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Calibrating Mortality Processes with Trend Changes to Multi-Population Data

- joint work with Matthias Börger and Justin Schoenfeld
- Johannes Schupp
- 2020 Living to 100 Symposium, Orlando
- January 14th

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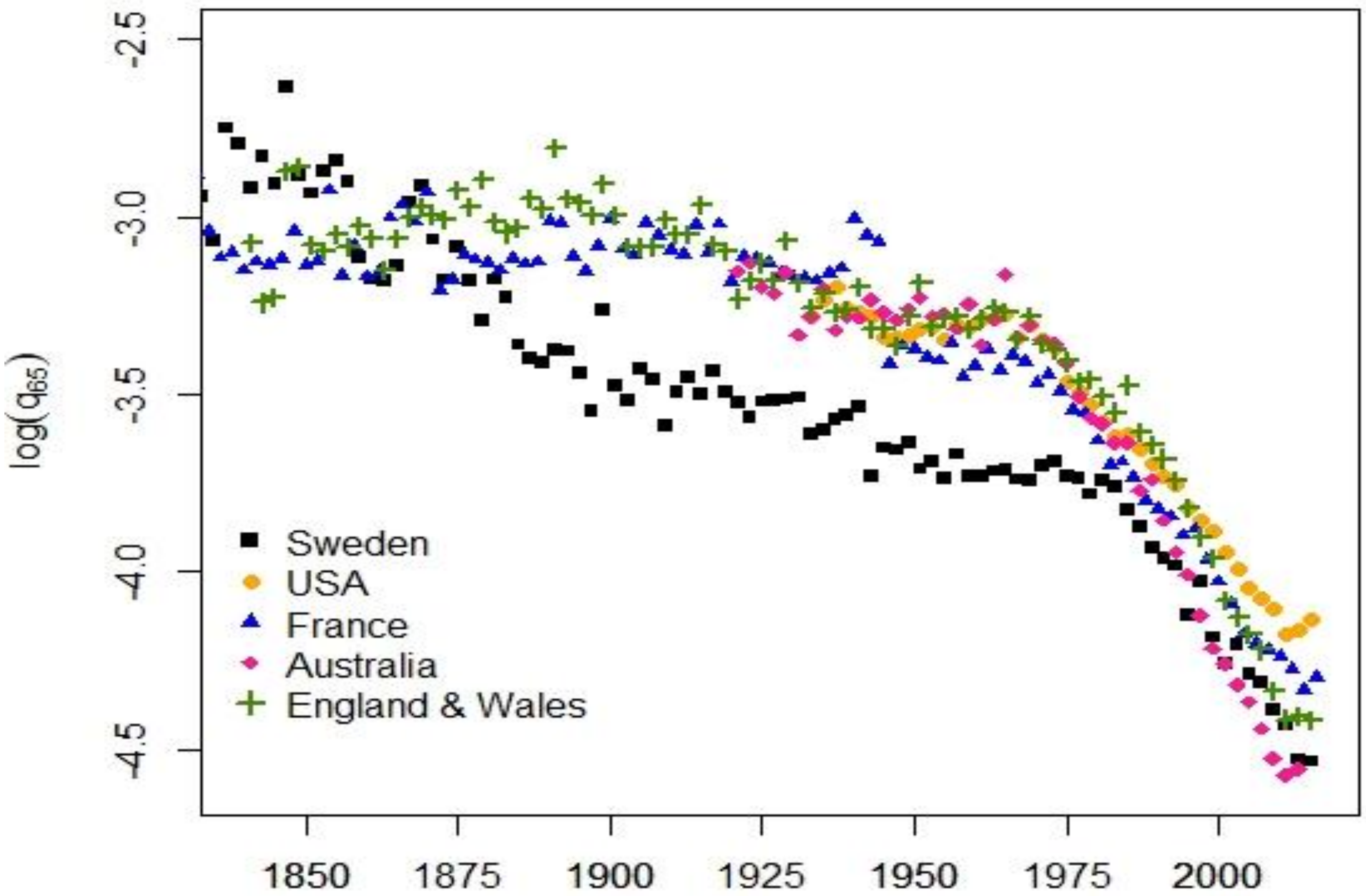
Introduction

Uncertainty about the evolution of mortality



Introduction

- Trend changes in mortality evolution have been identified by several authors for many countries:
  - Li et al. (2011), Coelho and Nunes (2011), Hainaut (2012), Sweeting (2011), Börger and Schupp (2018),…
  - Typically, only a few number of trend changes can be observed (2-6 trend changes).
  - significant amount of uncertainty → Combine data from different countries!
- Reliable trend changes can't be identified for countries, pension funds, with short data histories or large volatility
  - However, the underlying drivers of mortality should be similar. → Use data from related countries!

