Multi-year non-life insurance risk
A case study

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BELTIOS P&C GmbH
About the speakers (1/2)

Institut für Finanz- und Aktuarwissenschaften (ifa)

- ifa is an independent actuarial consulting firm.
- Our consulting services in all lines of insurance business include:
  - typical actuarial tasks and actuarial modelling
  - insurance product development
  - risk management, Solvency II, asset liability management
  - data analytics
  - market entries (cross-border business, setup of new insurance companies, Fintechs)
  - professional education
  - academic research on actuarial topics of practical relevance
- located in Ulm, Germany
- currently about 30 consultants
- academic cooperation with the University of Ulm (offering the largest actuarial program in Germany)

Lukas J. Hahn
Consultant

- joined ifa in 2013
- main working areas: data analytics, actuarial modelling, Solvency II
- Master of Science (Mathematics and Management, University of Ulm, 2015)
- Master of Mathematics (Statistics, University of Waterloo, 2014)
- Ph.D. candidate (University of Ulm)
- completed qualification to become certified actuary (German Association of Actuaries DAV)
About the speakers (2/2)

BELTIOS P&C GmbH

- BELTIOS P&C is an independent actuarial consulting firm founded in 2014 by a well-established team
- Broad range of actuarial services for non-life-insurers in the German-speaking area and the European area
  - Actuarial consultancy in loss reserving and pricing
  - Capital and risk modelling
  - Implementation of regulatory requirements
- Loss Reserving Tool “RESTA”
- Located in Cologne and Munich, Germany

Marc Linde
Senior Manager

- More than 10 years working experience in the insurance sector
- Main working areas: Loss Reserving, Capital Modelling, Solvency II
- joined BELTIOS P&C in 2014
- Diploma in Mathematics (University of Duisburg-Essen, 2007)
- Certified Non-Life actuary (German Association of Actuaries DAV)
- Member of the working group „Internal models in non-life insurance“ within the DAV
- Teacher for the final DAV exam for non-life insurance on „Internal models“
Agenda

- Motivation
  - Multi-year non-life insurance risk
  - Quantification of multi-year non-life insurance risk

- Bootstrap approach

- Case Study
  - Data and Setup
  - Results

- Conclusion
Multi-year non-life insurance risk (1/3)

- Balance sheet approach for risks in a multi-year view: change in own funds (OF) over $m$ future years

- Focus on reserve and premium risks

- Solvency Capital Requirement (SCR)
- Own Risk and Solvency Assessment (ORSA):
  - overall solvency needs
  - undertaking-specific risk profile, tolerance, and business plans
  - medium or long-term perspective (usually horizon of 3-5 years)
- Derivation of risk margins and risk loadings
Multi-year non-life insurance risk (2/3)

- **Claims development result** (CDR): difference in best estimate ultimates over time

\[
T = n 
\]

\[
T = n + m 
\]

**Single Portfolio**

- **Reserve risk**: Prior accident years
- **Premium risk**: Future accident years
- **Non-life insurance risk**: All prior and future accident years

\[
(n \rightarrow n + m)_{\text{CDR}}
\]
Multi-year non-life insurance risk (3/3)

Dependencies in claims development among portfolios:
- Portfolio C
- Portfolio D
- Portfolio E

Dependencies in CDR among portfolios:
- $C^{CDR}_{n\rightarrow n+m}$
- $D^{CDR}_{n\rightarrow n+m}$
- $E^{CDR}_{n\rightarrow n+m}$

Aggregated CDR:
- $C^{CDR}_{n\rightarrow n+m}$

Multivariate view
Quantification of multi-year non-life insurance risk

Quantification of the aggregated multi-year non-life insurance risk for multiple lines of business

Analytical Approaches

- Closed-form analytical formulae
- Only provides moments up to 2nd order, i.e. prediction error (= standard deviation of the predictive distribution)
- Fast computation
- Available for most common reserving models
- Consistent with well-known one-year and ultimo formulae

Bootstrap Approaches

- Simulation-based
- Leads to a full predictive distribution (provides also higher moments and risk measures like VaR and TVaR)
- More time-consuming, subject to simulation error
- Applicable to most common reserving models
- Consistent with well-known bootstrap approaches
Bootstrap Approach (1/10)

- The **reserving methods** used for best-estimate prediction determine the **stochastic model**.

