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It Takes Two: Why Mortality Trend Modeling is more than modeling one Mortality Trend

- Joint work with Matthias Börger and Jochen Russ
- Johannes Schupp
- September 13, 2019
- Longevity 15 Conference

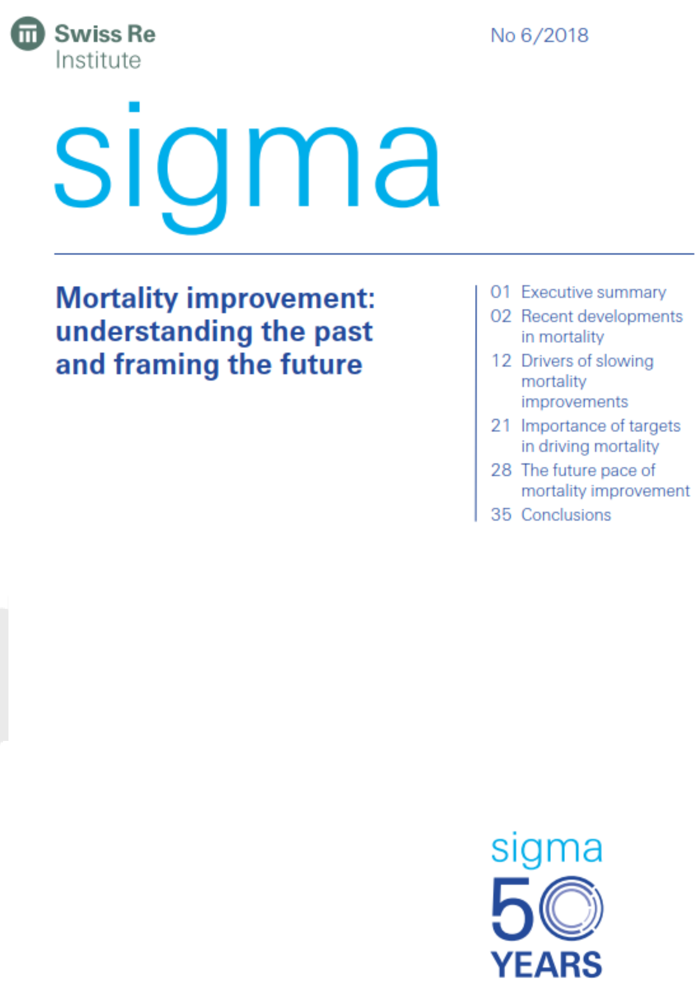
Agenda

- **Introduction: Why two mortality trends?**
  - Actual Mortality Trend (AMT)
  - Estimated Mortality Trend (EMT)
  - Some examples
- A combined model for AMT & EMT
  - AMT component
  - EMT component
- Applications
- Conclusion

Introduction: Why two mortality trends?

Highlights of recent SwissRe sigma study on „Mortality Improvements“

- “Mortality improvement has slowed unusually in the US, UK, Germany, the Netherlands and Taiwan.”
- “… but the recent slowdown is typically not statistically significant.”
- “Extrapolating future mortality trends solely from recent experience can be misleading unless we believe there has been a structural break.”
- “The ability to distinguish between shifts in the underlying mortality trend and short-term variability is crucial because a change in mortality trend is an aggregate risk that cannot be easily diversified away nor perfectly hedged.”


