

Allowance for Investment Expenses under Market-Consistent Valuation

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Introduction and motivation

Illustration of the problem

There can be cash flows associated with holding an asset, positive cash flows (e.g., dividends) as well as negative cash flows (e.g., expenses).

Market-consistent, arbitrage-free valuation usually relies on risk-neutral valuation.

- The treatment of expenses associated with holding an asset in the context of risk-neutral valuation is not as straight-forward as it may seem.

Well-known starting point: assume that there are no expenses associated with holding an asset.

- Let us assume we want to model an asset whose spot price's dynamics are described by the following SDE under the real-world measure P :

- $dS_t/S_t = \mu_t dt + \sigma_t dW_t^P$

- In a risk-neutral valuation, the dynamics under the risk-neutral measure Q become:

- $dS_t/S_t = r_t dt + \sigma_t dW_t^Q$

Today, we will focus on this part of the SDE.

Usually this is the more interesting part of the SDE (modelling of volatility, correlation, jumps, ...).

What changes if there are expenses associated with holding the asset?

Introduction and motivation

Illustration of the problem

Let us now assume there are continuous expenses associated with holding the asset (denoted by c_t).

- In a **real-world projection**, the drift parameter / expected return of e.g. a real-estate investment could be modelled and parameterised in two ways:
 - as **gross return** before expenses, which is **explicitly reduced by c_t** : $dS_t/S_t = (\mu_t^g - c_t)dt + \dots$,
 - or as **net return** (income less expected expenses associated with the investment): $dS_t/S_t = \mu_t^n dt + \dots$.
- Both variants lead to the same modelled investment performance in the real-world projection.
- If we apply both variants in a **risk-neutral valuation**, however, differences emerge:
 - Without further thinking, the **net return** will be replaced by the risk-free short rate r_t .
 - The modelled asset has drift r_t , i.e. $dS_t/S_t = r_t dt + \dots$.
 - In the variant based on the **gross return**, without further thinking, expenses would also be deducted in the risk-neutral projection.
 - The modelled asset has drift $r_t - c_t$, i.e. $dS_t/S_t = (r_t - c_t)dt + \dots$.
- **In the market-consistent valuation of liabilities, usually the result will differ depending on which variant is used.**
 - Which variant is the correct one and will lead to the “right” valuation? Is one of the two variants illustrated above correct or do we need a third variant?
 - And: Is there a general answer to this question or do we need to assess it on a case-by-case basis?

Introduction and motivation

Illustrative example

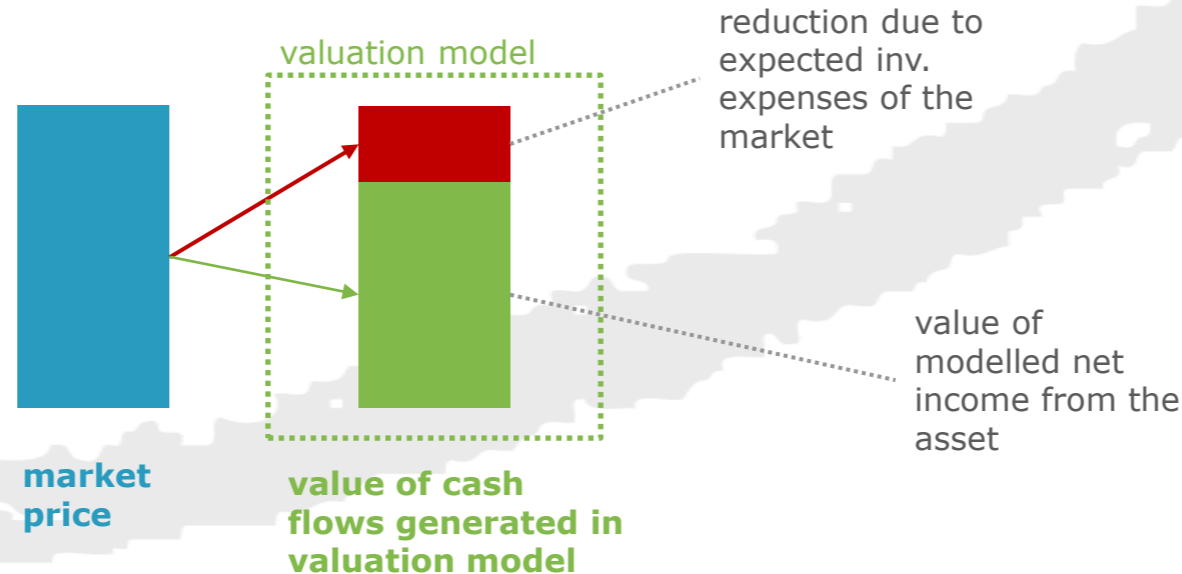
Possible first reaction (of an actuary): **“There are investment expenses in the real world, so they need to be considered and accounted for in the valuation.”**

