Stochastic Profit Testing

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- Convention A
- September 2022
Demographic trends
Increase in world-wide life expectancy is not a temporary trend...

Source: Oeppen and Vaupel (2002), extended by own calculations
Deterministic Mortality Projection

Projection

Derivation of adequate mortality projections is fairly complex.

- Heat charts are a helpful tool for the analysis of existing and the derivation of new projections,
- here: mortality improvements depending on age and time

vertical structures: time dependent effects
horizontal structures: age dependent effects
diagonal structures: cohort effects
Implications for Life Insurance Business

New Tasks for Actuaries

Longevity and Mortality Risk are key risk drivers of life insurance business
- the insurance company carries **interest rate risk** and **longevity risk**.
- We expect an increasing importance of retirement products for the securitization of an individuals' longevity risk.

Variety in the guarantee at annuitization, e.g.:
- guaranteed annuity option of NAV (c.f. **GAO** in the example)
- guaranteed annuity option with limit (c.f. **GAOWL**)
- guaranteed lifelong annuity if the contract is annuitized within a certain period. (c.f. **GMIB**)
- many variants and combinations

Innovative retirement phases
- **new traditional products**
  - adopt and adjust solvency optimization concepts from the deferment period
- **unit-linked products**
  - products with a higher **flexibility**, e.g. unit-linked until 80; traditional beyond
- Enhanced Annuities
- Annuity Pools or Mortality Indexed Annuities
Implications for Life Insurance Business

Typical Tasks for Actuaries in Pricing

Those innovations imply a variety of new tasks for actuaries

- **Economic Quantification and Pricing**
  - What is the economic fair value of a guaranteed annuitization rate?
  - Is the guaranteed annuitization rate at annuitization at or out of the money?
    - Is there a need for additional reservation?
  - What is the economic value of options and guarantees during the retirement phase?

- **Surplus participation based on different guarantees**
  - Which surplus participation is appropriate so that a product with a lower/different type of guarantee (e.g., due to a modified annuitization) has the same economic value from the customer’s perspective?

- **Profit Testing**
  - How does a new product design (e.g., a modified annuitization) affect the expected profitability of an insurer? What does a probability distribution of a future profitability of a product look like?

Those questions can only be answered properly within a framework with **stochastic capital market and stochastic mortality**.
Stochastic Profit Testing

Agenda

Motivation

Stochastic Mortality Models for Life Insurance

Model Selection

Lee-Carter Model

Cairns-Blake-Dowd Model

Case Study: Analysis of guarantees at annuitization

Summary

Contact and Literature
Stochastic Mortality Modeling
Model Selection

- What part of the distribution is relevant? E.g. only 99.5%-quantile?
- What structures in historical mortality should be incorporated, e.g., cohort effects?
- What is the relevant age range?
- Should we focus on the uncertainty in realized mortality or in estimated mortality?
- Analysis of a Run-off or only a limited time interval (e.g., 1 year)?
- What is the relevant age range?
- Should we focus on the uncertainty in realized mortality or in estimated mortality?
- Data sources available for model calibration
- Do we need a multi-population model?
- Robustness of the model
- Complexity of the model
Stochastic Mortality Modeling

Lee-Carter Model

The model of Lee und Carter (1992) was the first parametric mortality model and is still widely used today.

- stochastic modeling of mortality rates and transformation to the death probabilities $q_{x,t}$
  \[ q_{x,t} = 1 - \exp(-m_{x,t}) \]

- The historic mortality rates $m_{x,t}$ are modelled with
  - time dependent parameters for each year
  - age dependent parameters for each age.

- extrapolation of the mortality rates with a stochastic simulation of future values of the time dependent parameters
Stochastic Mortality Modeling
Lee-Carter Model

- stochastic extrapolation of the mortality rate $m_{x,t}$
  \[
  \ln(m_{x,t}) = \alpha_x + \beta_x \cdot \kappa_t
  \]

- $\alpha_x$ describes the age dependent base level (a base table in principle).
- $\kappa_t$ describes the changes in mortality over time.
- $\beta_x$ describes the impact of the changes on different ages.
- Subsequently, the further evolution of $\kappa_t$ is simulated with time series models, e.g. with a Random Walk with Drift or AR(1)-processes.

source: own calibration
Another widely used mortality model is the model of Cairns, Blake und Dowd (2006).

observation: In higher ages, the log of the death probabilities is almost a perfect straight line.

