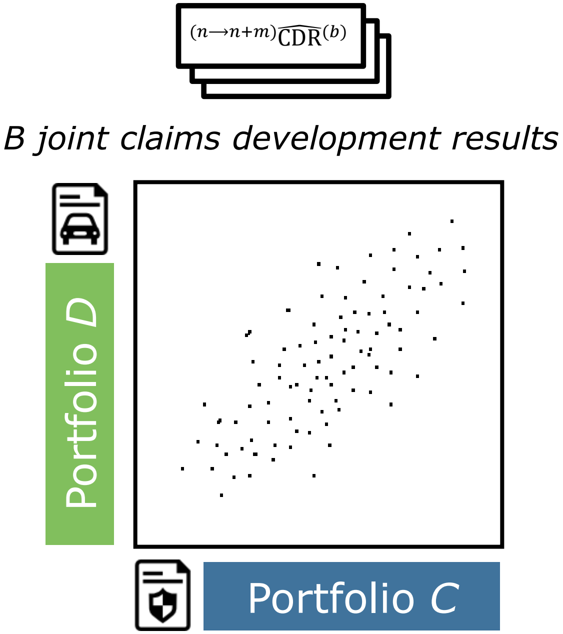
${}^{(n+m)}_{C}\widehat{U}_i^{(b)}$ ${}^{(n+m)}_{D}\widehat{U}_i^{(b)}$ for all accident years $i = 1, \dots, n + m$

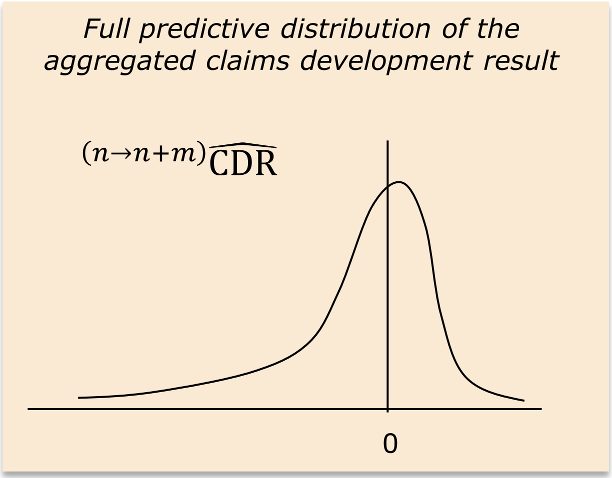
Sets of m-year claims development results

${}^{(n\to n+m)}_{C}\widehat{\mathrm{CDR}}^{(b)}$ ${}^{(n\to n+m)}_{D}\widehat{\mathrm{CDR}}^{(b)}$ with ${}^{(n\to n+m)}_{p}\widehat{\mathrm{CDR}}^{(b)} := \sum_i \left({}^{(n)}_{p}\widehat{U}_i - {}^{(n+m)}_{p}\widehat{U}_i^{(b)}\right)$ per portfolio $p = C, D$

Bootstrap Approach (10/10)

- Model Calibration
- Bootstrap Process
- Simulation of Future Payments
- Stochastic Re-reserving
- **Portfolio Aggregation**





Case study: Data and Setup (1/3)

