# Stochastic Profit Testing

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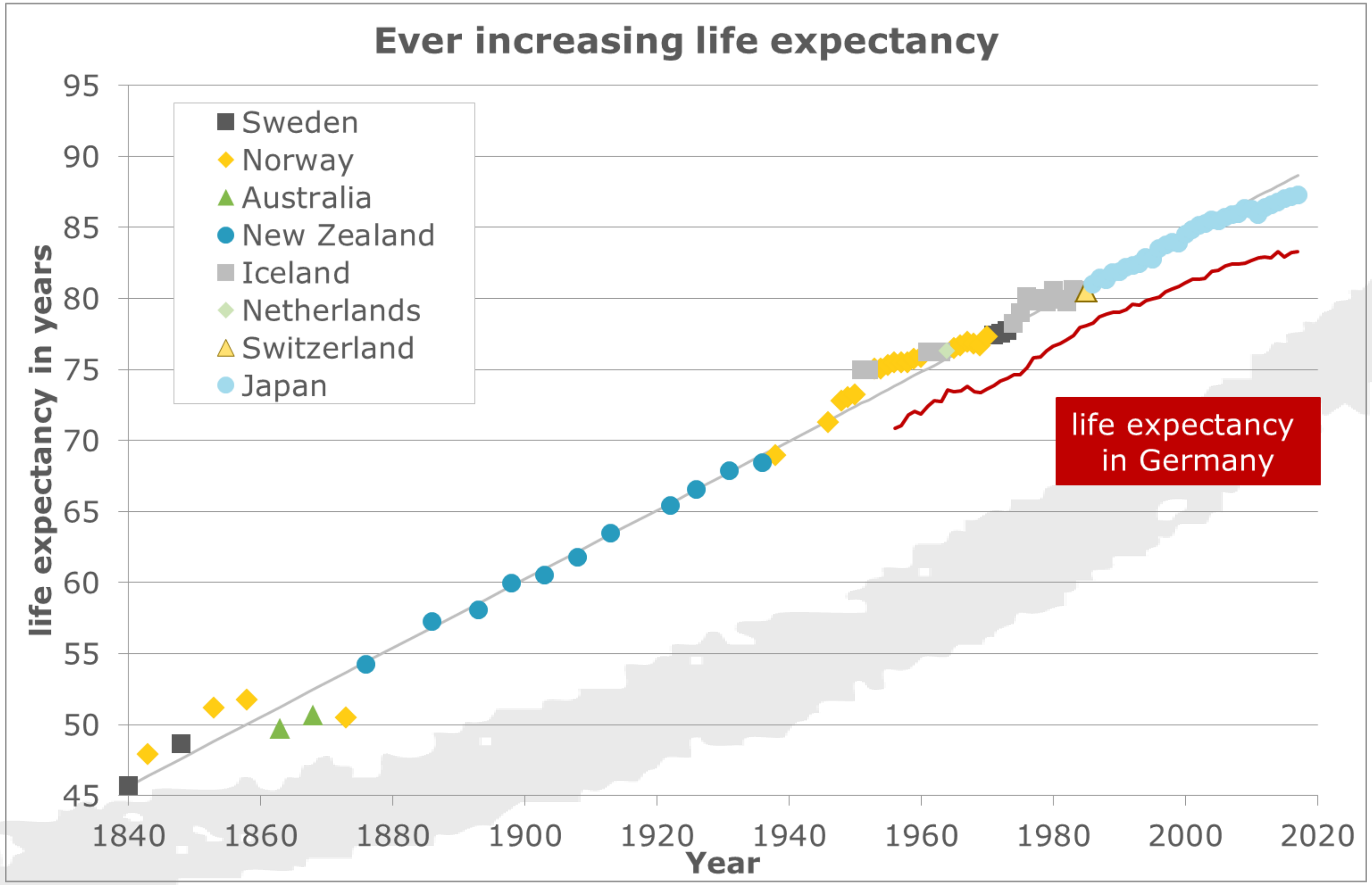
- Convention A
- September 2022