Swiss Re Institute

No 6/2018

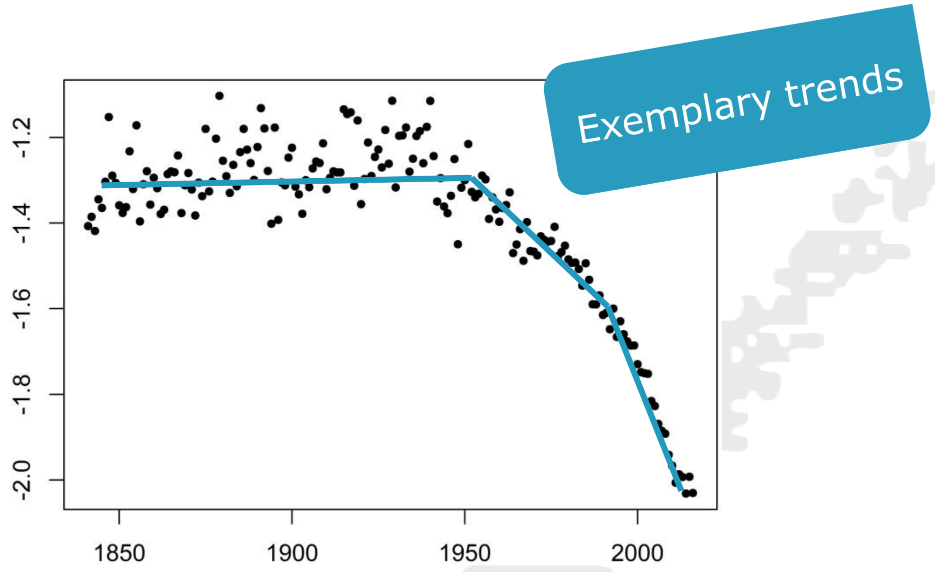
sigma

Mortality improvement: understanding the past and framing the future

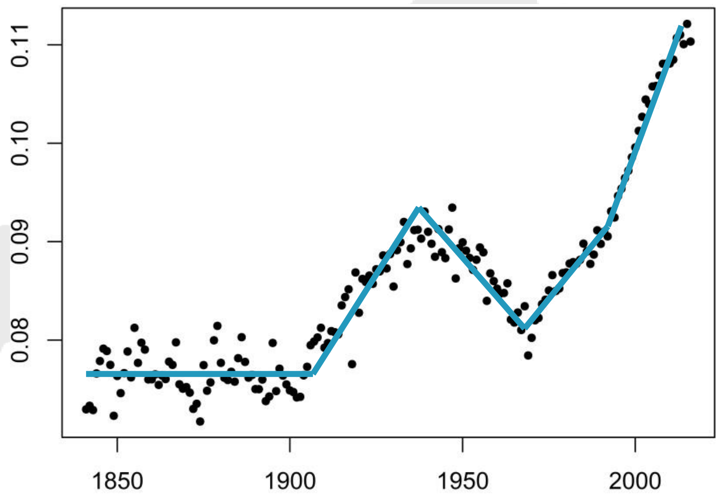
sigma 50 YEARS

Introduction: Why two mortality trends?

- two parameter processes (Cairns et al. (2006))
  - $\log\left(\frac{q_{x,t}}{1-q_{x,t}}\right) = \kappa_t^1 + \kappa_t^2 \cdot (x - \bar{x})$
  - parameters calibrated for English and Welsh males older than 60
- classical simulation approach: Random Walk with drift



- historical trend changed once in a while
  - only a piecewise linear trend with random changes in the trends slope
  - random fluctuation around the prevailing trend
- In principle, our approach can be applied to any changing mortality trend model.



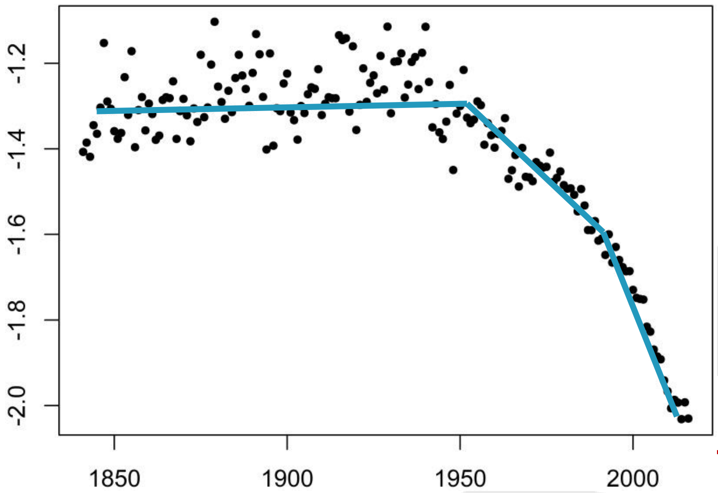
Introduction: Why two mortality trends?