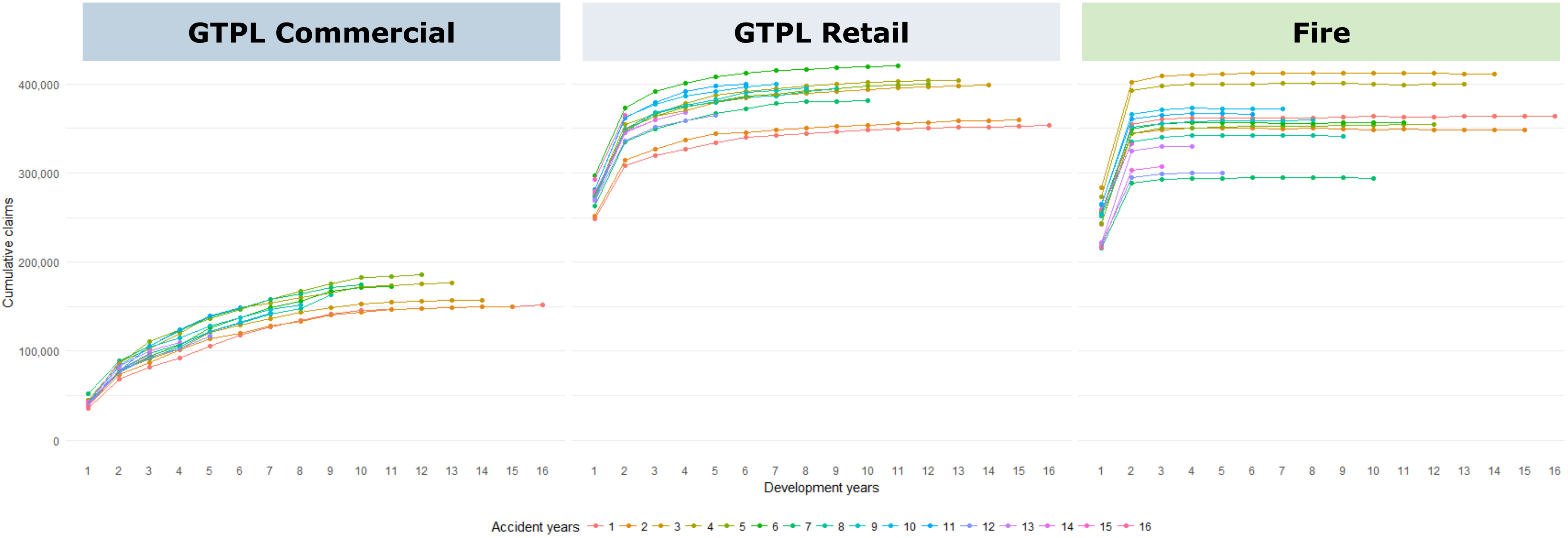
Fictional German non-life insurance company with **three portfolios** within **two lines of business**:

Line of business	General Third Party Liability (GTPL)		Fire
Portfolio	**Commercial**	**Retail**	**Fire**
Characteristics	▪ long tail ▪ uncertainty mainly driven by long-term development of claims	▪ medium tail ▪ uncertainty jointly driven by short- and long-term types of claims	▪ short tail ▪ uncertainty mainly driven by occurrence of large short-term claims
Reserving method	**Incremental Loss Ratio**	**Chain-Ladder**	

- **Model:** joint stochastic model through pairwise bivariate additive loss reserving / Mack chain-ladder models
- **Data:** paid claims triangles and number of contracts for accident years 2002-2017 based on realistic data
- **Horizon:** one year of future business
- **Simulation:** B=10’000 samples, log-normal marginals, Gaussian copula

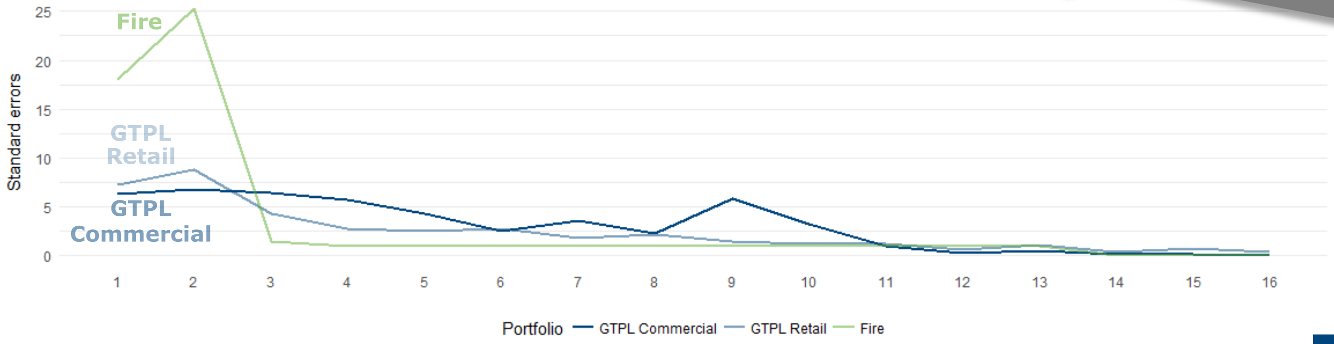
Case study: Data and Setup (2/3)

