Risk analysis of annuity conversion options with a special focus on decomposing risk

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About the speaker

Dr. Katja Schilling

- Master of Science (Mathematics, Illinois State University, USA, 2010)
- Diploma (Mathematics and Management, University of Ulm, 2011)
- Ph.D. (University of Ulm, 2017)
- since 2015 consultant at Allianz Pension Consult

  (Allianz Pension Consult is a subsidiary of Allianz Lebensversicherungs-AG, the largest life insurance company in Germany, with main focus on structuring and arranging tailor-made solutions in the field of occupational pensions for medium-sized and large companies)
- junior member of the German Society for Actuarial and Financial Mathematics (DGVFM)
- candidate for the membership of the German Society of Actuaries (DAV)

About the speaker

Dr. Alexander Kling

Institut für Finanz- und Aktuarwissenschaften (ifa)

- ifa is an independent actuarial consulting firm.
- Our consulting services in all lines of insurance business include:
  - typical actuarial tasks and actuarial modelling
  - insurance product development
  - risk management, Solvency II, asset liability management
  - data analytics
  - market entries (cross-border business, setup of new insurance companies, Fintechs)
  - professional education
  - academic research on actuarial topics of practical relevance
- located in Ulm, Germany
- currently about 30 consultants
- academic cooperation with the University of Ulm (offering the largest actuarial program in Germany)

- joined ifa in 2003
- qualified actuary (German Association of Actuaries DAV, 2007)
- Master of Science (University of Wisconsin, Milwaukee, 2002)
- Master of Science (Mathematics and Management, University of Ulm, 2003)
- Ph.D. (University of Ulm, 2007)
- lecturer at Ludwig-Maximilians-Universität München, University of Ulm, German Actuarial Academy (DAA), European Actuarial Academy (EAA)

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Risk analysis of annuity conversion options

Risk decomposition methods from literature

MRT decomposition

Application of MRT decomposition to annuity conversion options

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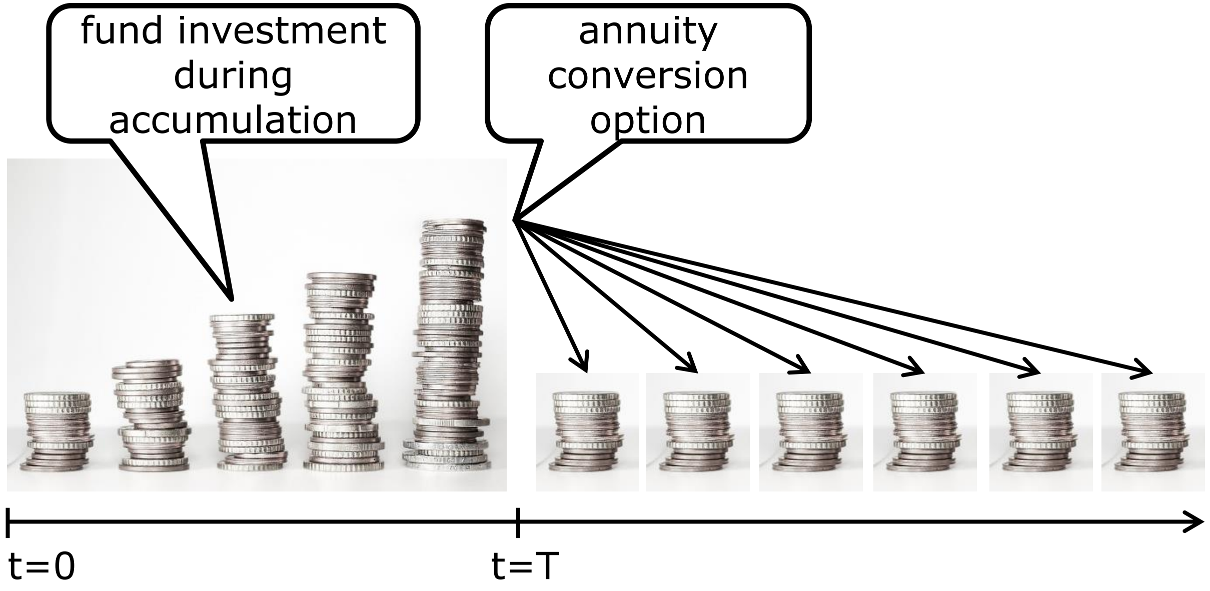
Introduction