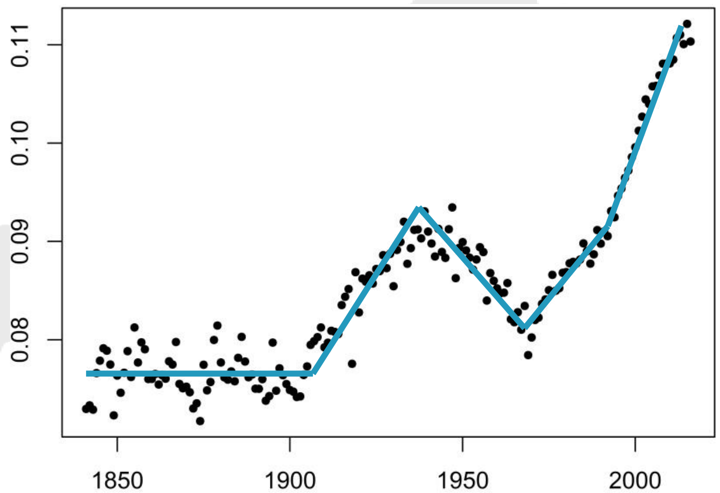
Actual Mortality Trend (AMT)

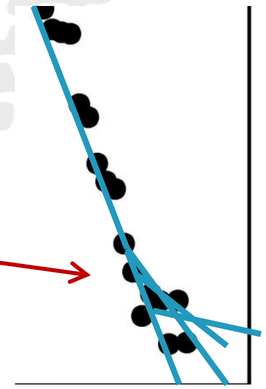
- The AMT describes realized mortality trends.
  - core of most existing mortality models
  - Time and magnitude of changes in the AMT and the error structure around the trend process need to be modeled.
- We have an idea of the historical AMT but it's not fully observable!
- We can't always distinguish between a recent trend change and "normal" random fluctuation around the prevailing trend.

→ possible undetected trend change in the recent years

- unknown current value and slope of the AMT
- **one model for the AMT**







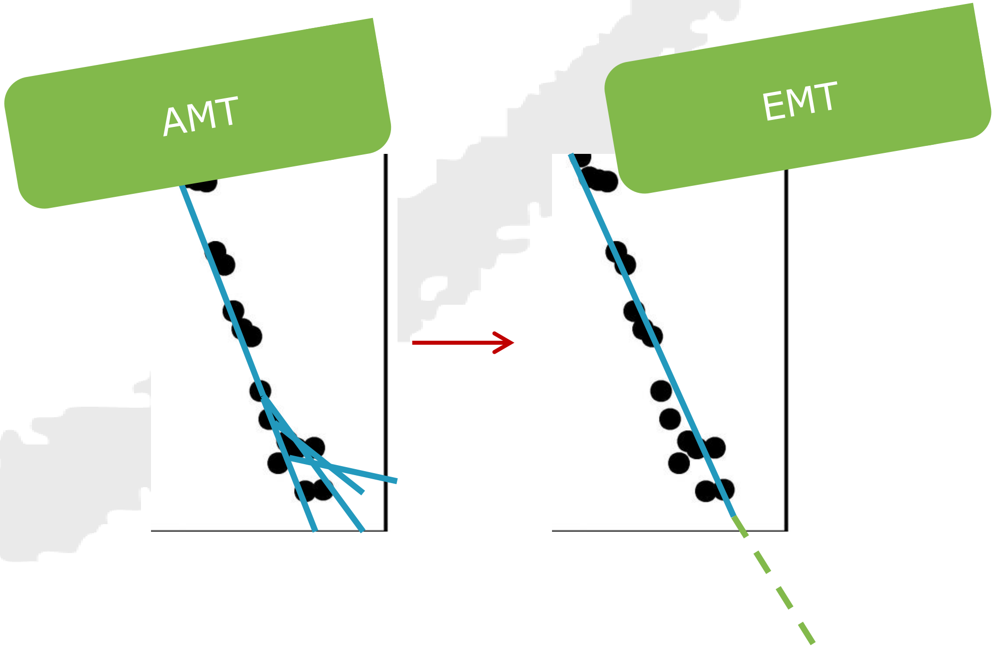
Introduction: Why two mortality trends?

