Institut für Finanz- und Aktuarwissenschaften

# The Future of Mortality

## Mortality Forecasting by Extrapolation of Deaths Curve Evolution Patterns

- 24th International Congress on Insurance: Mathematics and Economics
- July, 7th 2021
- Martin Genz
- Joint work with Matthias Börger and Jochen Ruß

# Agenda

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# Motivation

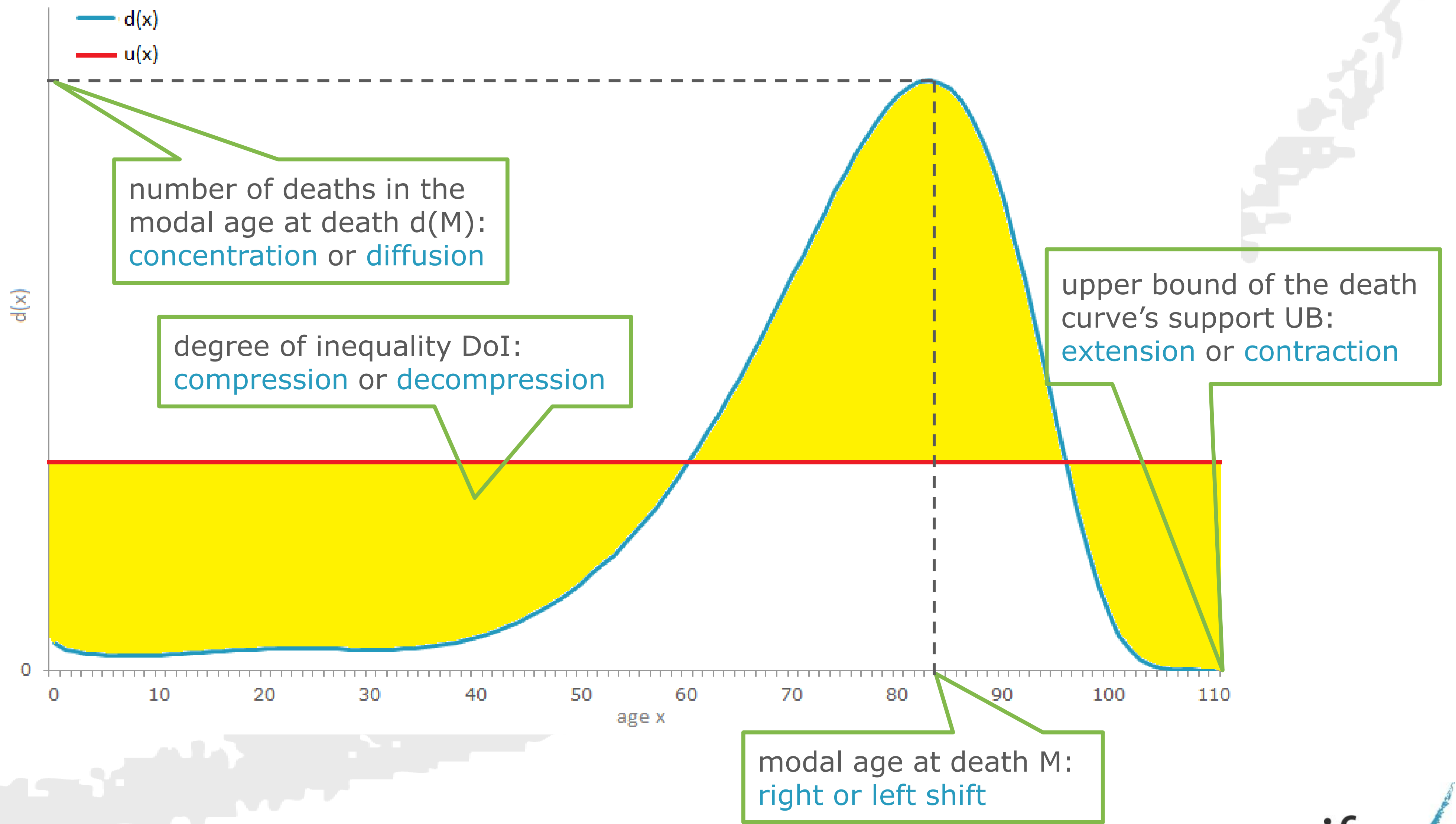
- Estimates for future mortality are important in many areas, for example:
  - projections of social security systems,
  - risk management in the private pension and life insurance sector, etc.
- **On the one hand**, there is a variety of mortality models, e.g.:
  - the Lee-Carter-Model (LC; Lee and Carter 1992),
  - the Cairns-Blake-Dowd-Model (CBD; Cairns et al. 2006).
- **However**:
  - Often the model parameters lack a clear demographic interpretation.
  - Mortality forecasts might not be plausible from a demographic perspective.
- **On the other hand**, also demographers forecast future mortality, e.g.:
  - Oeppen and Vaupel (2002) forecast the world record life expectancy.
  - Dong et al. (2016) "suggest that the maximum lifespan of humans is fixed".
- **However**:
  - Demographic forecasts often focus on single aspects of the mortality evolution.
  - They typically do not (quantitatively) forecast complete distributions of deaths.