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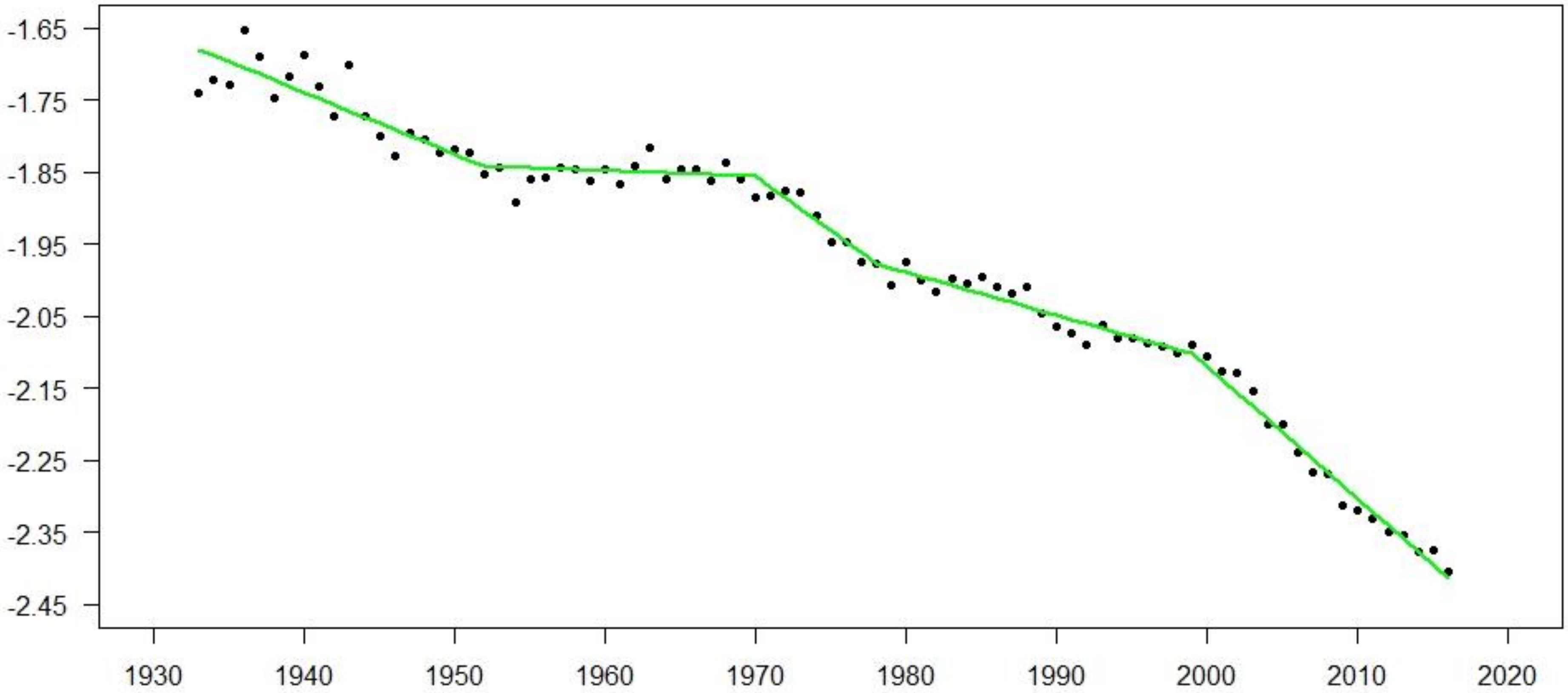
Trend change process

Underlying CBD mortality model

- Throughout, we assume that the trend process of Börger and Schupp (2018) is applied to project the period effects in the CBD mortality model of Cairns et al. (2006).
- structure of the CBD model:
  - $logit(q_{x,t}) := \log\left(\frac{q_{x,t}}{1-q_{x,t}}\right) = \kappa_t^{(1)} + \kappa_t^{(2)} \cdot (x - \bar{x})$
  - $\bar{x}$ is some medium age
- Each period effect $\kappa_t^{(i)}, i = 1,2$ is projected by a separate instance of the trend process.

Specification of a Trend Model

Optimal historical trend realization for US males $\kappa_t^{(1)}$



Trend change process

Specification

- continuous piecewise linear trend, with random changes in the slope and random fluctuation around the trend
  - Model the trend process with random noise →$\kappa_t^{(i)} = \hat{\kappa}_t^{(i)} + \varepsilon_t^{(i)}$; $\epsilon_t \sim \mathcal{N}(0, \Sigma)$.
  - Extrapolate the most recent actual mortality trend →$\hat{\kappa}_t^{(i)} = \hat{\kappa}_{t-1}^{(i)} + \hat{d}_t^{(i)}$.
  - In every year, there is a possible change in the mortality trend with probability p.

$$\rightarrow \hat{d}_t^{(i)} = \begin{cases} \hat{d}_{t-1}^{(i)} & , \textit{without trend change in } t-1 \textit{ with probability } 1-p^{(i)} \\ \hat{d}_{t-1}^{(i)} + \lambda_{t-1}^{(i)} & , \textit{with trend change } \lambda_{t-1}^{(i)} \textit{ in } t-1 \textit{ with probability } p^{(i)} \end{cases}$$

  - in the case of a trend change →$\lambda_t^{(i)} = S_t^{(i)} \cdot M_t^{(i)}$
    - with absolute magnitude of the trend change $M_t^{(i)} \sim LN(\mu^{(i)}, \sigma^{(i)})$,
    - sign of the trend change $S_t^{(i)}$bernoulli distributed with values -1, 1 each with probability $\frac{1}{2}$
- For details on the calibration, see Börger and Schupp (2018) and Schupp (2019)

parameters to be estimated for projections starting in t_0:

$p^{(i)}, \mu^{(i)}, \sigma^{(i)}, \hat{\kappa}_{t_0}^{(i)}, \hat{d}_{t_0}^{(i)}, \Sigma$

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Parameter uncertainty in case of one population

Comparison of trend process parameters

Magnitude and relevancy of uncertainty in different sets of trend process parameters

- **trend change parameters $p^{(i)}, \mu^{(i)}, \sigma^{(i)}$**
  - substantial amount of parameter uncertainty
    - typically estimated from a small number of trend changes
  - parameters have a high influence on forecasts
- **starting values $\hat{\kappa}_{t_0}^{(i)}, \hat{d}_{t_0}^{(i)}$**
  - highly case specific amount of parameter uncertainty
    - can be substantial for some period effects and almost irrelevant for other cases
  - parameters have potentially a high influence on forecasts
- covariance matrix $\boldsymbol{\Sigma}$ of the two-dimensional noise vector
  - small amount of parameter uncertainty
    - estimated from a large sample of errors
  - hardly relevant in long-time forecasts
    - moderate impact in 1-year forecasts

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Parameter uncertainty in case of one population

Sources of uncertainty

Two main sources of uncertainty arise from trend process estimation

The two main sources of uncertainty are:

- The actual number of trend changes is unknown due to randomness in the data.
  - The actual number of trend changes may have a weight (based on relative likelihood) significantly different from zero, but it may be far from 1.
  - However, we can use weights to assign probabilities to different sets of parameters estimated from trend curves with varying numbers of trend changes.
- Assuming the actual number of trend changes to be known, significant uncertainty still remains due to parameter estimation from only a small number of trend changes.
  - Here, the maximum likelihood estimation provides covariance matrices of (approximate) standard errors for each value of k trend changes.