- Focus on two of the most common reserving methods:
  - **Chain-Ladder method (CL)** $\implies$ Mack chain-ladder model
  - **Incremental Loss Ratio method (ILR)** $\implies$ Additive loss reserving model

- Estimate model parameters:
  - CL: Chain-ladder factors, Mack volatility parameters
  - ILR: Incremental loss ratios, volatility parameters

- Predict best estimates for ultimates per accident year.
Bootstrap Approach (2/10)

Model Calibration

Bootstrap Process

Simulation of Future Payments

Stochastic Re-reserving

Portfolio Aggregation

Portfolio C

Portfolio D

Marginal Mack chain-ladder model
- Independence of accident years
  - $\mathbb{E}[\hat{c}_{k|i}|c_{i0}, ..., c_{i,k-1}, d_{i0}, ..., d_{i,k-1}] = f_k$
  - $\mathbb{V}[\hat{c}_{k|i}|c_{i0}, ..., c_{i,k-1}] = a_k^2/C_{i,k-1}$

Marginal Mack chain-ladder model
- Independence of accident years
  - $\mathbb{E}[\hat{c}_{k|i}|c_{i0}, ..., c_{i,k-1}] = g_k$
  - $\mathbb{V}[\hat{c}_{k|i}|c_{i0}, ..., c_{i,k-1}] = r_k^2/D_{i,k-1}$

$T = n$

$n$ historical accident years

$m$ future accident years

$\text{Cov}[\hat{c}_{i,k}, \hat{c}_{j,k}|c_{i0}, ..., c_{i,k-1}, d_{i0}, ..., d_{i,k-1}] = c_D \rho_{ij} \sqrt{C_{i,k-1} D_{j,k-1}}$
Bootstrap Approach (3/10)

Model Calibration

Bootstrap Process

Simulation of Future Payments

Stochastic Re-reserving

Portfolio Aggregation

Portfolio C

Portfolio D

$T = n$

$n$ historical accident years

$m$ future accident years

$\text{Cov}[M_{i,k}, N_{i,k}] = c_{i} \sigma_{k} \sqrt{\psi_{i} \mu_{i}}$

Marginal additive reserving model

- Independence of incremental payments
- $\mathbb{E}[M_{i,k}] = m_{k}$
- $\mathbb{V}[M_{i,k}] = s_{i}^{2} / c_{i} \psi_{i}$

Marginal additive reserving model

- Independence of incremental payments
- $\mathbb{E}[N_{i,k}] = n_{k}$
- $\mathbb{V}[N_{i,k}] = t_{i}^{2} / d_{i} \psi_{i}$
Bootstrap Approach (4/10)

Portfolio C

Portfolio D

Model Calibration

Bootstrap Process

Simulation of Future Payments

Stochastic Re-reserving

Portfolio Aggregation

Marginal Mack chain-ladder model
- Independence of accident years
- Cov\(R_{ik}, N_{ik}|C_{ik-1} = f_k\)
- \(\text{Var}\{R_{ik}|C_{ik-1} = \sigma_k^2/C_{ik-1}\}

Marginal additive reserving model
- Independence of incremental payments
- \(\text{E}\{N_{ik}\} = n_k\)
- \(\text{Var}\{N_{ik}\} = \gamma_k/n_k^2\)
Bootstrap Approach (5/10)

Model Calibration

Portfolio Aggregation

Bootstrap Process

Simulation of Future Payments

Stochastic Re-reserving

Portfolio Aggregation

\[ T = n \]

Portfolio C

Portfolio D

Combination chain-ladder and additive models
Bootstrap Approach (6/10)

Portfolio Aggregation

Bootstrap Process

Stochastic Re-reserving

Model Calibration

1. Portfolio C

- Marginal Mack chain-ladder model
  - Independence of accident years
  - $\mathbb{E}[C_{ik}|C_{i0}, \ldots, C_{i(k-1)}] = f_k$
  - $\mathbb{V}[C_{ik}|C_{i0}, \ldots, C_{i(k-1)}] = \sigma_k^2 / C_{i(k-1)}$

2. Portfolio D

- Marginal additive reserve model
  - Independence of incremental payments
  - $\mathbb{E}[N_{ik}] = n_k$
  - $\mathbb{V}[N_{ik}] = t_k^2 / \delta_k$

Sets of resampled pseudo triangles

Combination chain-ladder and additive models

Bootstrap joint residuals from original triangles
Bootstrap Approach (7/10)

Portfolio Aggregation

Model Calibration

Bootstrap Process

Simulation of Future Payments

Stochastic Re-reserving

Combination chain-ladder and additive models

For each set of pseudo triangles

Sets of joint pseudo chain ladder factors / incremental loss ratios

$T = n$
Bootstrap Approach (8/10)