Annuity conversion options

(Unit-linked) deferred annuities



- Money is allocated in some fund during a deferment period.
- At the end of the deferment period, the accumulated fund value is converted into a lifelong annuity.

Different annuity conversion options

- Guaranteed annuity option (GAO)
- GAO on a limited amount (Limit)
- Guaranteed minimum income benefit (GMIB)

Annuity conversion options are influenced by various risk sources such as

- equity, interest rate, and mortality

Introduction

Annuity conversion options

Existing literature on annuity conversion options

- measure the total risk by advanced stochastic models
- typically no decomposition of the total risk into risk factors

Our research interests

- (1) Theory:
  - How can the randomness of liabilities be allocated to different risk sources?
- (2) Application to annuity conversion options:
  - What is the dominating risk in annuity conversion options?
  - What is the relative importance of different risk sources?
- (3) Risk management of annuity conversion options:
  - How can the single risks be managed by product design or internal hedging?

Our contributions

- **Risk analysis of annuity conversion options in a stochastic mortality environment**
  - Katja Schilling, Alexander Kling, Jochen Ruß (2014)
  - ASTIN Bulletin 44 (2), 197 - 236.
- **Decomposing life insurance liabilities into risk factors**
  - Katja Schilling, Daniel Bauer, Marcus C. Christiansen, Alexander Kling (2018)
  - under review: Management Science
- **Comparing financial and biometric risks in annuity conversion options via the MRT decomposition**
  - Katja Schilling (2018)
  - under review: Insurance: Mathematics and Economics

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Risk analysis of annuity conversion options

Different annuity conversion options

Guaranteed annuity option (GAO)

- minimum conversion rate g for converting the account value into a lifelong annuity at time T

$$L_T^{GAO,i} = \mathbb{I}_{\{\tau_x^i > T\}} \cdot g \cdot A_T \cdot \max\{a_T - \frac{1}{g}, 0\}$$

GAO with limit (Limit)

- upper bound L (limit) to which the conversion rate g at most applies

$$L_T^{Limit,i} = \mathbb{I}_{\{\tau_x^i > T\}} \cdot g \cdot \min\{A_T; L\} \cdot \max\{a_T - \frac{1}{g}, 0\}$$

Guaranteed minimum income benefit (GMIB)

- fixed minimum annuity amount $M (= g \cdot G)$

$$L_T^{GMIB,i} = \mathbb{I}_{\{\tau_x^i > T\}} \cdot \max\{g \cdot G \cdot a_T - A_T, 0\}$$

Notation

- T: deferment period/retirement date
- x: policyholder’s age at inception of the contract (t = 0)
- τ_x: remaining lifetime
- A_T: account value at the end of the deferment period
- a_T: present value of an immediate annuity of amount 1 p.a.

Risk analysis of annuity conversion options

Insurer's strategies and stochastic model

Risk management strategies

	No hedging	Hedging
No option fee	Strategy A	-
Option fee	Strategy B	Strategy C

- **Strategy A**
  - Insurer charges no option fee and does not hedge.
- **Strategy B**
  - Insurer charges an option fee which is simply invested in money market instruments (no hedging).
- **Strategy C**
  - Insurer charges an option fee to buy a static hedge against the financial risk during the deferment period.
  - Assumption: option fee = hedging costs

Stochastic model

Fund

- Geometric Brownian motion

$$dS(t) = (r(t) + \lambda_S) \cdot S(t)dt + \sigma_S \cdot S(t)dW_S(t), \qquad S(0) > 0$$