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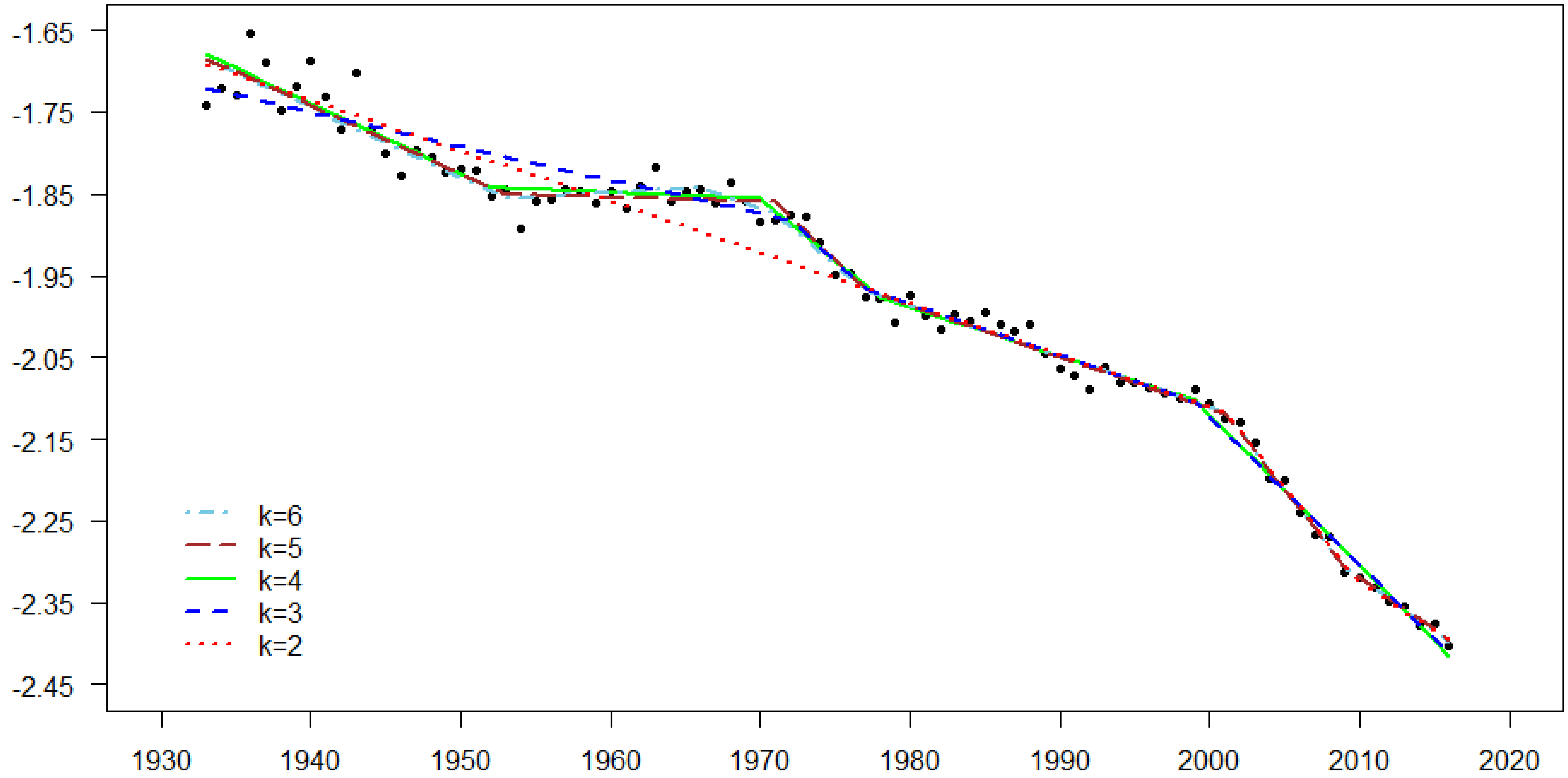
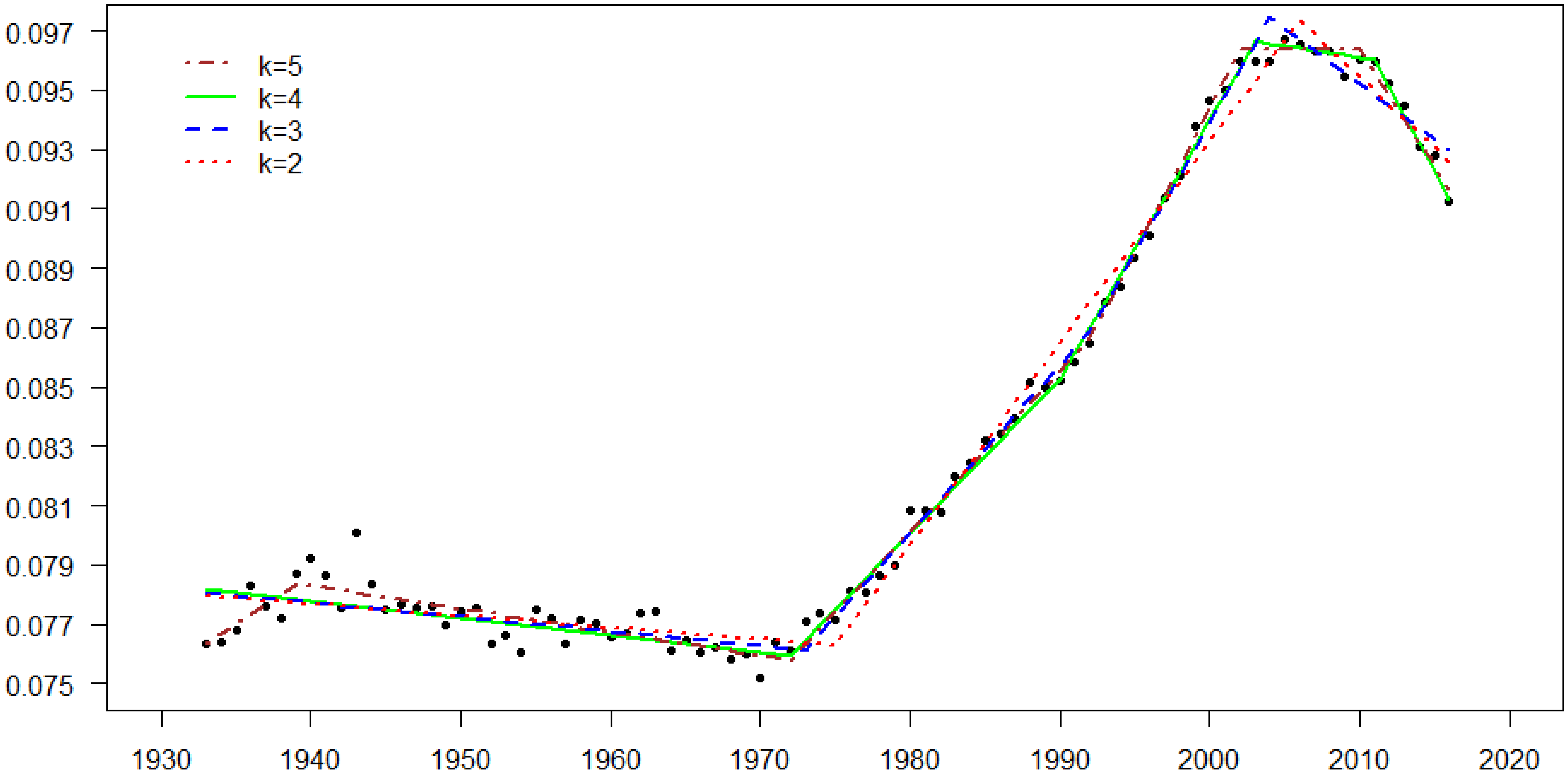
Specification of a Trend Model