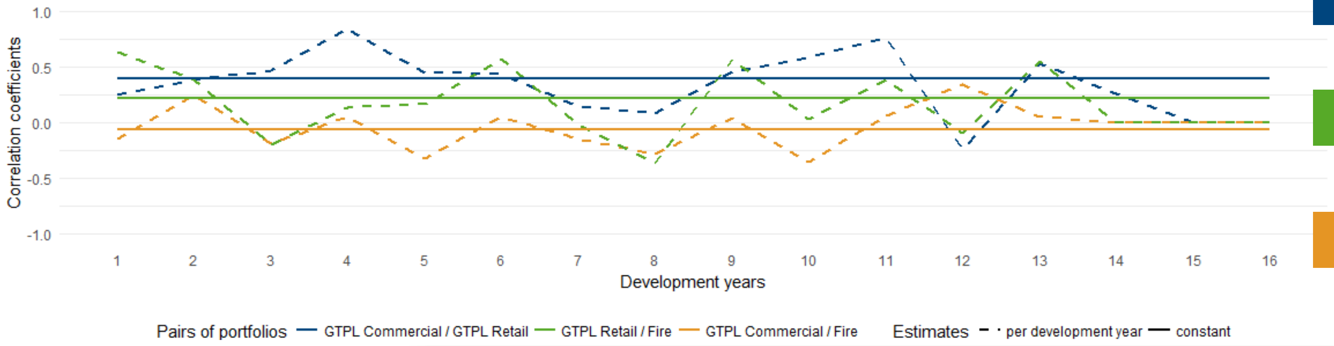
Development patterns



Case study: Data and Setup (3/3)

Second moment estimates





GTPL Commercial / GTPL Retail

40.11%

GTPL Retail / Fire

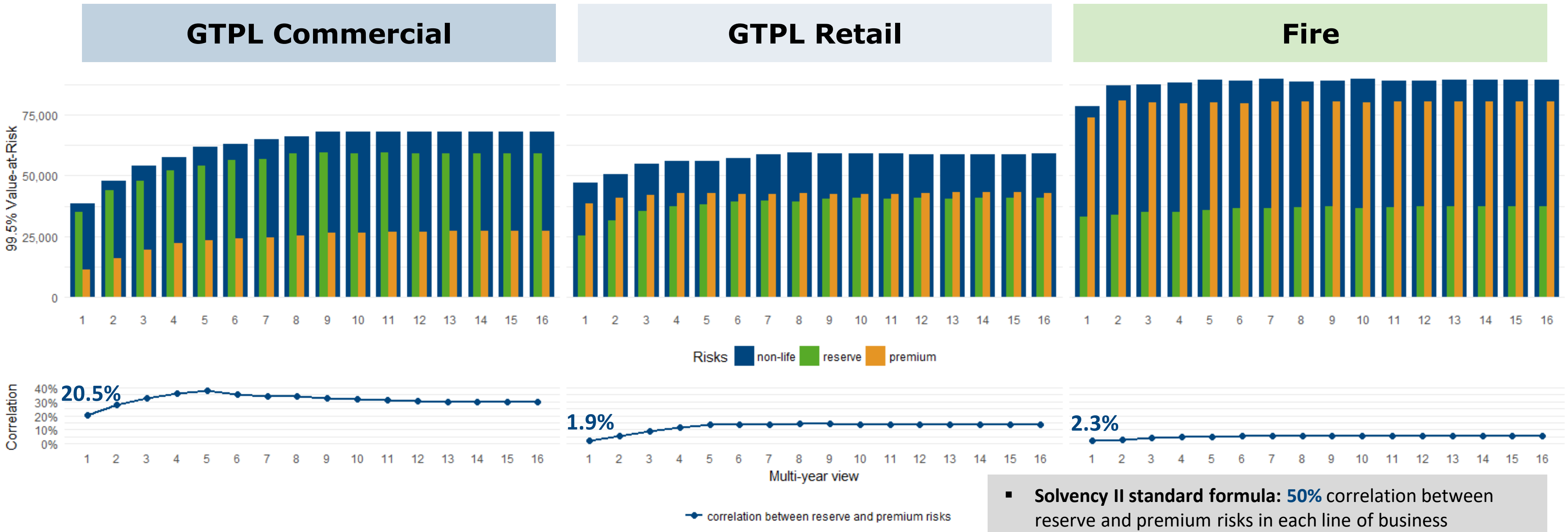
21.64%

GTPL Commercial / Fire

-5.61%

Case study: Results (1/5)