# Demographic trends

## Increase in world-wide life expectancy is not a temporary trend…



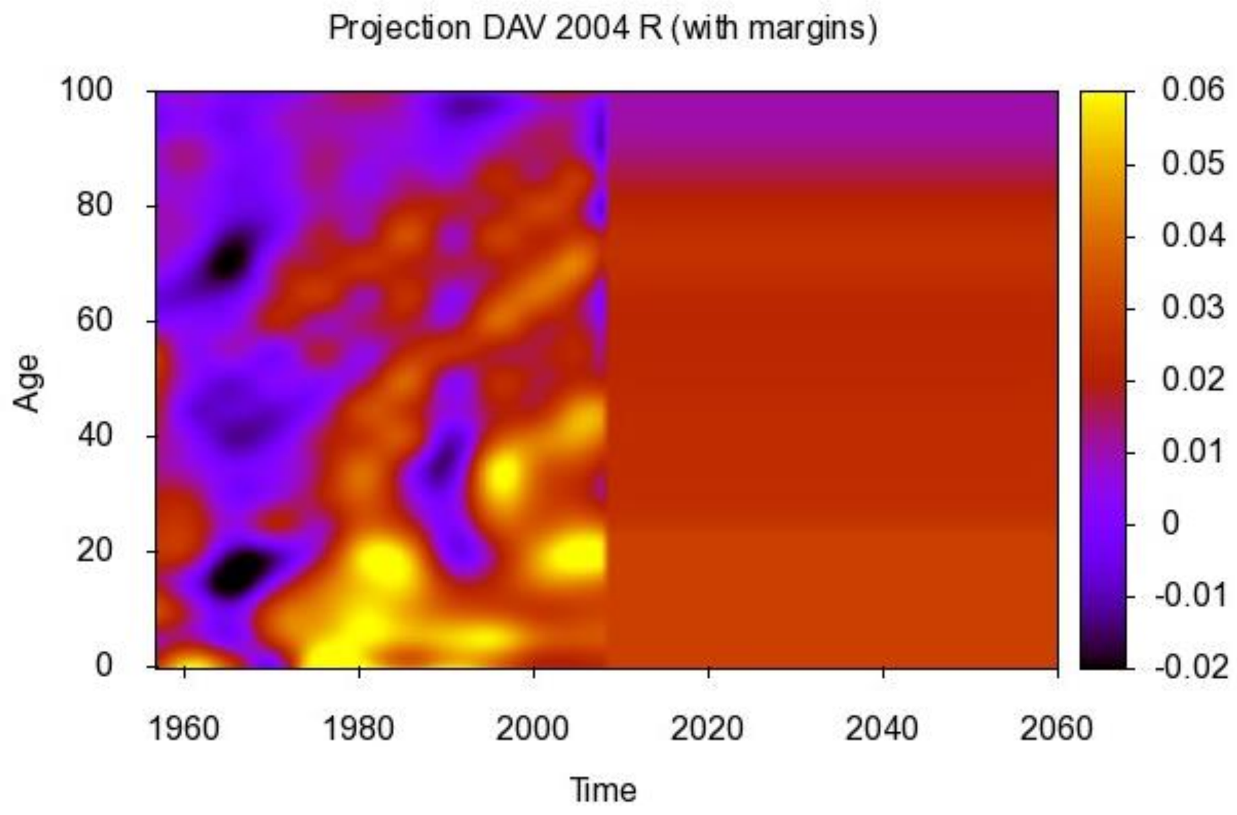

Source: Oeppen and Vaupel (2002), extended by own calculations

# Deterministic Mortality Projection

## Projection

Derivation of adequate mortality projections is fairly complex.

- Heat charts are a helpful tool for the analysis of existing and the derivation of new projections,
  - here: mortality improvements depending on age and time




vertical structures: time dependent effects

horizontal structures: age dependent effects

diagonal structures: cohort effects

# Implications for Life Insurance Business

## New Tasks for Actuaries

Longevity and Mortality Risk are key risk drivers of life insurance business

- the insurance company carries **interest rate risk** and **longevity risk.**
- We expect an increasing importance of retirement products for the securitization of an individuals‵ longevity risk.

Variety in the guarantee at annuitization, e.g.:

- guaranteed annuity option of NAV (c.f. **GAO** in the example)
- guaranteed annuity option with limit (c.f. **GAOWL**)
- guaranteed lifelong annuity if the contract is annuitized within a certain period. (c.f. **GMIB**)
- many variants and combinations

innovative retirement phases

- **new traditional products**
  - adopt and adjust solvency optimization concepts from the deferment period
- **unit-linked products**
- products with a higher **flexibility**, e.g. unit-linked until 80; traditional beyond
- Enhanced Annuities
- Annuity Pools or Mortality Indexed Annuities

# Implications for Life Insurance Business

## Typical Tasks for Actuaries in Pricing

Those innovations imply a variety of new tasks for actuaries

- **Economic Quantification and Pricing**
  - What is the economic fair value of a guaranteed annuitization rate?
  - Is the guaranteed annuitization rate at annuitization at or out of the money?
    - Is there a need for additional reservation?
  - What is the economic value of options and guarantees during the retirement phase?
- **Surplus participation based on different guarantees**
  - Which surplus participation is appropriate so that a product with a lower/different type of guarantee (e.g., due to a modified annuitization) has the same economic value from the customer‘s perspective?
- **Profit Testing**
  - How does a new product design (e.g., a modified annuitization) affect the expected profitability of an insurer? What does a probability distribution of a future profitability of a product look like?