Parameter estimation for US males $\kappa_t^{(1)}$



Specification of a Trend Model

Parameter estimation for US males $\kappa_t^{(2)}$



Numerical example for individual populations

Overview

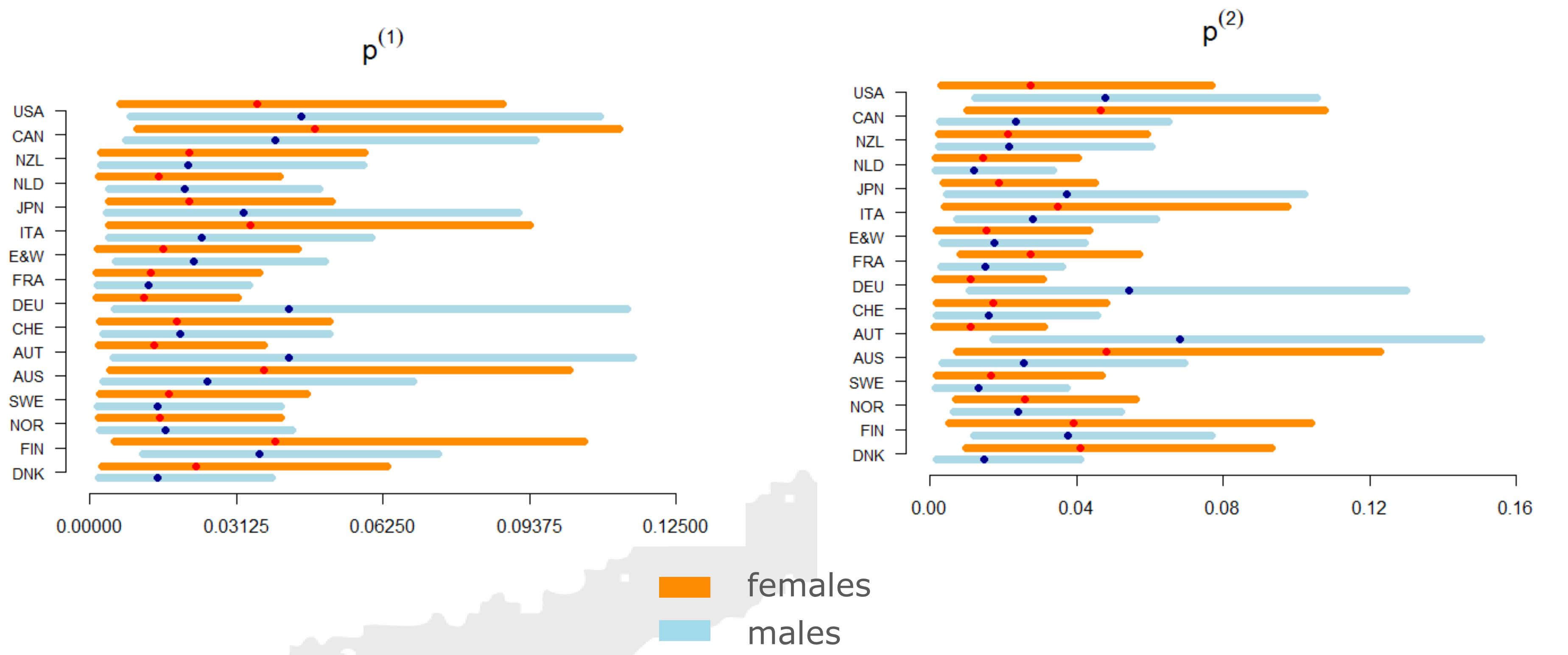
Derivation and comparison of parameter estimates for a large set of populations