Interest rate

- Cox-Ingersoll-Ross model

$$dr(t) = \kappa \cdot \big(\theta - r(t)\big)dt + \sigma_r \cdot \sqrt{r(t)}dW_r(t), \qquad r(0) > 0$$

Stochastic mortality

- 6-factor forward model (cf. Bauer et al., 2008)

$$d\mu(t,T,x) = \alpha(t,T,x)\, dt + \sigma(t,T,x)\, dW_\mu(t), \qquad \mu(0,T,x) > 0$$

Risk analysis of annuity conversion options

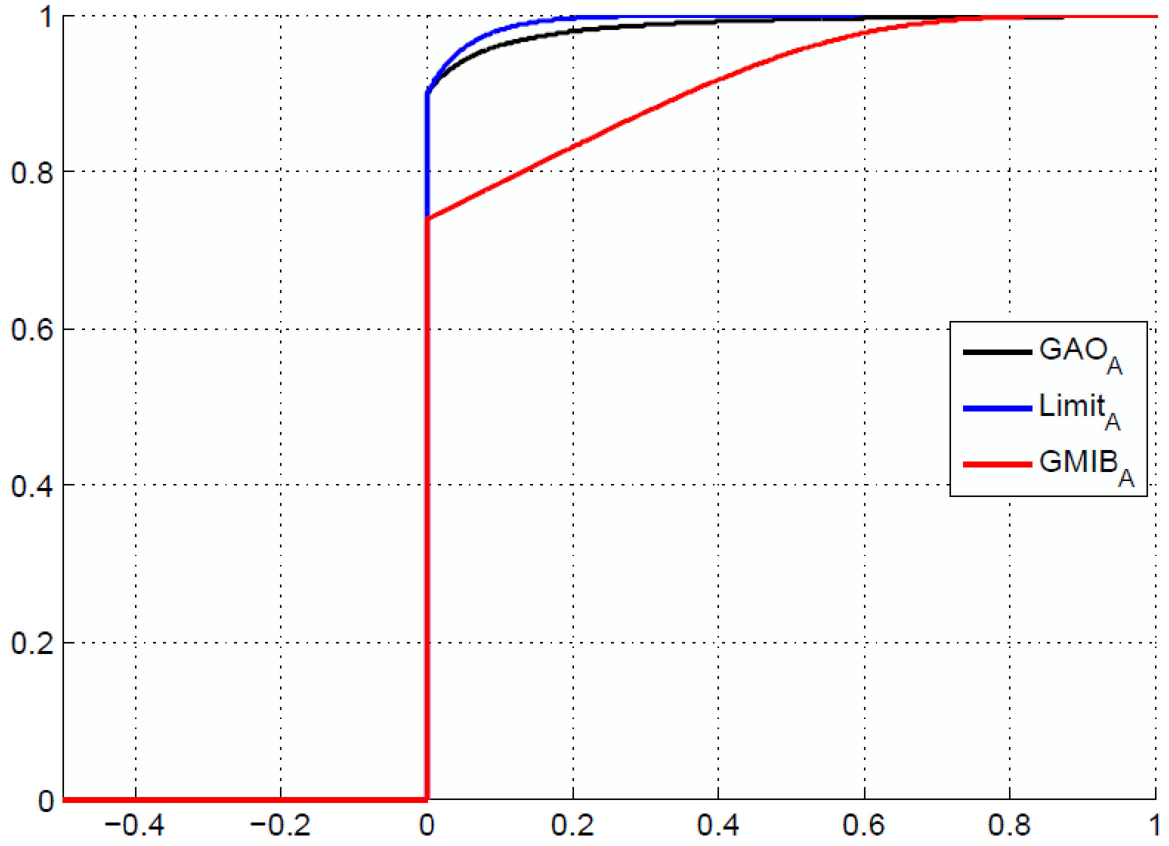
Sample results

Risk of different annuity conversion options **without hedging (under strategy A)**