- It seems worthwhile to combine the purely statistical approach of mortality models with the demographic expertise.

# The Classification Framework of Börger et al. (1)

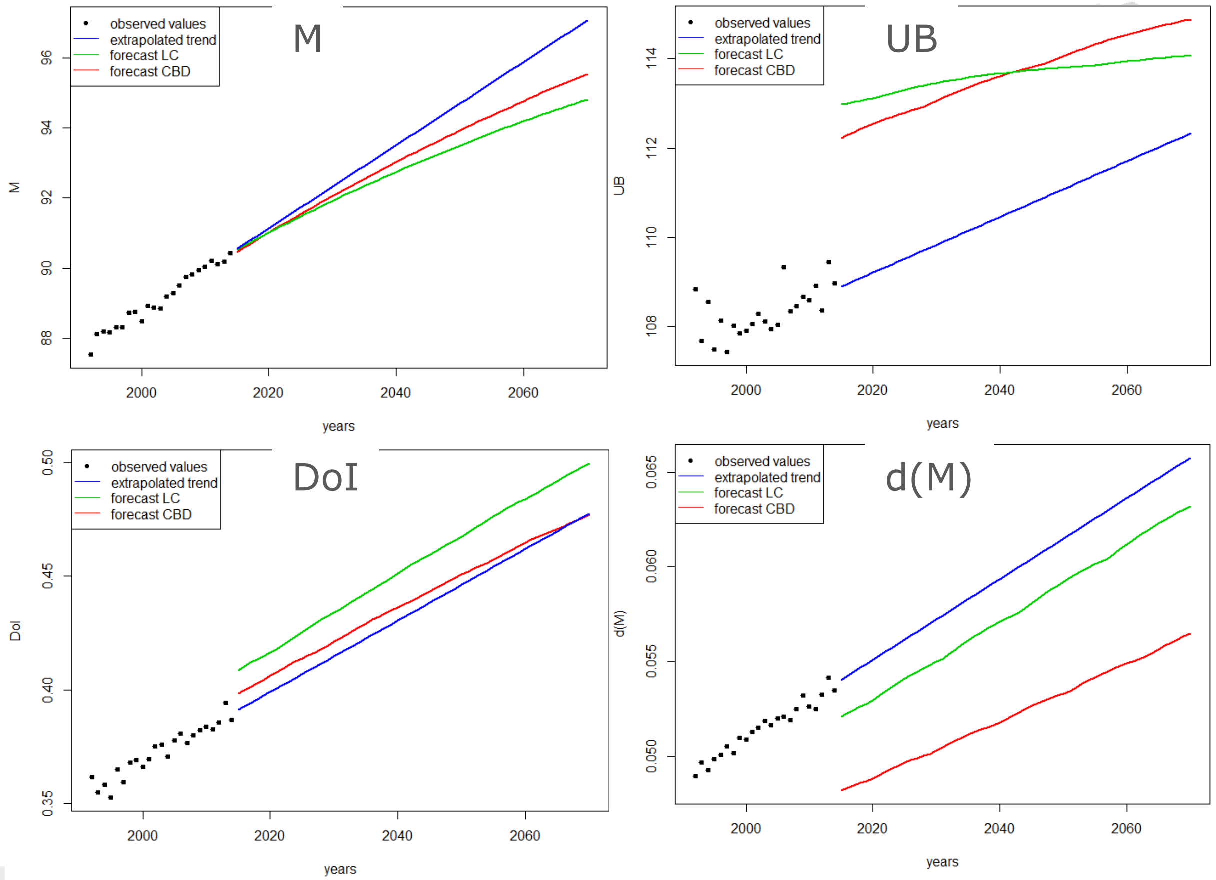
- The basis for the model we suggest is a unique classification framework for mortality evolution patterns by Börger et al. (2018).
  - This framework is based on four statistics on the deaths curve.
  - It gives a comprehensive picture of the deaths curve's evolution in the past.