Estimated Mortality Trend (EMT)

- The EMT describes the expectation of an actuary/demographer about the AMT, i.e. the current slope and value of the mortality trend at some point in time.
- based on most recent historical, observed mortality evolution and updated as soon as new observations become available
- The EMT is the basis for mortality projections, (generational) mortality tables, reserves, etc.
- **one model for the EMT**

"… but the recent slowdown is typically not statistically significant."

"Extrapolating future mortality trends solely from recent experience can be misleading unless we believe there has been a structural break."



Introduction: Why two mortality trends?

Some examples

Why another trend?

- Requirement for AMT and/or EMT depends on application:
  - reserves for a portfolio → EMT today
  - capital for a portfolio run-off → AMT over the run-off
  - reserves for a portfolio after 10 years → AMT over the 10 years, EMT after 10 years
  - payout of a mortality derivative → AMT up to maturity, EMT at maturity
  - analyze the hedge effectiveness of the previous derivative → EMT at maturity, AMT beyond

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  - EMT component
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A Combined model for AMT/EMT

AMT component

- continuous piecewise linear trend, with random changes in the slope and random fluctuation around the trend
- AMT model specification:
  - Model the trend process with random noise → $\kappa_t = \hat{\kappa}_t + \epsilon_t;\ \epsilon_t \sim \mathcal{N}(0, \sigma_\epsilon^2)$.
  - Extrapolate the most recent actual mortality trend → $\hat{\kappa}_t = \hat{\kappa}_{t-1} + \hat{d}_t$.
  - In every year, there is a possible change in the mortality trend with probability p.

    $$\rightarrow \hat{d}_t = \begin{Bmatrix} \hat{d}_{t-1} & \text{with probability } 1-p \\ \hat{d}_{t-1} + \lambda_t & \text{with probability } p \end{Bmatrix}$$
  - in the case of a trend change → $\lambda_t = M_t \cdot S_t$
    - with absolute magnitude of the trend change $M_t \sim \mathcal{LN}(\mu, \sigma^2)$
    - sign of the trend change S_t bernoulli distributed with values -1, 1 each with probability $\frac{1}{2}$

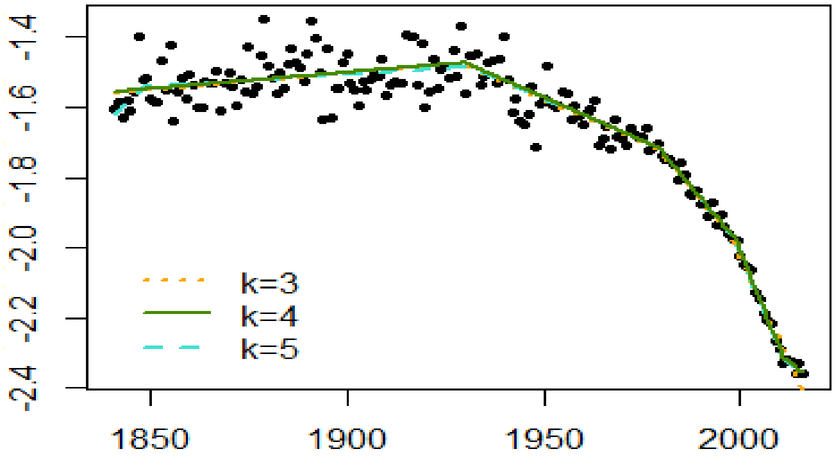
parameters to be estimated for projections:

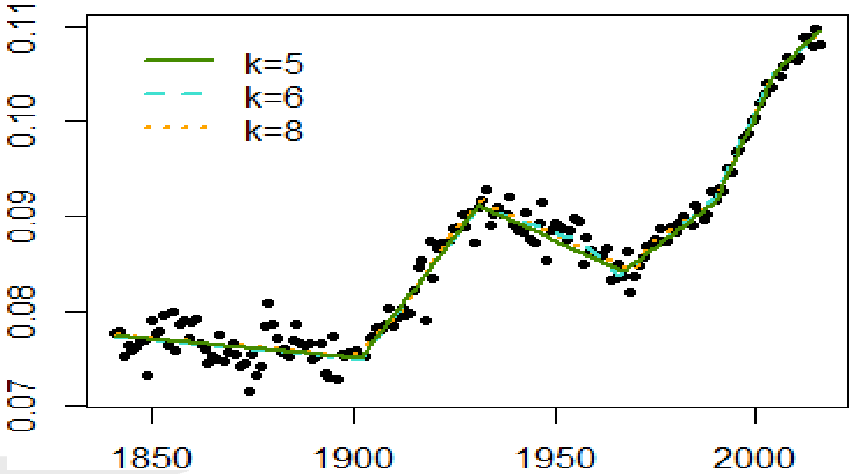
$p, \sigma_\epsilon^2, \mu, \sigma^2, \hat{d}_n, \hat{\kappa}_n$

A Combined model for AMT/EMT

AMT component

Idea: Use historic trends to estimate the parameters $p, \sigma_\epsilon^2, \mu, \sigma^2, \hat{d}_n, \hat{\kappa}_n$.



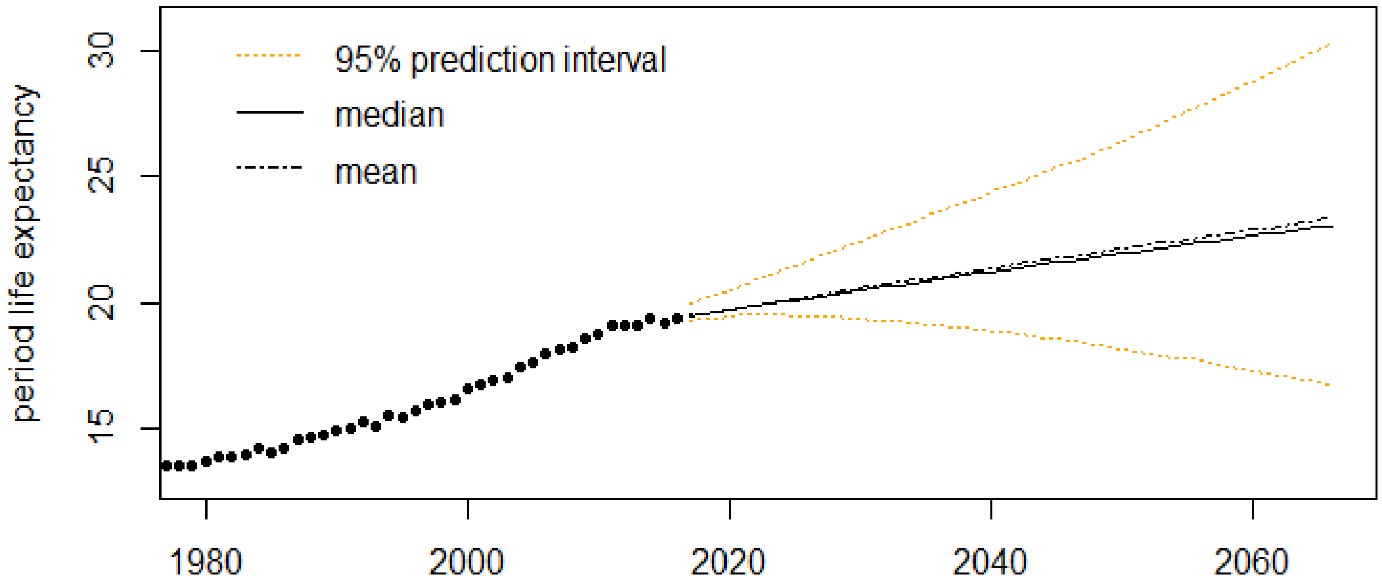


For details on the calibration we refer to Börger and Schupp (2018) and Schupp (2019). Parameter uncertainty is included.