	GAO	Limit	GMIB
risk ($TVaR_{0,99}$)	0.7362	0.2214	0.7739
option value	0.0167	0.0096	0.1361

- Loss probability for GMIB much higher than for the other annuity conversion options
- Risk ($TVaR_{0,99}$) similar for GMIB and GAO
  - Limit has a much lower risk
- Option value does not reflect the risk of the annuity conversion option

Cumulative distribution function of insurer's loss



Risk analysis of annuity conversion options

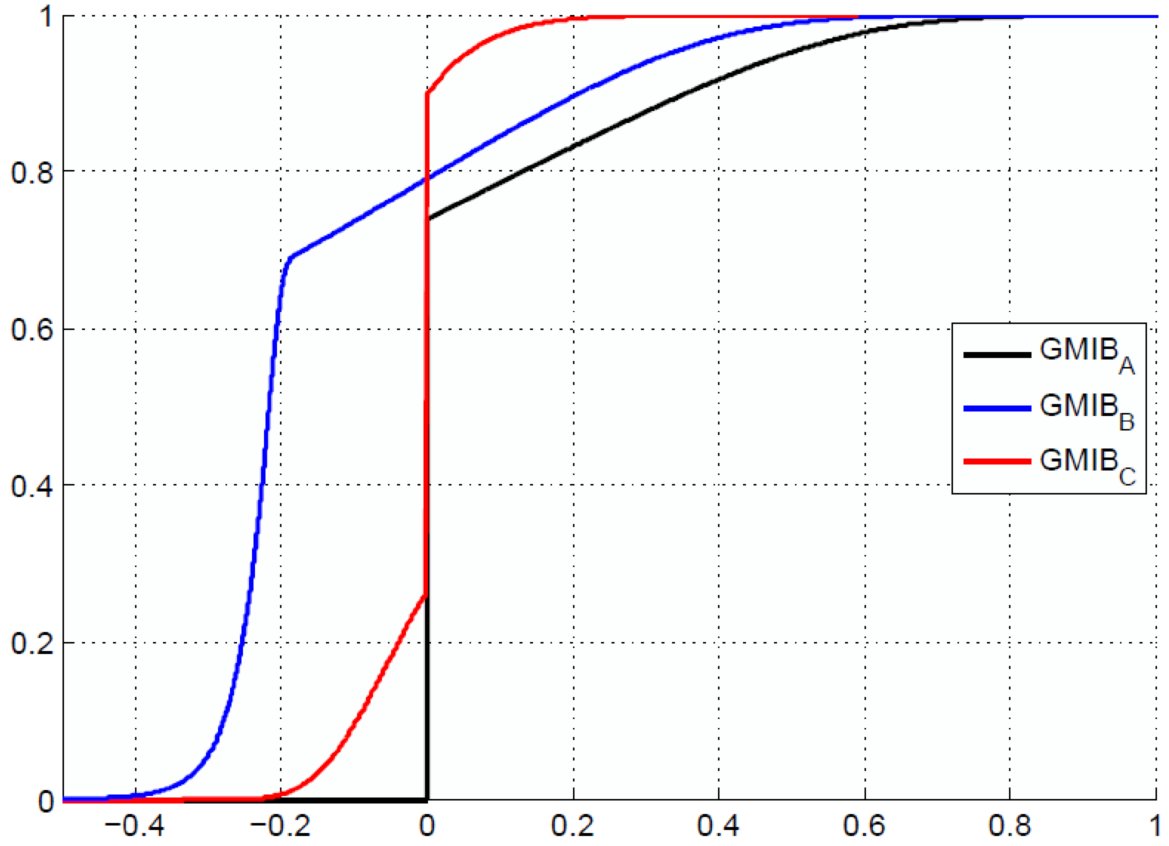
Sample results

Risk of **GMIB** guarantee under **different risk management strategies**

	strategy A	strategy B	strategy C
risk ($TVaR_{0,99}$)	0.7739	0.5962	0.2385
option value	0.1361	0.1708	0.1708