Model Calibration

Bootstrap Process

Simulation of Future Payments

Stochastic Re-reserving

Portfolio Aggregation
Bootstrap Approach (9/10)

Model Calibration

Bootstrap Process

Simulation of Future Payments

Stochastic Re-reserving

Portfolio Aggregation

Combination chain-ladder and additive models

For each set of future diagonals for original triangles

Sets of joint best estimate ultimates at $T = n + m$

Sets of $m$-year claims development results

with $\sum_{i=1}^{n} \sum_{j=1}^{m} A_{ij} = \sum_{i=1}^{n} (A_{ij} - T_{ij})^{(C)}$ per portfolio $p = C, D$
Bootstrap Approach (10/10)

- Model Calibration
- Bootstrap Process
- Stochastic Re-reserving
- Portfolio Aggregation

B joint claims development results

Portfolio D

Portfolio C

Full predictive distribution of the aggregated claims development result

\[(n \rightarrow n + m) CDR^R\]
Case study: Data and Setup (1/3)

Fictional German non-life insurance company with three portfolios within two lines of business:

<table>
<thead>
<tr>
<th>Line of business</th>
<th>General Third Party Liability (GTPL)</th>
<th>Fire</th>
</tr>
</thead>
<tbody>
<tr>
<td>Portfolio</td>
<td>Commercial</td>
<td>Retail</td>
</tr>
<tr>
<td>Characteristics</td>
<td>▪ long tail</td>
<td>▪ medium tail</td>
</tr>
<tr>
<td></td>
<td>▪ uncertainty mainly driven by long-term development of claims</td>
<td>▪ uncertainty jointly driven by short- and long-term types of claims</td>
</tr>
</tbody>
</table>

- **Model**: joint stochastic model through pairwise bivariate additive loss reserving / Mack chain-ladder models
- **Data**: paid claims triangles and number of contracts for accident years 2002-2017 based on realistic data
- **Horizon**: one year of future business
- **Simulation**: $B=10’000$ samples, log-normal marginals, Gaussian copula
Case study: Data and Setup (2/3)

GTPL Commercial

GTPL Retail

Fire
Case study: Data and Setup (3/3)

GTPL Commercial / GTPL Retail
40.11%

GTPL Retail / Fire
21.64%

GTPL Commercial / Fire
-5.61%
Case study: Results (1/5)

- **GTPL Commercial**
- **GTPL Retail**
- **Fire**

- **Solvency II standard formula:** 50% correlation between reserve and premium risks in each line of business.
Case study: Results (2/5)

General Third Party Liability (GTPL)

Company (GTPL & Fire)

13.3% correlation between GTPL and Fire (for combined premium and reserve risks)

5.4%

- SII standard formula: 25%
Case study: Results (3/5)

Company level (GTPL & Fire)
## Case study: Results (4/5)

### Prediction errors for $m = 1$

<table>
<thead>
<tr>
<th>Prediction errors for</th>
<th>Parametric (lognormal/Gaussian)*</th>
<th>Parametric (normal/Gaussian)*</th>
<th>Non-parametric</th>
<th>Analytical**</th>
<th>Solvency II standard formula***</th>
</tr>
</thead>
<tbody>
<tr>
<td>Company level</td>
<td>40.899</td>
<td>40.947</td>
<td>41.024</td>
<td>192.070</td>
<td></td>
</tr>
<tr>
<td>Correlation GTPL / Fire</td>
<td>13.3%</td>
<td>13.7%</td>
<td>12.6%</td>
<td>25.0%</td>
<td></td>
</tr>
<tr>
<td>Aggregated risk in GTPL</td>
<td>25.231</td>
<td>25.166</td>
<td>25.379</td>
<td>178.397</td>
<td></td>
</tr>
<tr>
<td>Corr. res/pre risk in GTPL</td>
<td>5.4%</td>
<td>6.8%</td>
<td>7.2%</td>
<td>50.0%</td>
<td></td>
</tr>
<tr>
<td>Reserve risk in GTPL</td>
<td>17.997</td>
<td>17.826</td>
<td>17.955</td>
<td>95.001</td>
<td></td>
</tr>
<tr>
<td>Premium risk in GTPL</td>
<td>16.736</td>
<td>16.597</td>
<td>16.695</td>
<td>110.792</td>
<td></td>
</tr>
<tr>
<td>Aggregated risk in Fire</td>
<td>29.013</td>
<td>29.034</td>
<td>29.186</td>
<td>39.392</td>
<td></td>
</tr>
<tr>
<td>Corr. res/pre risk in Fire</td>
<td>2.3%</td>
<td>4.2%</td>
<td>2.9%</td>
<td>50.0%</td>
<td></td>
</tr>
<tr>
<td>Reserve risk in Fire</td>
<td>12.860</td>
<td>12.797</td>
<td>12.904</td>
<td>9.851</td>
<td></td>
</tr>
<tr>
<td>Premium risk in Fire</td>
<td>25.714</td>
<td>25.526</td>
<td>25.807</td>
<td>33.531</td>
<td></td>
</tr>
</tbody>
</table>