A Combined model for AMT/EMT

AMT component

Period life expectancies for 65-year old males in England and Wales



Ongoing of recent improvements and also slowdown of mortality improvements incorporated

A Combined model for AMT/EMT

EMT component

- We don't know today's AMT, but we want a model to estimate it: $\mathbb{E}(\hat{d}_t) = \tilde{d}_t$ and $\mathbb{E}(\hat{\kappa}_t) = \tilde{\kappa}_t$.
- Calculation of EMT is complex as all potential evolutions of unknown AMT need to be incorporated. Especially, in a simulation this is not feasible.
  - path-dependent calculation of the EMT
  - path-dependent recalibration of whole AMT

- piecewise linear trend process with symmetric changes in the AMT

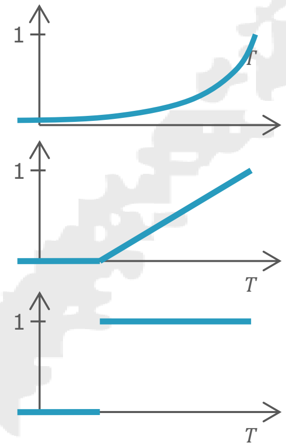
  → Calibrate the EMT with a weighted linear regression on most recent data.

- How many years should be included in the regression?
  - too many → delayed reaction of EMT on trend changes in the AMT
  - too little → EMT is vulnerable to random noise in the AMT

A Combined model for AMT/EMT

EMT component

- Higher influence of most recent data in the estimation of the regression.
  - weighted exponential regression in year T : $w_{exp}(t,T) = \frac{1}{\left(1+{}^{1}/_{h_{exp}}\right)^{T-t}}, t \leq T.$
  - weighted linear regression in year T : $w_{lin}(t,T) = \max\left(0; 1 - \frac{1}{h_{lin}}(T-t)\right), t \leq T,$
  - weighted constant regression in year T : $w_{const}(t,T) = \begin{cases} 1 & , if\ T - h_{const} < t \leq T \\ 0 & , if\ t \leq T - h_{const} \end{cases}.$



Calibration of the weighting parameters

- calibrate the AMT model
- Simulate the future evolution of the AMT 100.000 times to avoid dependencies on fixed historical trends.
- EMT calibration
  - After T=40 years, calculate the optimal weighting parameters based on:
    - EMTs cohort life expectancy of 65-year old males $\tilde{e}_{65,T}$ close to AMTs $\hat{e}_{65,t_\omega}$

A Combined model for AMT/EMT

EMT component – results

- EMT calibration
  - After T=40 years, calculate the optimal weighting parameters based on:
    - EMTs cohort life expectancy of 65-year old males $\tilde{e}_{65,T}$ close to AMTs $\hat{e}_{65,t_\omega}$

weights	constant	linear	exponential	fixed period
$h_\cdot^{(1)}$	10.0	11.0	2.2	30
$h_\cdot^{(2)}$	8.0	10.5	2.1	30
mse	1.59	1.56	1.58	3.04
$P(\tilde{e}_{65,T} < 95\% \cdot \hat{e}_{65,t_\omega})$	8.7%	8.5%	8.7%	15.0%
$P(\tilde{e}_{65,T} > 105\% \cdot \hat{e}_{65,t_\omega})$	7.3%	7.1%	7.0%	11.1%