Univariate results

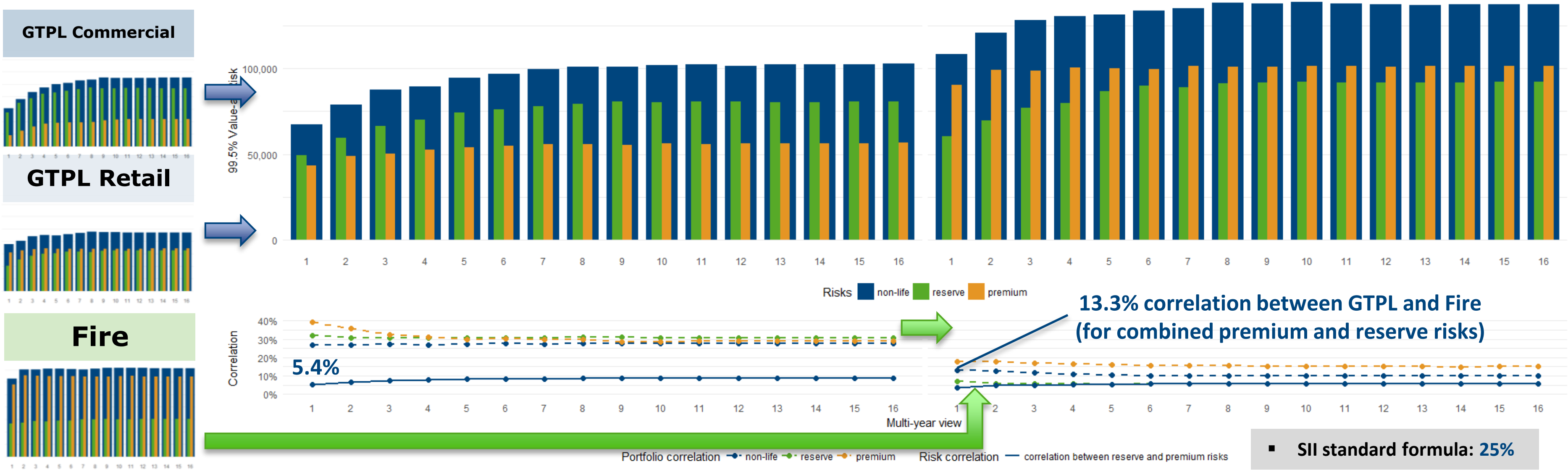


- **Solvency II standard formula: 50%** correlation between reserve and premium risks in each line of business

Case study: Results (2/5)

Portfolio aggregation

General Third Party Liability (GTPL) | **Company (GTPL & Fire)**

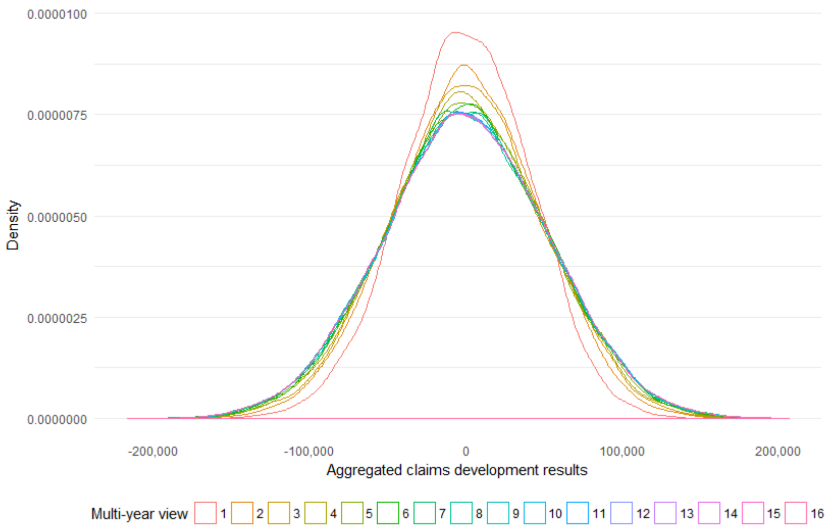


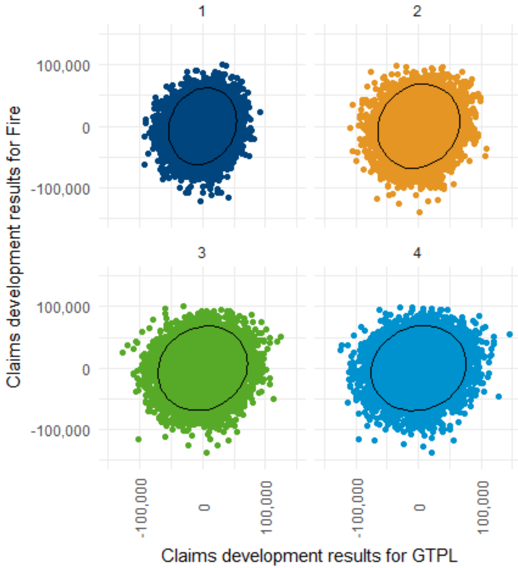
- SII standard formula: 25%

Case study: Results (3/5)