- Let us look at two unit-linked contracts with guarantees: Contract **A** and Contract **B**
 - In both contracts, the policyholder receives the value of a fund investment at the maturity of the contract, but at least a minimum accumulation benefit.
 - Contract **A** invests in Asset **A** with expenses of 0.5% p.a. and a real-world expected return of 5.5% p.a.
 - Contract **B** invests in Asset **B** with expenses of 1.0% p.a. and a real-world expected return of 6.0% p.a.
 - Asset **A** and Asset **B** shall have the exact same stochastic distribution of returns after deduction of expenses.
- Let us assume we consider all investment expenses in the risk-neutral valuation:
 - With the full consideration of the investment expenses of both assets, the valuation of the technical provision for Contract **B** will be higher than that for Contract **A** (due to higher expenses associated with Asset B).
- **However, from an economic perspective, there appears no obvious reason why the technical provisions of the two contracts should differ - regardless of the insurer’s hedging strategy.**

Introduction and motivation

Illustrative example

- Consequences of considering all expenses: **Is the valuation market-consistent?**
 - In order to be market-consistent, the valuation model must replicate market prices (at least of relevant assets).
 - With the modelling of expected future expenses, the modelled benefits from holding the asset are reduced.
 - The market price of the asset is implicitly split between the present value of future expenses (red) and the present value of future cash flows to the holder (green).
 - The value of owning the asset calculated by the valuation model (green) is smaller than the actual market price of the asset (blue).



Can this be considered as replicating the market price?

Introduction and motivation

Intuition: It does not seem to be appropriate to explicitly consider all investment expenses since some of them are already implicitly included in the valuation.

So, what is the appropriate way of considering investment expenses?

- In the following, we will present two approaches to address the problem in a technically sound manner.

Relevance

- The extent to which investment expenses are considered in the valuation of technical provisions can have a material impact on the calculated market-consistent value.
 - In particular, technical provisions for life insurance are affected.
 - The question how investment expenses are to be treated is especially relevant for the modelling of less liquid investments that often come with high associated expenses.
- There was (and still is) a discussion in the German market about the appropriate modelling of investment expenses in the calculation of technical provisions under the local regulatory framework (Solvency II in the EU).
 - Under Solvency II, a best estimate of the insurer's liabilities has to be calculated with the requirements of market consistency and absence of arbitrage.
- In July 2022, the German Association of Actuaries (DAV) published a report on the findings of an ad-hoc working group on this topic (download link to be found under References).