- male and female populations in 16 countries
- data obtained from the Human Mortality Database (HMD)
- age range 60–109
- focus on trend change parameters
  - uncertainty in starting values is highly case specific and thus not directly comparable between populations
  - uncertainty in covariance matrix of noise vector appears negligible in general

Numerical example for individual populations

Uncertainty in trend change parameters

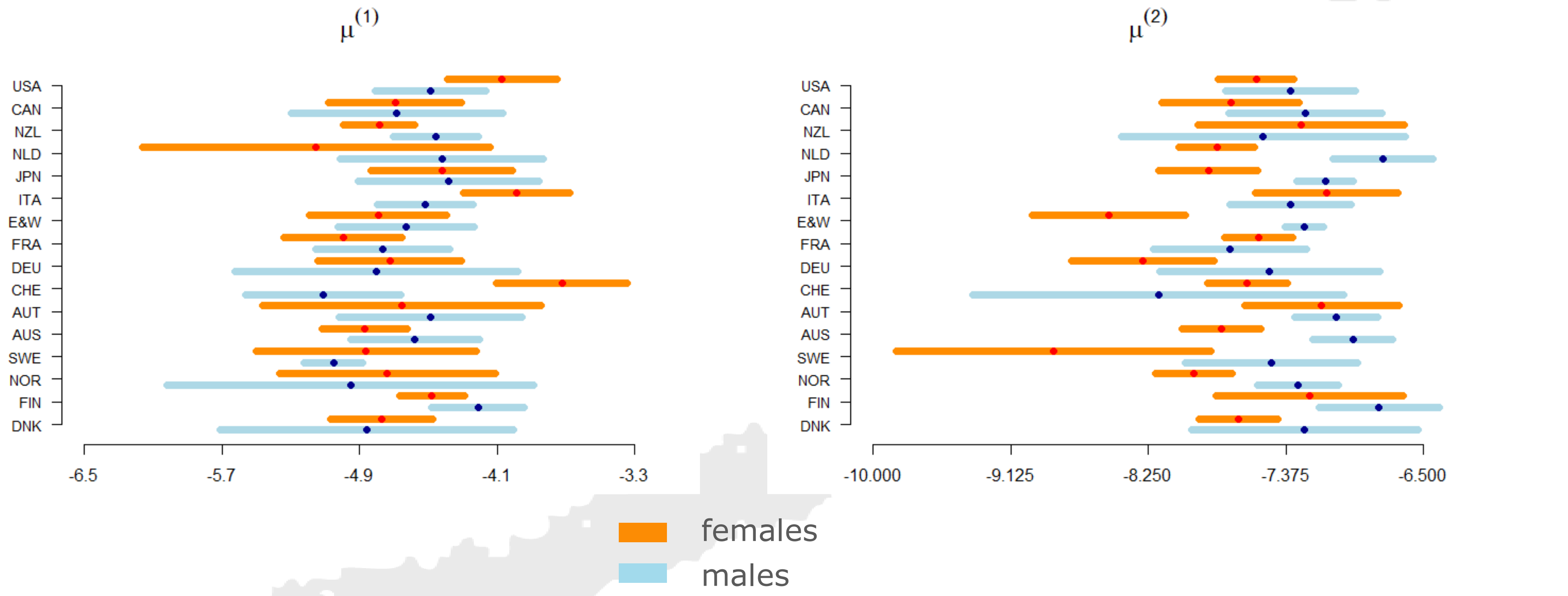
Central parameter estimates and 95% confidence intervals for the trend change probability



Numerical example for individual populations

Uncertainty in trend change parameters

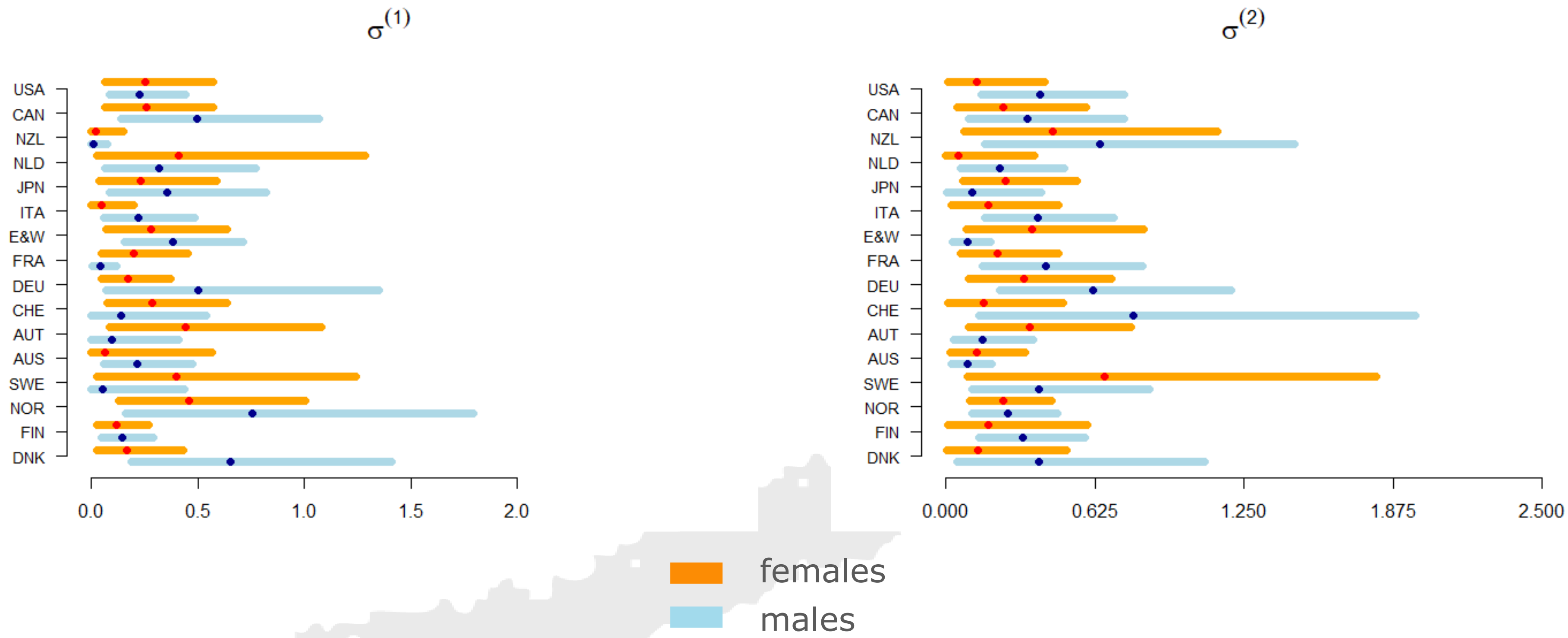
Central parameter estimates and 95% confidence intervals for the trend change magnitudes