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Applications
Overview

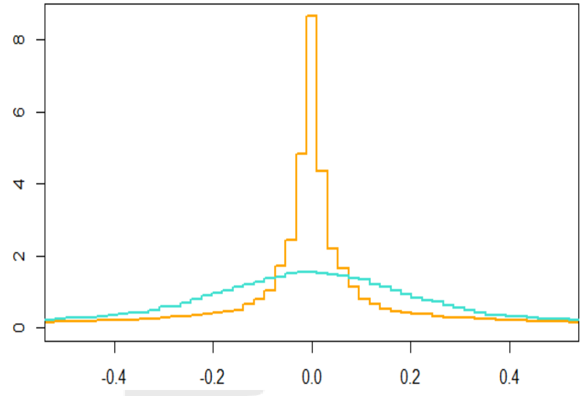
- 1. Hedge Effectiveness of a Value Hedge
- 2. SCR for Longevity Risk

- common assumptions
  - deterministic and constant interest rate 2%
  - annuitants'/pensioners' mortality rates are exactly as for males in England and Wales
  - portfolios are large enough → no unsystematic mortality risk
  - For the EMT's, we use linear weighting based on life expectancy optimization.

Applications

Example 1

Hedge Effectiveness of a Value Hedge



- pension fund with members aged 45 in t_0
- Hedge provider offers value hedge when they retire at $T = 20 + t_0$.
- If necessary, hedge fills up fund's liabilities at expiry.
- two Risks:
  - AMT changes after T
  - AMT assumption at T is inaccurate
- Unfortunately, the pension fund's trustees do not distinguish between AMT and EMT. They assume, that the current AMT is observable. Thus, they think their remaining risk is
  - $PV_T\left(pension\ payouts \mid AMT_{t_\omega}\right) - PV_T(pension\ payouts \mid AMT_T)$ (yellow).
  - hedge effectiveness: $1 - \frac{Var(Risk\ after\ hedge)}{Var(without\ hedge)} = \mathbf{92,1\%}$
- However, the actual risk is
  - $PV_T\left(pension\ payouts \mid AMT_{t_\omega}\right) - PV_T(pension\ payouts \mid EMT_T)$ (blue).
  - true hedge effectiveness: $1 - \frac{Var(Risk\ after\ hedge)}{Var(without\ hedge)} = \mathbf{87,2\%}$

hedge looks better than it is

Applications

Example 2

SCR for Longevity Risk

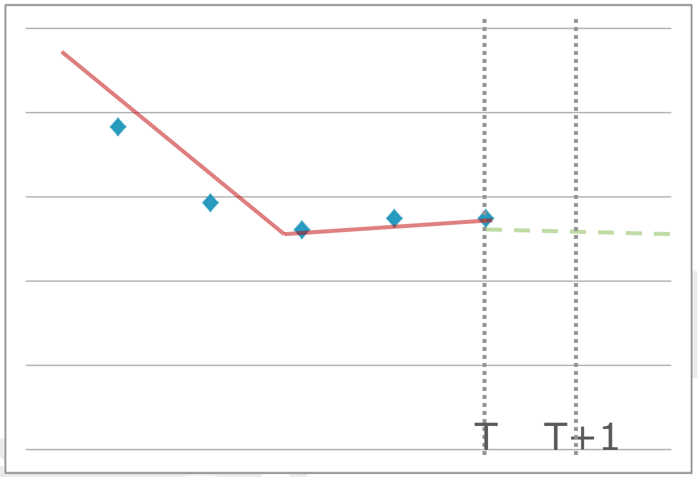
- Consider a portfolio of 75-year old annuitants at $T = 20 + t_0$ (no costs, no premiums).
- Insurer with internal model calculates the SCR as the 99,5% percentile of (see Börger (2010)):
- $\Delta BEL_{T+1} = (BEL_{T+1} + CF_{T+1}) \cdot \frac{1}{1+r} - BEL_T$
  - CF_{T+1} actual cashflow
    - realized mortality evolution over 1-year horizon
    - →AMT component
  - BEL_t best-estimate of liabilities
    - influence of the additional one year observation
    - →EMT component with optimal weighting