Full predictive distributions

Company level (GTPL & Fire)





Case study: Results (4/5)

Comparison of SCR for different approaches

ICA CIA BERLIN 2018

m	Parametric (lognormal/ Gaussian)*	Parametric (normal/ Gaussian)*	Non-parametric	Analytical**	Solvency II standard formula***
1	108.483	105.476	95.913	105.843	576.209
2	121.048	119.596	115.334	120.644	
3	128.182	126.741	119.625	126.199	
4	130.661	126.226	125.087	129.911	
5	131.421	128.994	127.447	132.497	
6	134.003	131.342	126.674	134.373	
7	135.562	132.646	128.204	135.779	
8	138.748	133.757	129.403	136.800	
9	138.105	135.357	130.567	137.558	
10	139.118	136.292	131.326	137.853	
11	138.337	135.668	131.210	137.976	
12	137.821	135.659	131.807	138.040	
13	137.407	136.098	130.703	138.084	
14	137.860	136.430	130.997	138.095	
15	137.709	135.571	131.149	138.102	
16	137.802	135.341	131.306	138.103	

Prediction errors for $m = 1$	Parametric (lognormal/ Gaussian)*	Parametric (normal/ Gaussian)*	Non-parametric	Analytical**	Solvency II standard formula***
Company level	40.899	40.899	40.947	41.024	192.070
Correlation GTPL / Fire	13,3%	13,3%	13,7%	12,6%	25,0%
Aggregated risk in GTPL	25.231	25.256	25.166	25.379	178.397
Corr. res/pre risk in GTPL	5,4%	5,3%	6,8%	7,2%	50,0%
Reserve risk in GTPL	17.997	18.039	17.826	17.955	95.001
Premium risk in GTPL	16.736	16.746	16.597	16.695	110.792
Aggregated risk in Fire	29.013	28.977	29.034	29.186	39.392
Corr. res/pre risk in Fire	2,3%	2,3%	4,2%	2,9%	50,0%
Reserve risk in Fire	12.860	12.855	12.797	12.904	9.851
Premium risk in Fire	25.714	25.673	25.526	25.807	33.531

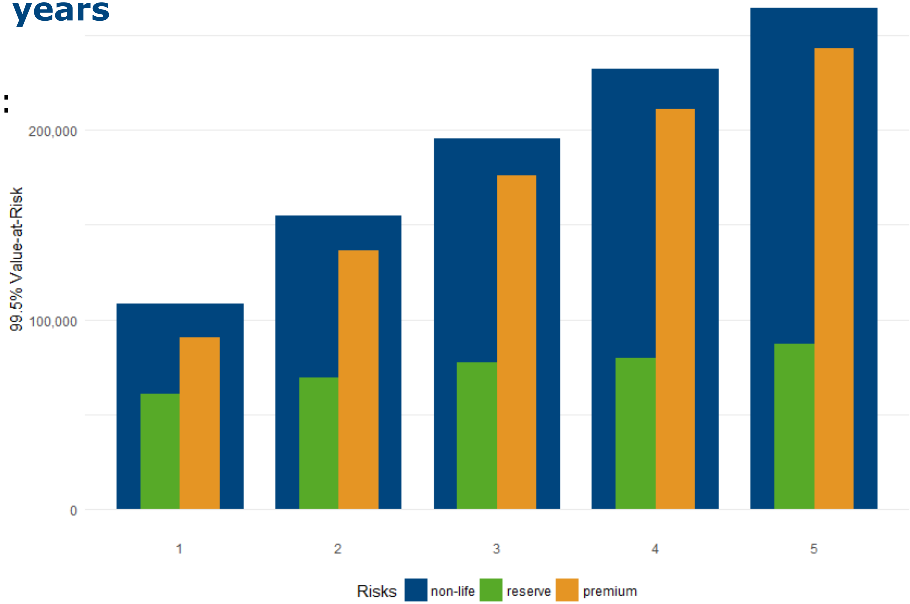
* Both parametric bootstraps are based on the same realizations of independent uniforms.
** The distribution-free analytical approach does not yield any higher moments than the estimate for the prediction error. The SCR is calculated as 2.58 times the prediction error, i.e. the 99.5% quantile under a normal assumption motivated by the bootstrap predictive distribution.
*** We use the best estimate reserves from the marginal reserving methods as the volumes for reserve risk. For the volume underlying the premium risk, we assume future earned premiums to be the historic average net premium multiplied by a cost, safety and profit margin of 30%.