Numerical example for individual populations

Uncertainty in trend change parameters

Central parameter estimates and 95% confidence intervals for the trend change magnitudes



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Parameter uncertainty in a multi-population setting

Overview

Combination of parameters from different populations

- trend change parameters
  - Given the similarities between parameter estimates for many populations and given the substantial parameter uncertainties, it seems reasonable to “combine” parameter calibrations from different populations.
  - different assumptions on “similarity” of parameters for different populations:
    - equal parameter values for all populations under consideration
    - population specific realizations of parameter values from one underlying distribution
    - similar parameters without any distributional assumptions
  - next slides contain details on five approaches for “common” parameter estimation

Parameter uncertainty in a multi-population setting

Notation

Necessary notation for explanations on the next slides

- We combine data from several populations to obtain calibrations for the trend change parameters $\theta^{(i)} = \left(p^{(i)}, \mu^{(i)}, \sigma^{(i)}\right)$.
- We analyze a set P of 32 populations (see example above), and the index $\cdot_p$ denotes specific parameter estimates etc. for population $p \in P$.
- Let N_p denote the number of data points for population p.
- Let $p^* \in P$ be the population whose future mortality evolution is to be projected in a practical application. We will consider US males to illustrate the different approaches in a numerical example below.
- Let v_p denote a weight for each population to account for population specific credibility, e.g. due to data reliability issues or longer/shorter data histories.
  - We will use $v_p = N_p / \sum_{q \in P} N_q$ in the numerical example.

Parameter uncertainty in a multi-population setting

Five approaches

1st approach: maximum likelihood

- assumption: **equal parameters** for all populations
- MLE of trend change parameters from the set of all observed trend changes.

2nd approach: weighted average

- assumption: **same but unknown distribution**.
- The common parameter estimates are

$$\theta^{(i)} = \sum_{p \in P} v_p \cdot \theta_p^{(i)}.$$

4th approach: credibility approach

- assumption: **similar parameter values** for different populations
- larger influence of population specific estimate: $v_{p^*} = 0.5$

3rd approach: parameter sampling

- assumption: parameters $\theta_p^{(i)}$ belong to the **same but unknown distribution**.
- approximation by empirical distribution $F_{\theta^{(i)}}$ derived from the population specific parameters $\theta_p^{(i)}$ and the weights v_p, i.e. $P(\theta_p^{(i)}) = v_p$ for $p \in P$ and zero otherwise.

5th approach: Bayesian approach

- assumption: **same but unknown distribution**.
- This distribution is approximated by the prior distribution $F_{\theta^{(i)}}$ (see above).
- The realized $\kappa_{p^*}^{(i)}$ processes provide additional information on likely parameter values $\theta_{p^*}^{(i)}$.

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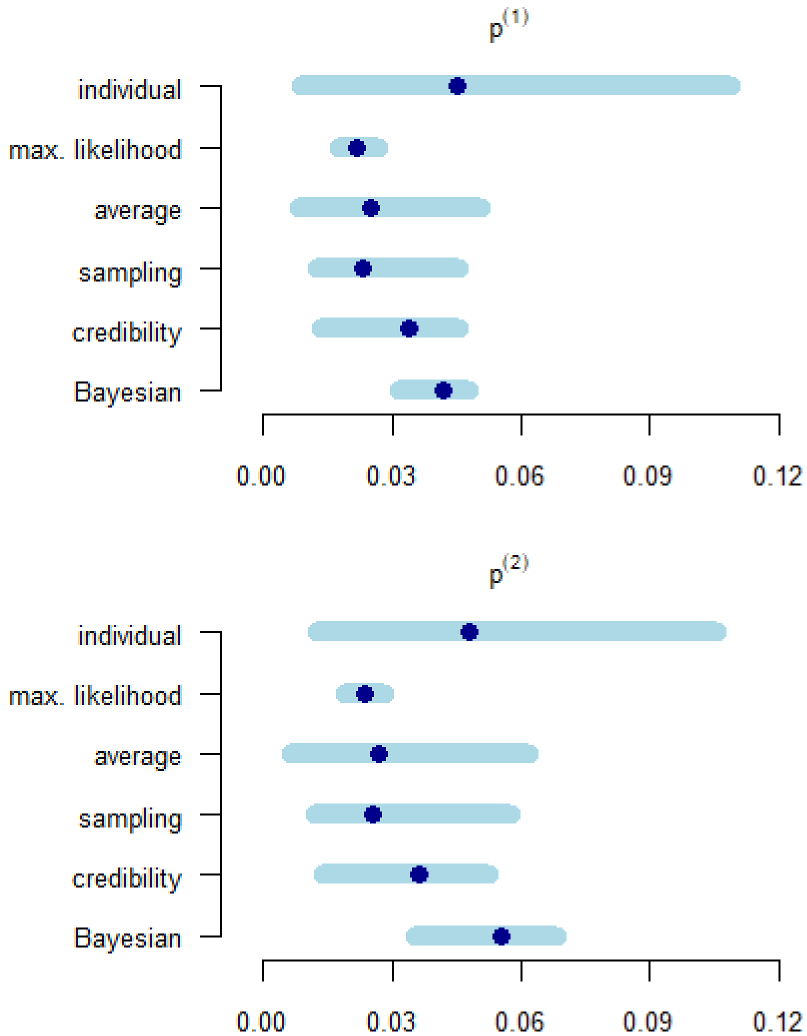
Numerical example for multi-population calibrations