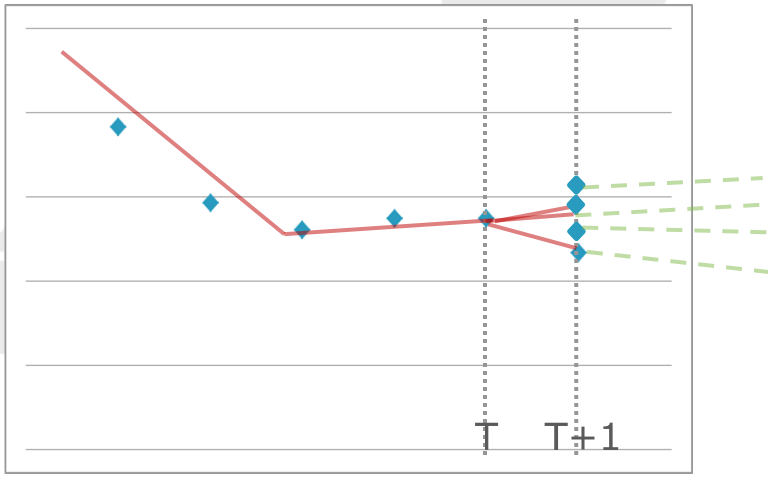
Applications
Example 2

SCR for Longevity Risk – continued

- $\Delta BEL_{T+1} = (BEL_{T+1} + CF_{T+1}) \cdot \frac{1}{1+r} - BEL_T$
- estimate AMT up to T 10.000 times (outer paths)
  - for each simulated AMT, simulate 10.000 inner 1-year paths
  - estimate 99.5% precentile of ΔBEL_{T+1} based on the 10.000 inner paths
- illustration: one outer path

$$\Delta BEL_{T+1} = (BEL_{T+1} + CF_{T+1}) \cdot \frac{1}{1+r} - BEL_T$$



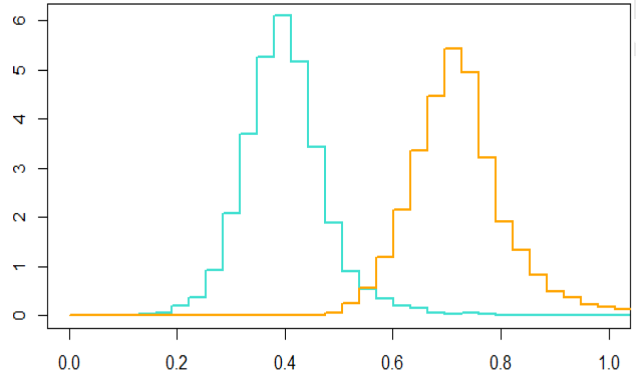


Applications
Example 2

SCR for Longevity Risk – continued

- If the insurer falsely assumes the AMT to be known, he would calculate the BEL_t based on the AMT.

→The SCR would be on average **0.73** (yellow).



- Instead, if the insurer recognizes the AMT to be unknown,

→the SCR would be on average **0.40** (blue).

- If the AMT is assumed to be known, the longevity risk would be **overestimated** in this example!
- Why?
  - AMT exhibits rather massive trend changes in one year
  - Annual changes in EMT are not that strong as the EMT does not pick up trend changes immediately

Conclusion

- Two trends need to be distinguished and modeled.
  - The actual mortality trend (AMT) is the prevailing, unobservable mortality trend.
  - The estimated mortality trend (EMT) is the estimate of the AMT.
- The trend to consider depends on the question in view.
- The AMT is modeled as a continuous and piecewise linear trend with random changes in the trend's slope.
- Choice of EMT approach is crucial in many practical situations.
  - A weighted regression approach seems reasonable.
  - Optimal regression weights can be determined in a practical setting.

- If the AMT is wrongfully assumed observable, risk is significantly misestimated in all our examples – sometimes underestimated, sometimes overestimated.

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