- Loss probability and risk can significantly be reduced by risk management (strategies B and C)
- Risk under strategy B (option fee but no hedging) still significant
- Significant risk reduction for strategy C (option fee and hedging)
- Option value is increased if a guarantee fee is charged

Cumulative distribution function of insurer's loss



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Risk decomposition methods from literature

Variance decomposition approach

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Definition (given two sources of risk X_1 and X_2)

The (stochastic) variance decomposition of the risk R is defined as

$$R = \underbrace{E^P(R|X_1)}_{R_1} + \underbrace{[R - E^P(R|X_1)]}_{R_2},$$

where the risk factors R_1 and R_2 are supposed to capture the randomness caused by the sources of risk $\mathrm{X}_1 = (X_1(t))_{0 \le t \le T}$ and $\mathrm{X}_2 = (X_2(t))_{0 \le t \le T}$, respectively.

It follows the well-known result (not focused here):

$$Var(R) = Var(R_1) + Var(R_2)$$

Simple example

Let the insurer's risk be equal to $R = X_1(T)X_2(T)$, where X_1, X_2 are two independent (standard) Brownian motions.

Then the variance decomposition yields two different results depending on the order of X_1 and X_2:

1) $R_1 = E^P(R|X_1) = X_1(T)E^P(X_2(T)) = 0$
$R_2 = R - E^P(R|X_1) = X_1(T)X_2(T) - 0 = X_1(T)X_2(T)$

2) $R_2 = E^P(R|X_2) = X_2(T)E^P(X_1(T)) = 0$
$R_1 = R - E^P(R|X_2) = X_1(T)X_2(T) - 0 = X_1(T)X_2(T)$

=> *Variance decomposition is not order-invariant!*

Risk decomposition methods from literature

Overview

Properties of risk decomposition methods from existing literature

	randomness	attribution	uniqueness	order invariance	scale invariance	aggregation	additive aggregation
Variance decomposition cf. Bühlmann (1995)	✓	✓	✓	✗	✓	✓	✓
Hoeffding decomposition cf. Rosen & Saunders (2010)	✓	✓	✓	✓	✓	✗	✗
Taylor expansion cf. Christiansen (2007)	✓	✓	✗	✓	✗	✗	✗
Solvency II approach cf. Gatzert & Wesker (2014)	✓	✗	✗	✓	✓	✗	✗
MRT decomposition	✓	✓	✓	✓	✓	✓	✓

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MRT decomposition

Modeling framework

Let all financial and demographic sources of risk be modeled by a $(\mathrm{k}-1)$-dimensional Itô process:

$$dX(t) = \theta(t)dt + \sigma(t)dW(t), \quad t \in [0,T], \quad X(0) = x_0 \in \mathbb{R}^{k-1}$$

The number of survivors is modeled by a doubly stochastic counting process $m - N(t)$

- $\left(N(t)\right)_{t\in[0,T]}$ number of deaths with jump intensity $\left(\mu(t)\right)_{t\in[0,T]}$

Notation

- $(\Omega, \mathcal{F}, \mathbb{F}, \mathbb{P})$ probability space
- T time horizon
- W d-dimensional Brownian motion with filtration $\mathbb{G} = \left(\mathcal{G}(t)\right)_{t\in[0,T]}$; $\mathbb{G}$ is a sub-filtration of $\mathbb{F}$
- $m \in \mathbb{N}$ denotes the initial number of policyholders
- τ_x^i remaining life time of insured $i = 1, \dots, m$

MRT decomposition

Definition and properties

Definition

The MRT decomposition of $R = L - E^P(L)$ is defined as

$$R = \underbrace{\sum_{i=1}^{k-1} \int_0^T \psi_i^W(t) dM_i^W(t)}_{\text{risk factor } R_i} + \underbrace{\int_0^T \psi^N(t) dM^N(t)}_{\text{risk factor } R_k}$$

for some $\mathbb{F}$-predictable processes $\psi_i^W(t)$ and $\psi^N(t)$.