Case study: Results (5/5)

- **ORSA process:** results for a business horizon of **five years**

- Assumptions on risk profile, tolerance, business plans:
  - linearly growing business in both GTPL portfolios
  - stable volumes in Fire portfolio

- Allows to
  - estimate multi-year overall solvency needs,
  - derive future one-year capital requirements (rolling-forward definition of reserve and premium risks),
  - calculate risk margins and safety loadings,
  - perform sensitivity analyses.



Conclusion

Practical value of bootstrap approach

The **bootstrap approach** allows to quantify premium and reserve risk

- among **lines of business** (or more granular segments)
- through various **risk measures**,
- over a time horizon of **multiple future accounting years**.

➔ input for **overall solvency needs** in the **ORSA process**

➔ understanding of how **dependencies** between the occurrence and settlement of claims among different loss portfolios influence the aggregated risk capital

➔ **suitability assessment** of the standard formula

➔ indicator for possible benefits from **undertaking-specific parameters**

➔ derivation of **risk capitals in future one-year view** to compute a risk margin under a run-off scenario

Selected features of implementation in R

- **chain-ladder** and **additive loss reserving models** subject to development year correlations
  - combinations and generalized versions possible
- **non-parametric** and **parametric** bootstraps/simulations of historic and future triangles
  - different marginals and copulae for parametric approach
- various analyses based on **full predictive distributions**
  - risk by accident years, reserve and premium risks
  - flexible (e.g. pairwise or complete) portfolio aggregations
  - split into estimation and process error
  - m-year and (updated) future one-year view
  - extensive plotting functions (model diagnostics, results)
- **closed-form analytical estimators** for benchmarking

Thank you very much for your attention!

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Appendix

CorrS	1	2	3	4	5	6	7	8	9	10	11
1: Motor vehicle liability	1										
2: Other motor	0,5	1									
3: MAT	0,5	0,25	1								
4: Fire	0,25	0,25	0,25	1							
5: 3rd party liability	0,5	0,25	0,25	0,25	1						
6: Credit	0,25	0,25	0,25	0,25	0,5	1					
7: Legal exp.	0,5	0,5	0,25	0,25	0,5	0,5	1				
8: Assistance	0,25	0,5	0,5	0,5	0,25	0,25	0,25	1			
9: Miscellaneous.	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	1		
10: Np reins. (casualty)	0,25	0,25	0,25	0,25	0,5	0,5	0,5	0,25	0,25	[illegible]	
11: Np reins. (MAT)	0,25	0,25	0,5	0,5	0,25	0,25	0,25	0,25	0,5	[illegible]	[illegible]
12: Np reins. (property)	0,25	0,25	0,25	0,5	0,25	0,25	0,25	0,5	[illegible]	[illegible]	[illegible]

Premium and Reserve Risk
$SCR_{PR} = 3 \cdot \sigma_{nl} \cdot V_{nl}$

- 99.5% VaR approximation (under normal assumption)

Aggregated risk factor
σ_{NL}

Correlation matrix
- Volume-weighted aggregation of risk factors of joint premium-reserve risk over all LoBs with a covariance approach
- Pre-determined correlation matrix

Aggregated Volume
$V_{nl} = \Sigma_s V_s$

- Summation

Risk factor per LoB
σ_s

Correlation=50%
- Volume-weighted aggregation of the risk factors for premium and reserve risks within a single LoB with a covariance approach
- Unique correlation of 50% between premium and reserve risks within each LoB

Diversified Exposure per LoB
$V_s = \left(V_{(prem,s)} + V_{(res,s)}\right) \cdot (0{,}75 + 0{,}25 \cdot DIV_s)$

- Aggregation through summation
- Geographical diversification depending on the business decomposition into regions

Risk factor of Premium Risk for LoB *s*
$\sigma_{(prem,s)}$
*Pre-determined parameter**

Risk factor of Reserve risk for LoB *s*
$\sigma_{(res,s)}$
*Pre-determined parameter**

Volume of Premium Risk for LoB *s*
$V_{(prem,s)}$
Future earned premium

Volume of Reserve Risk for LoB *s*
$V_{(res,s)}$
Outstanding claims reserve

**may be replaced by undertaking specific parameters (USP) using standardized methods upon regulator's approval*