Trend change probabilities

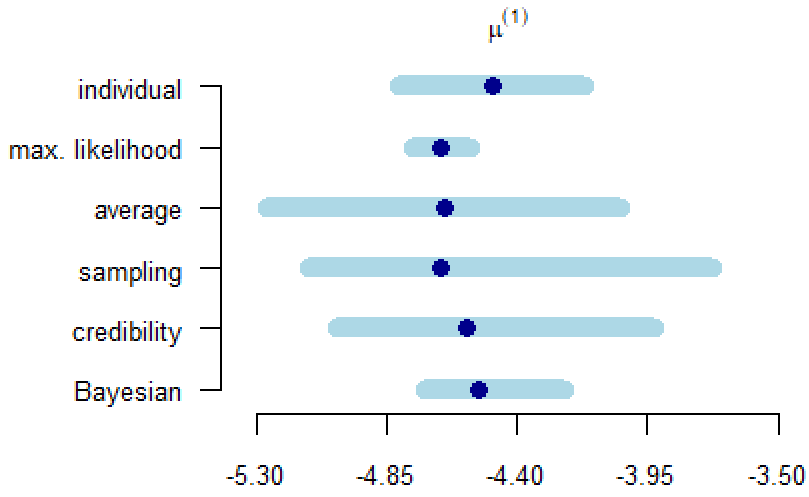


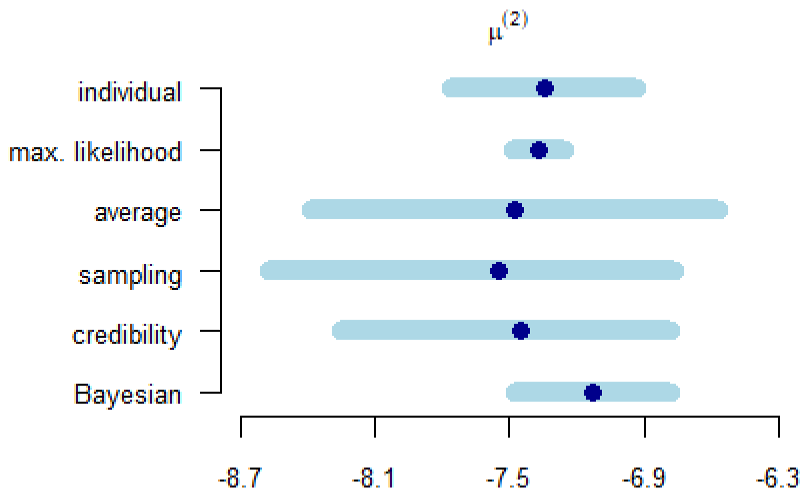
Trend change probabilities

- Central parameter estimates mostly decrease in multi-population approaches.
  - compensation for comparably large number of trend changes for US males
- uncertainty in the maximum likelihood approach considerably smaller than in all other approaches, in particular the individual case.
  - impact of data aggregation very obvious
- as expected, similar results for average and sampling approach
- credibility and Bayesian approaches somewhere in between the individual calibration and the averaging/sampling.
  - interpretation as sampling with overweighting of individual (and in Bayesian case also similar) parameter estimates