Agenda

Introduction and motivation

Discussion of solutions

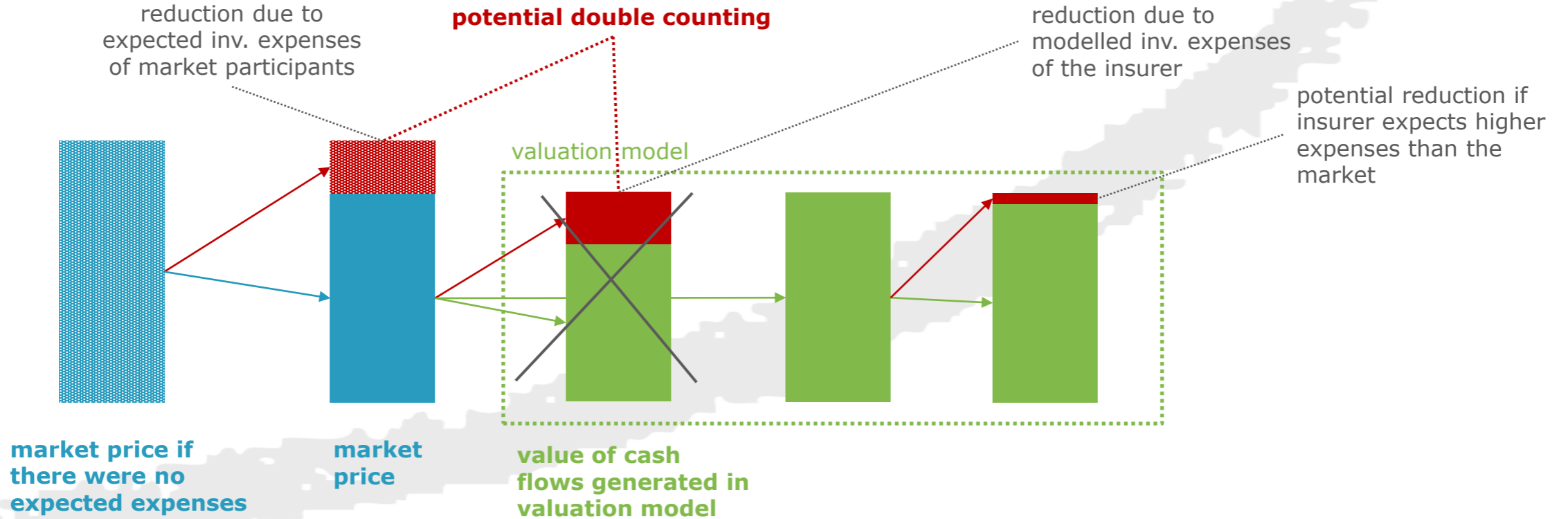
Conclusion and outlook

References and Contact Information

Discussion of solutions

Equilibrium theory: expenses implicitly incorporated in market price

- Approach: If all market participants expect future expenses to be associated with holding the asset, these expected expenses will have an impact on their decision-making and, thus, will reduce the market price of the asset.
- see Gossner and Florig (2021, 2022) for a detailed derivation



Discussion of solutions

Cost-of-carry and the analogy to dividend modelling

- In the context of **commodities** and **commodity derivatives**, expenses that occur while holding an asset are standard and well-described in the corresponding literature. They are part of the so-called "**cost of carry**".
 - Examples for costs: storage cost, necessary insurance while holding the asset
 - The cost-of-carry is accounted for in the pricing of commodity derivatives and in the calculation of forward prices (cf. Geman (2009)).
 - The same holds for dividends in the context of stocks.
- **BUT: Both cost-of-carry and dividends are accounted for in a different way than one might intuitively expect.**
 - With **dividends**, the drift of the spot price is **reduced**, but there is a separate **positive** cash flow (the dividends).
 - In sum (of spot price and dividends), the investment has the drift r_t .
 - With **expenses**, the drift of the spot price is **increased**, but there is a separate **negative** cash flow (the expenses).
 - In sum (of spot price and expenses), the investment has the drift r_t .
- While **expected dividends reduce** the forward price, **expected expenses increase** it.
 - Derivation via replication argument in combination with the no-arbitrage assumption:
 - Replication: borrow money, buy asset, sell at maturity for forward price
 - Dividends earned reduce the overall replication costs. → Forward price can be lower for break-even.
 - Expenses increase the overall replication costs. → Forward price must be higher for break-even.

Discussion of solutions

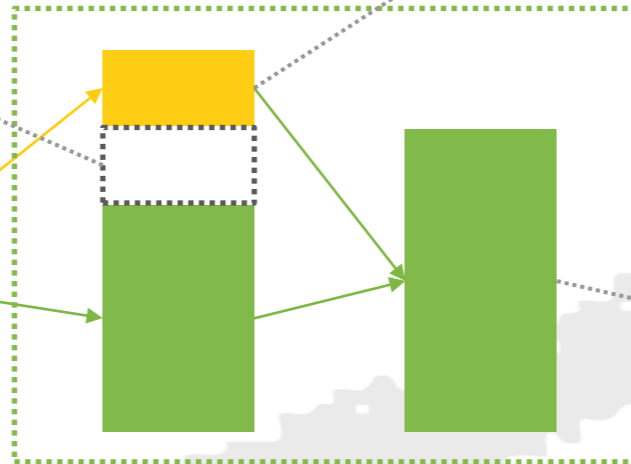
Cost-of-carry and the analogy to dividend modelling