# The Classification Framework of Börger et al. (2)



# Consistency Issues in Existing Forecasting Models

- A mortality forecast can be regarded as demographically reasonable, if the most recent trends in these four (and other demographic) statistics are smoothly continued.
- We forecast mortality with the LC and the CBD mortality models. As an example, we use HMD data for Swiss females from 1992 to 2014.
- From these forecasts we determine the four aforementioned statistics:




- Three of the four statistics exhibit significant jumps at the transition from the calibration to the forecasting period in both mortality models.
- This indicates that the extrapolated deaths curves from the LC and the CBD model in this example do not exhibit smooth changes at this transition.



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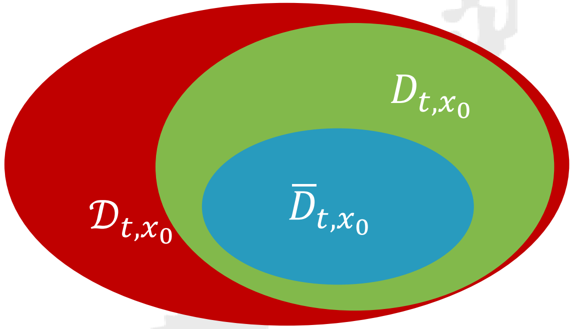
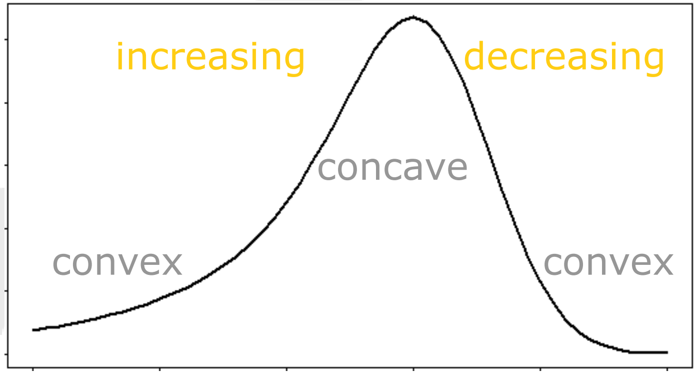
# A New Best Estimate Mortality Model

## Theoretical Concept

- We assume that a prediction $(M_t, UB_t, DoI_t, d(M)_t)$ of the four statistics for any year $t$ of the forecasting period is given.
- We start at the space of all density functions on the interval between the starting age $x_0$ and $UB_t$. We denote it by $\mathcal{D}_{t,x_0}$.




- We observed, that the deaths curve has a typical shape – for example it is…
  - … unimodal, …
  - … monotonically increasing left of the mode, …
  - … monotonically decreasing right of the mode, …
  - … convex at the left and right tail, …
  - … concave around the mode.




- Thus, we choose a forecast from a subset $D_{t,x_0} \subset \mathcal{D}_{t,x_0}$, which contains deaths curves with a reasonable shape.
- Finally, the deaths curve's forecast must fit the statistics' projection.
- We therefore further restrict the set of potential deaths curve's forecasts to the subset $\overline{D}_{t,x_0} \subset D_{t,x_0}$, which contains deaths curves which fit the tupel $(M_t, UB_t, DoI_t, d(M)_t)$.