Numerical example for multi-population calibrations

Trend change magnitudes



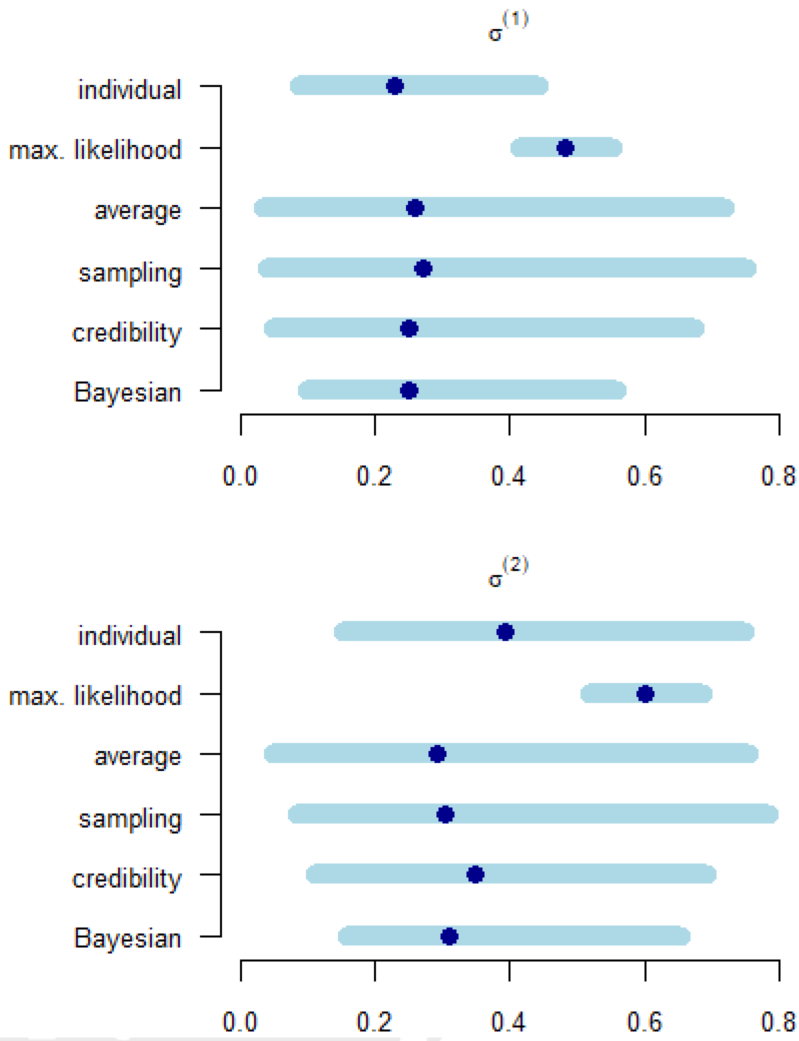


Means of trend change magnitudes

- Again, uncertainty in the maximum likelihood approach is considerably smaller than in the other cases.
- again, similar results for average and sampling approach
  - Uncertainty remains or even increases compared to the individual calibration.
  - This is primarily systematic uncertainty arising from the assumption of a distribution for population specific parameter values, as opposed to unsystematic uncertainty from the parameter estimation.

Numerical example for multi-population calibrations

Trend change magnitudes



Standard deviations of trend change magnitudes

- Central estimates in maximum likelihood approach increase as variability increases due to data aggregation
  - uncertainty is again considerably smaller than in the individual case.
- Uncertainty for the other approaches remains or even increases again compared to the individual calibration due to systematic uncertainty.

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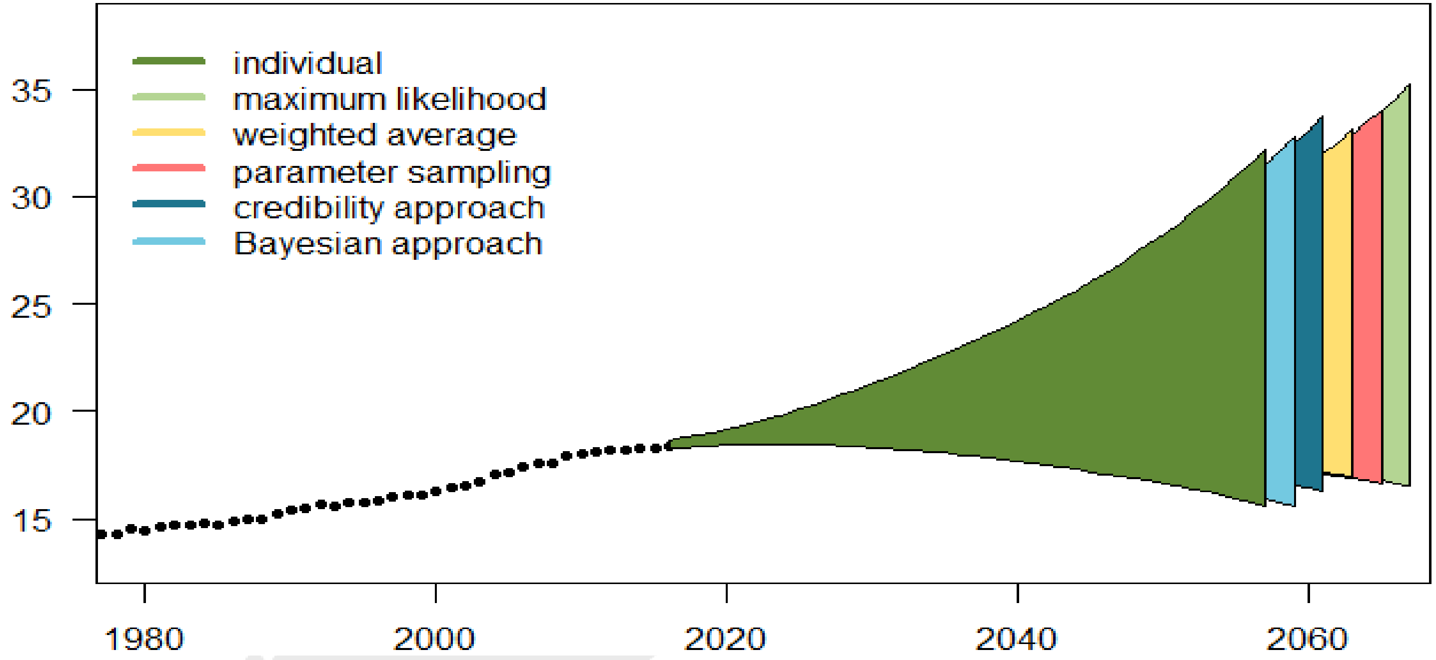
Numerical example for multi-population calibrations

Trend change parameters

Comparison based on remaining period life expectancies for 65-year old US males

- In each case, we use the same approach for the starting values and the covariance matrix of the errors.



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Conclusion

- Somehow similar trend changes in mortality for many populations worldwide
- Uncertainty in trend change parameters can be better understood when data from several countries is combined.
- Data aggregation reduces parameter uncertainty substantially in the example of US males, even though the reduction clearly depends on the aggregation approach.
- For other populations with comparably small individual parameter estimates and thus possibly underestimated uncertainty, the opposite may be observed.

Literature

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