Cairns et al. (2009):

Modeling of the logit-death probabilities for a single year with a straight line.
Another widely used mortality model is the model of Cairns, Blake und Dowd (2006).

\[
\logit(q_{x,t}) = \ln \left( \frac{q_{x,t}}{1 - q_{x,t}} \right) = \kappa_t^{(1)} + \kappa_t^{(2)} \cdot (x - \bar{x})
\]

- \( \kappa_t^{(1)} \) describes the general level of mortality over time.
- \( \kappa_t^{(2)} \) describes the evolution of the slope of the mortality curve over time for different ages \( x \).
- \( \bar{x} \) is the median age in the considered age span.
- Again, stochastic scenarios of the future evolution of mortality can be generated by a simulation of the further evolution of both time dependent parameters \( \kappa_t^{(1)} \) and \( \kappa_t^{(2)} \), e.g., with a two-dimensional Random Walk with Drift.

source: own calibration
Stochastic Profit Testing

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Stochastic Mortality Models for Life Insurance

Case Study: Analysis of guarantees at annuitization

Summary

Contact and Literature
Case Study: Analysis of guarantees at annuitization

Overview

- The analysis of different guarantees requires a probability distribution of the (unknown) annuitization factor after the deferment period.
- Depending on the (simulated) capital market and the (simulated) prevailing mortality, there is a different annuitization factor.
Case Study: Analysis of guarantees at annuitization

Wrapup: assumptions and parameters

- Black-Scholes for assets, Shadow-Rate interest rate model
- Independence between capital market and mortality
- Guaranteed interest rate based on returns of 10-year government bonds
- Data for the total population; adjusted to fit insureds mortality
- LC and CBD model
- (Multidimensional) Random Walk with drift; Simulation of realized mortality numbers with Binomial / Poisson distr. see Börger, Ruß und Schupp (2020)
- Similar approach as proposed by the DAV

**Loss for guarantee GAO:**

\[
L_T^{GA} = g \cdot A_T \cdot \max\left(\frac{1}{R_{FT}} - \frac{1}{g}, 0\right)
\]

**Loss for guarantee GAOWL:**

\[
L_T^{GAWL} = g \cdot \min\{A_T; L\} \cdot \max\left(\frac{1}{R_{FT}} - \frac{1}{g}, 0\right)
\]

**Loss for guarantee GMIB:**

\[
L_T^{GMIB} = \max\left\{g \cdot G \cdot \frac{1}{R_{FT}} - A_T, 0\right\}
\]
Case Study: Analysis of guarantees at annuitization

Results

Comparison for the insurers risk for the three different guarantees

decomposition of the TVaR in financial and longevity risk

<table>
<thead>
<tr>
<th></th>
<th>GAO</th>
<th>GAOWL</th>
<th>GMIB</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loss probability</td>
<td>12,6%</td>
<td>12,6%</td>
<td>20,4%</td>
</tr>
<tr>
<td>Expected loss</td>
<td>2.140</td>
<td>540</td>
<td>4.990</td>
</tr>
<tr>
<td>VaR (99,5%)</td>
<td>78.900</td>
<td>15.600</td>
<td>71.840</td>
</tr>
<tr>
<td>TVaR (99%)</td>
<td>107.680</td>
<td>17.360</td>
<td>75.110</td>
</tr>
<tr>
<td>Proportion (TVaR) longevity risk</td>
<td>33,3%</td>
<td>45,4%</td>
<td>0,9%</td>
</tr>
<tr>
<td>Proportion (TVaR) financial risk</td>
<td>66,7%</td>
<td>54,6%</td>
<td>99,1%</td>
</tr>
</tbody>
</table>

In thousands

Financial Risk
Longevity Risk
Loss probability

Longevity risk reduced by 80%
Longevity risk further reduced by 90%
Case Study: Analysis of guarantees at annuitization

Results

Comparison for the insurers risk for the three different guarantees analysed with the **CBD model** and the **LC model**

![Comparison graph]

- **GMIB** only has financial risk that can be handled by product design or hedging
- Choice of the mortality model is crucial for a thorough understanding of the insurers risk.
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Summary

**Longevity** and **mortality risk** are key risk drivers of life insurance business
- increasing importance of retirement products and higher retirement rates of deferred annuities
- We observe a variety of new retirement products and guarantees

**A sound risk management** requires to properly **quantify** and **model** the risk of products and guarantees.
- **This is only possible with a stochastic mortality model.**
- We have seen, that the choice of the mortality model is crucial for a thorough understanding of the insurer's risk.
  - A deep understanding of the mortality models is necessary to identify a suitable model.