Modelling of dividends

reduction in projected spot price due to reduction of drift due to dividends

value of dividends

valuation model



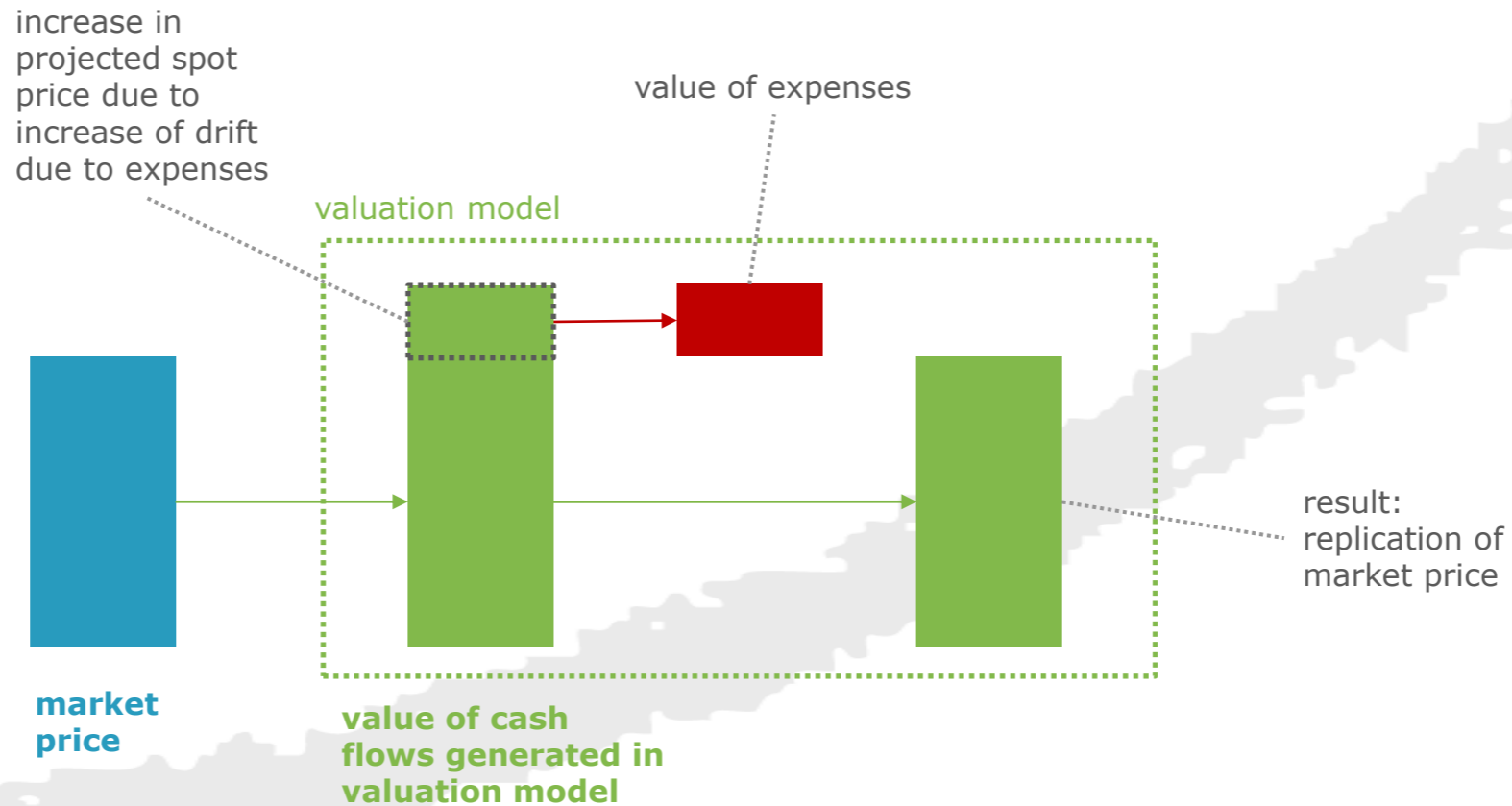
value of cash flows generated in valuation model

in sum: replication of market price

Discussion of solutions

Cost-of-carry and the analogy to dividend modelling

Modelling of **cost-of-carry**



Discussion of solutions

■ **Consequence: How to account for investment expenses in the risk-neutral valuation?**

■ Possible solutions:

- Only model that part of investment expenses that is specific to the insurer, leave out the expenses expected by the market.
- Or: Increase the drift of the asset by the expenses expected by the market, then explicitly deduct the expenses from the insurer's viewpoint.

■ **Challenge:** parameter estimation of "market expectation" of future expenses associated with the asset

- The extent to which expenses are to be taken into account will have to be assessed on a sufficiently granular level, e.g., on (sub)-asset-class basis.

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Conclusion and outlook

Current state of the discussion in the German market

- In July 2022, the German Association of Actuaries (DAV) published a report on the findings of an ad-hoc working group on this topic.
 - Result (shortened and simplified): Yes, there are investment expenses that do not reduce the return of an asset in a risk-neutral valuation.
 - Follow-up work for a further result report: parameter estimation and the specific modelling
- Further discussions, especially with the regulator, seem necessary.

Conclusion and outlook

Conclusion and outlook

- The extent to which investment expenses are considered in the valuation can have a material impact on the calculated market-consistent value of technical provisions.
 - This is especially relevant with respect to less liquid investments that often come with high associated expenses.
- There are at least two approaches that show that it might not be appropriate to explicitly consider all investment expenses in all cases.
- We are currently running further analyses on calibration and justification of assumptions.

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References

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