# A New Best Estimate Mortality Model

## Implementation

- For an implementation of this procedure a feasible representation of the deaths curve must be chosen. We use a B-Spline representation of the deaths curve with 21 B-splines of fifth degree, such that the deaths curve can be written as

$$\hat{d}_t(x) = \sum_{j=1}^{21} a_t^{(j)} \cdot b_t^{(j)}(x) = B_t(x) * a_t,$$

  where $B_t\colon \mathbb{R} \to \mathbb{R}^{21}$ is a vector-valued function and $a_t = \left(a_t^{(1)}, \dots, a_t^{(21)}\right)^T$ is a vector of spline weights.

- Each spline $b_t^{(j)}(x)$, $j \in \{1, \dots, 21\}$ is centered at its so-called knot $k_t^{(j)}$ and symmetric around this knot. Furthermore, it is different from zero only on a certain interval around this knot.
- The algorithm always starts in a year $t_0$, where we are given a deaths curve $d_{t_0}$ and its B-spline representation with $k_{t_0}^{(j)}$, $j \in \{1, \dots, 21\}$ and $a_{t_0}$.

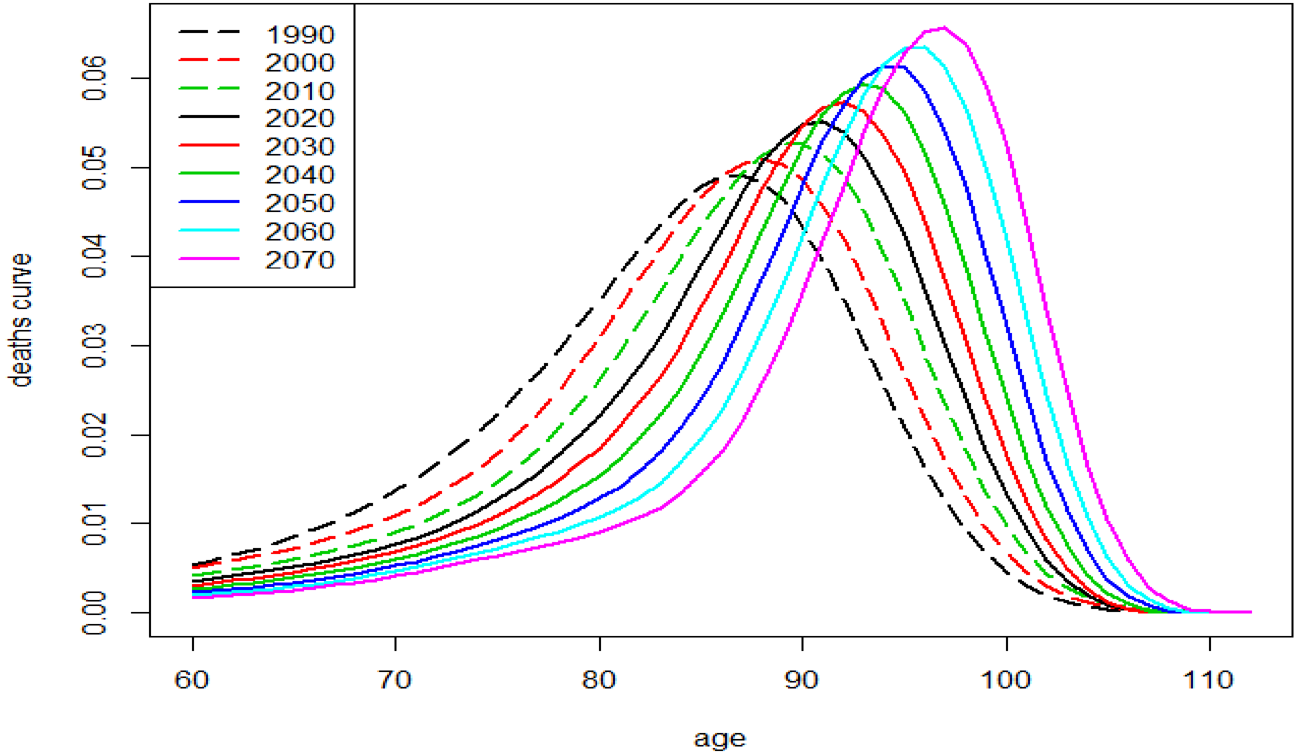
To specify a concrete spline representation of a deaths curve for a future year $t$, we need to determine:

- the spline knot positions $k_t^{(j)}$, $j \in \{1, \dots, 21\}$
- the spline weights $a_t = \left(a_t^{(1)}, \dots, a_t^{(21)}\right)^T$

# Examples (1)

## Extrapolation of the Most Recent Trend

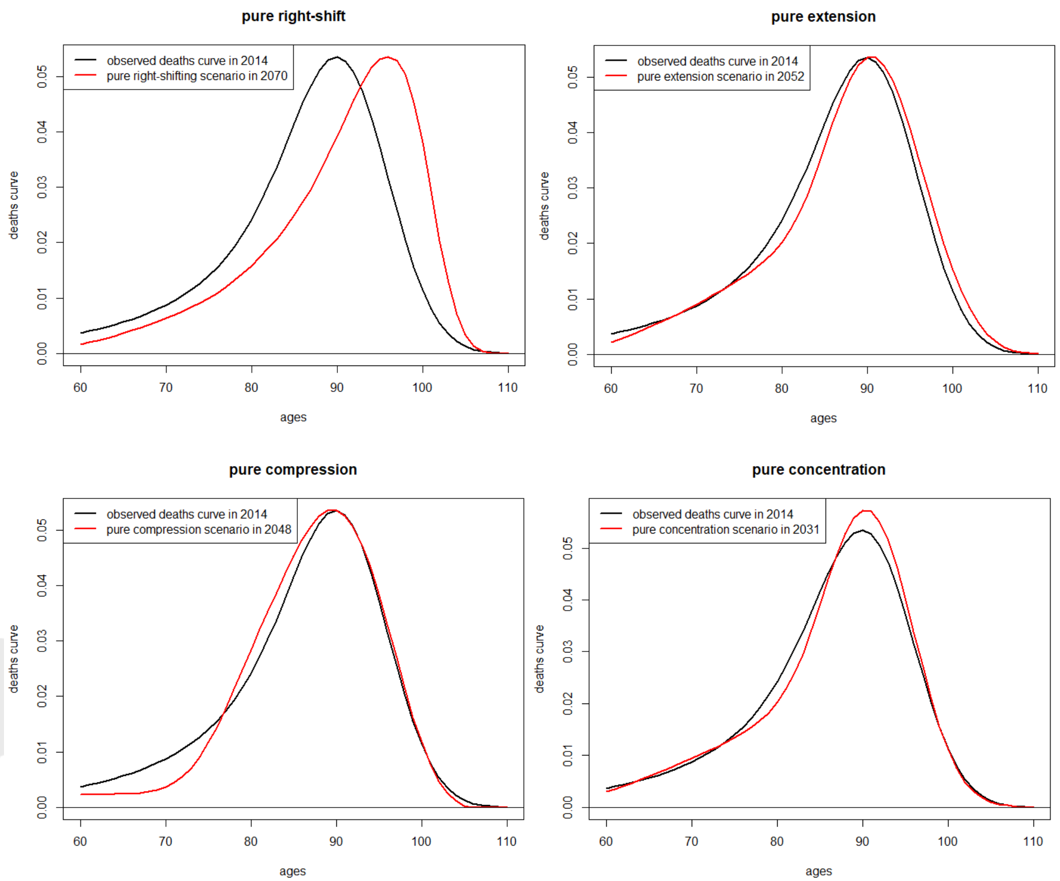
- For the examples in what follows we use HMD data for Swiss females.
- In a first example, we linearly extrapolated the most recent trend for each statistic until 2070 and determined a deaths curve for each calendar year:



# Examples (2)

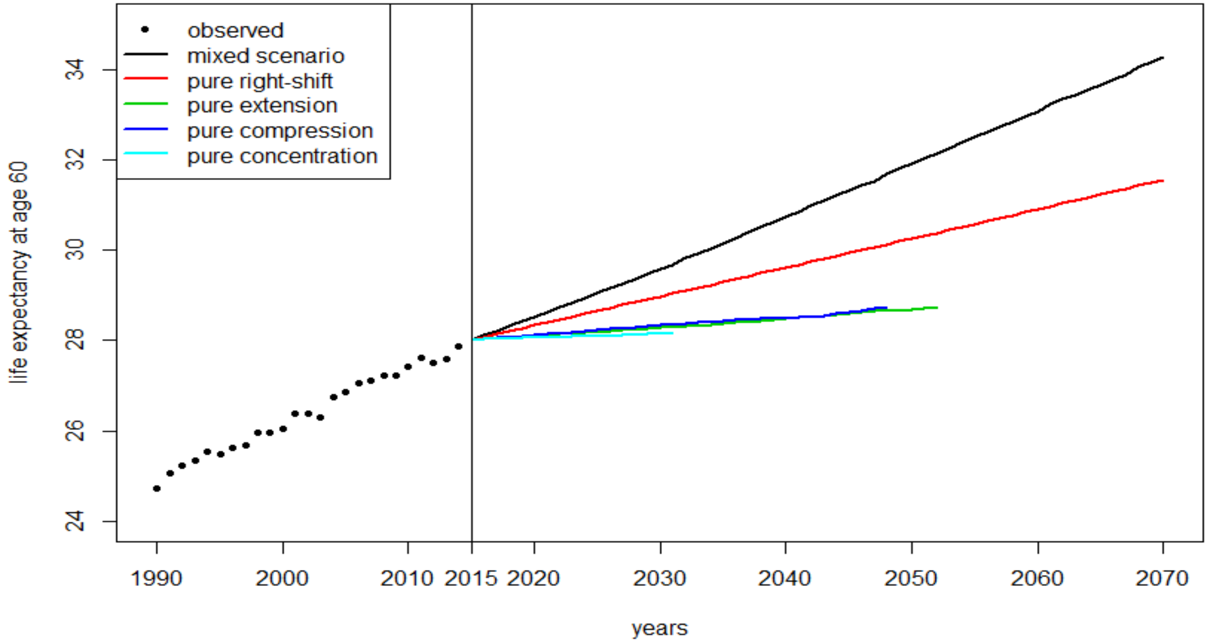
## Pure Scenarios

- In a second example we **analyze so-called "pure scenarios"**, i.e. scenarios where only one component continues its trend while the other three components stay neutral.
- For each pure scenario we compare the deaths curve in 2014 to the last future year where we still have reasonable deaths curves:
  - Pure right-shift (until 2070)
  - Pure extension (until 2052)
  - Pure compression (until 2048)
  - Pure concentration (until 2031)
- Note that "pure scenarios" usually cannot happen on a long-run without violating the shape requirements.



# Examples (3)
## Pure scenarios

For each pure scenario as well as for the base scenario (i.e. extrapolation of the most recent trend) we determined the remaining period life expectancy at the starting age 60 $e(60)$:




- In each pure scenario, the increase of the forecasts of $e(60)$ is slower than in the mixed scenario.
- The increase in $e(60)$ in the pure right-shifting scenario is faster than in any other pure scenario.

# Examples (4)

## Stress scenario

- In a third example we **analyze a stress-scenario**, where the trend of one or more statistics is intensified.
- To this end, we use the base scenario but double the intensity of right-shifting mortality and extension from 2015 to 2070.
  - Demographically this expert scenario means that the position of the deaths curve will change twice as fast in the future than recently observed, while trends in the deaths curve's shape remain unaltered.
  - In this scenario we obtained reasonably shaped deaths curves until 2070.
- To illustrate the differences between the base scenario and the stress scenario we calculated the cohort life expectancy of a 60-year old Swiss female in 2015 (i.e. with year of birth 1954).
- This figure increases from 30.4 years in the base scenario to 32.1 years in the stress scenario which is an increase of more than 5.4%.

# Conclusion

- In the paper we discuss the plausibility of mortality forecasts from a demographic perspective:
  - Trends in key demographic figures should be reasonably extrapolated into the future.
  - We illustrate this with two well-known mortality models: the LC and the CBD mortality model.
- We develop a new deterministic mortality model which …
  - … builds on four demographic statistics on the deaths curve, …
  - … forecasts future mortality in a demographically reasonable manner, and …
  - … allows for the incorporation of expert’s opinions.
- We discuss issues in the practical implementation of this model.
- We illustrate the benefits of the model with different forecast mortality scenarios.

# Thank you for your attention!

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