The processes $M_i^W(t)$ and $M^N(t)$ denote the martingale part of $X_i(t)$ and $N(t)$, respectively.

Theorem

Let L be $\mathcal{F}_T$-measurable and square integrable, $k - 1 = d$, and $\det(\sigma(t)) \neq 0$ for all t.
Then the MRT decomposition of $R = L - E^P(L)$ exists and satisfies all properties (i.e. randomness, attribution, uniqueness, order invariance, scale invariance, aggregation, additive aggregation).

Further properties

- Applicability: Explicit formulas for the integrands $\psi_1^W(t), \dots, \psi(t)_{k-1}^W(t), \psi^N(t)$ are derived (within a life insurance context)
- Convergence: Unsystematic mortality risk factor (R_k) is diversifiable; all other risk factors converge to a non-zero limit as the portfolio size increases

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Application of MRT decomposition to annuity conversion options

MRT decomposition for a GAO

For the insurer's (discounted) loss $L = e^{-\int_0^T r(s)ds}\big(m - N(T)\big) g A_T \max\{a_T - \frac{1}{g}, 0\}$ from a GAO it holds (given a slightly modified stochastic model compared to slide 10):

- There exists a **measurable function** $\boldsymbol{h}$ such that $h\big(S(T), r(T), \mu(T)\big) = g A_T \max\{a_T - \frac{1}{g}, 0$
- The function $f\big(t, S(t), r(t), \mu(t)\big) := E^P\left(e^{-\int_t^T (r(s)+\mu(s))ds} h\big(S(T), r(T), \mu(T)\big|\mathcal{G}_t\right)$ is in $\boldsymbol{C^{1,2}}$

The **unique MRT risk factors** of the insurer's risk $R = L - E^P(L) = R_1 + R_2 + R_3 + R_4$ are given by

$$R_1 = \int_0^T (m - N(t-))\ e^{-\int_0^t r(s)ds} \frac{\partial f}{\partial x_1}\big(t, X(t)\big)\sigma_S S(t) dW_S(t)$$ (fund risk)

$$R_2 = \int_0^T (m - N(t-))\ e^{-\int_0^t r(s)ds} \frac{\partial f}{\partial x_2}\big(t, X(t)\big)\sigma_r \sqrt{r(t)} dW_r(t)$$ (interest risk)

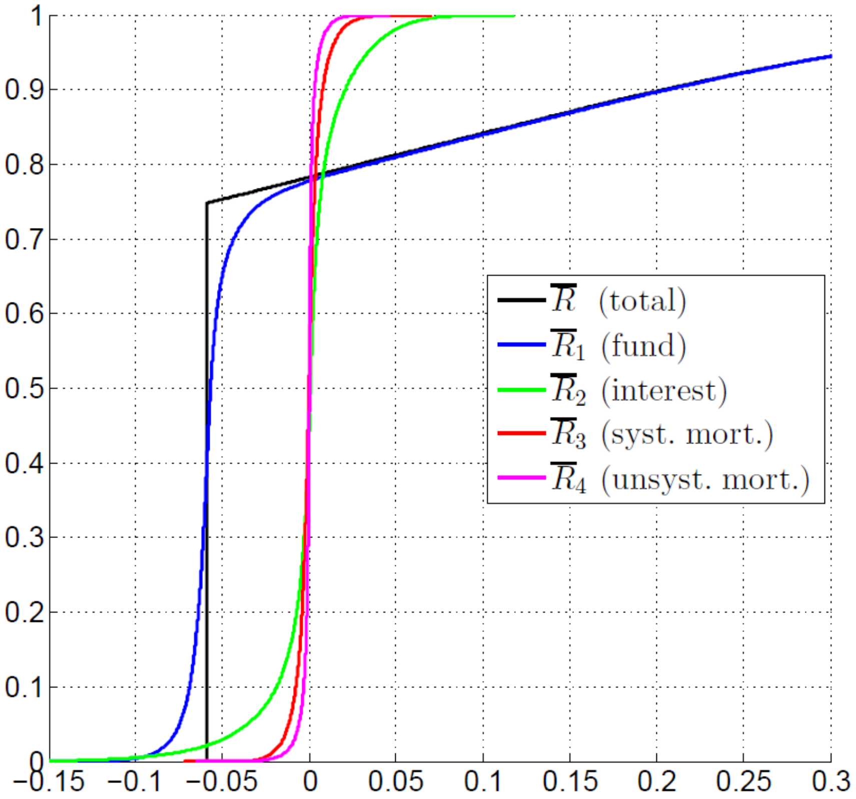
$$R_3 = \int_0^T (m - N(t-))\ e^{-\int_0^t r(s)ds} \frac{\partial f}{\partial x_3}\big(t, X(t)\big)\sigma_\mu(t) \sqrt{\mu(t)} dW_\mu(t)$$ (systematic mortality risk)