* Both parametric bootstraps are based on the same realizations of independent uniforms.

** The distribution-free analytical approach does not yield any higher moments than the estimate for the prediction error.

The SCR is calculated as 2.58 times the prediction error, i.e. the 99.5% quantile under a normal assumption motivated by the bootstrap predictive distribution.

*** We use the best estimate reserves from the marginal reserving methods as the volumes for reserve risk. For the volume underlying the premium risk, we assume future earned premiums to be the historic average net premium multiplied by a cost, safety and profit margin of 30%.
Case study: Results (5/5)

- **ORSA process**: results for a business horizon of **five years**

- Assumptions on risk profile, tolerance, business plans:
  - linearly growing business in both GTPL portfolios
  - stable volumes in Fire portfolio

- Allows to
  - estimate multi-year overall solvency needs,
  - derive future one-year capital requirements (rolling-forward definition of reserve and premium risks),
  - calculate risk margins and safety loadings,
  - perform sensitivity analyses.
Conclusion

The **bootstrap approach** allows to quantify premium and reserve risk
- among **lines of business** (or more granular segments)
- through various **risk measures**,
- over a time horizon of **multiple future accounting years**.
- input for **overall solvency needs** in the **ORSA process**
- understanding of how **dependencies** between the occurrence and settlement of claims among different loss portfolios influence the aggregated risk capital
- **suitability assessment** of the standard formula
- indicator for possible benefits from **undertaking-specific parameters**
- derivation of **risk capitals in future one-year view** to compute a risk margin under a run-off scenario

**Selected features of implementation in R**

- **chain-ladder and additive loss reserving models** subject to development year correlations
  - combinations and generalized versions possible
- **non-parametric** and **parametric** bootstraps/simulations of historic and future triangles
  - different marginals and copulae for parametric approach
- various analyses based on **full predictive distributions**
  - risk by accident years, reserve and premium risks
  - flexible (e.g. pairwise or complete) portfolio aggregations
  - split into estimation and process error
  - $m$-year and (updated) future one-year view
  - extensive plotting functions (model diagnostics, results)
- **closed-form analytical estimators** for benchmarking
Thank you very much for your attention!

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Appendix

Premium and Reserve Risk

$$SCR_{PR} = 3 \cdot \sigma_{nl} \cdot V_{nl}$$

- 99.5% VaR approximation (under normal assumption)

Aggregated risk factor

$$\sigma_{NL}$$

- Volume-weighted aggregation of risk factors of joint premium-reserve risk over all LoBs with correlation approach
- Pre-determined correlation matrix

Correlation matrix

Aggregated Volume

$$V_{nl} = \sum_s V_s$$

- Summation

Risk factor per LoB

$$\sigma_s$$

- Volume-weighted aggregation of risk factors for premium and reserve risks within a single LoB with a covariance approach
- Unique correlation of 50% between premium and reserve risks within each LoB

Correlation=50%

Risk factor of Premium Risk for LoB s

$$\sigma_{(prem,s)}$$

Pre-determined parameter*

Volume of Premium Risk for LoB s

$$V_{(prem,s)}$$

Future earned premium

Risk factor of Reserve risk for LoB s

$$\sigma_{(res,s)}$$

Pre-determined parameter*

Volume of Reserve Risk for LoB s

$$V_{(res,s)}$$

Outstanding claims reserve

Diversified Exposure per LoB

$$V_s = (V_{(prem,s)} + V_{(res,s)}) \cdot (0.75 + 0.25 \cdot DI\%_s)$$

- Aggregation through summation
- Geographical diversification depending on the business decomposition into regions

*may be replaced by undertaking specific parameters (USP) using standardized methods upon regulator’s approval