$$R_4 = \int_{0+}^T e^{-\int_0^t r(s)ds} f\big(t, X(t)\big) dM^N(t)$$ (unsystematic mortality risk)

Application of MRT decomposition to annuity conversion options

Numerical results for a GMIB

Cumulative distribution functions of the total risk and the four risk factors



Relative risk contributions of the four sources of risk under **different risk measures**

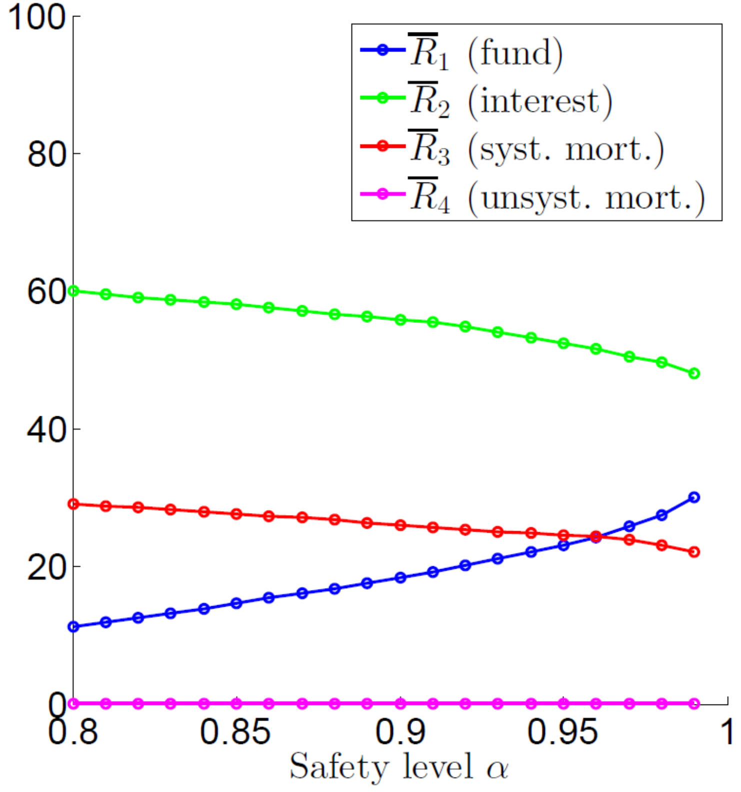
	$TVaR_{0,99}$	Standard deviation
Fund	89.8 %	96.5 %
Interest	8.3 %	3.0 %
Syst. mortality	1.2 %	0.4 %
Unsyst. mortality	0.7 %	0.1 %

- Fund risk dominates total risk
- Interest risk is slightly relevant in the tail
- Mortality risks are negligible

Application of MRT decomposition to annuity conversion options

Numerical results for a GAO

Relative risk contributions of the four sources of risk under $\boldsymbol{TVaR_\alpha}$ (in %) for different safety levels α



- Interest risk dominates total risk
- Systematic mortality is significant
- Fund risk is particularly responsible for high risk outcomes
- Unsystematic mortality risk is negligible

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Literature

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