Those questions can only be answered properly within a framework with **stochastic capital market and stochastic mortality.**

# Stochastic Profit Testing

## Agenda


**Motivation**

**Stochastic Mortality Models for Life Insurance**

Model Selection

Lee-Carter Model

Cairns-Blake-Dowd Model

**Case Study: Analysis of guarantees at annuitization**

**Summary**

**Contact and Literature**

# Stochastic Mortality Modeling

## Model Selection

What part of the distribution is relevant? E.g. only 99,5%-quantile?

What structures in historical mortality should be incorporated, e.g., cohort effects?

What is the relevant age range?

robustness of the model

Analysis of a Run-off or only a limited time interval (e.g., 1 year)?

Should we focus on the uncertainty in realized mortality or in estimated mortality?

complexity of the model

data sources available for model calibration

Do we need a multi-population model?

# Stochastic Mortality Modeling

## Lee-Carter Model

The model of Lee und Carter (1992) was the first parametric mortality model and is still widely used today.

- stochastic modeling of mortality rates and transformation to the death probabilities $q_{x,t}$

$$q_{x,t} = 1 - \exp(-m_{x,t})$$

- The historic mortality rates $m_{x,t}$ are modelled with
  - time dependent parameters for each year
  - age dependent parameters for each age.
- extrapolation of the mortality rates with a stochastic simulation of future values of the time dependent parameters

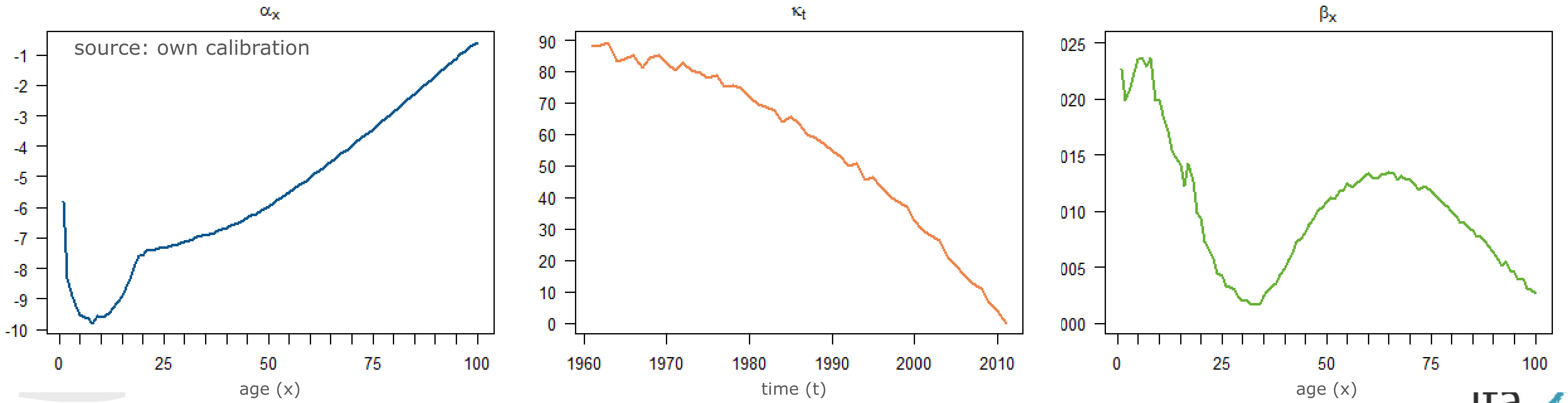
# Stochastic Mortality Modeling

## Lee-Carter Model

- stochastic extrapolation of the mortality rate $m_{x,t}$

$$\ln(m_{x,t}) = \alpha_x + \beta_x \cdot \kappa_t$$

- $\alpha_x$ describes the age dependent base level (a base table in principle).
- $\kappa_t$ describes the changes in mortality over time.
- $\beta_x$ describes the impact of the changes on different ages.
- Subsequently, the further evolution of $\kappa_t$ is simulated with time series models, e.g. with a Random Walk with Drift or AR(1)-processes.

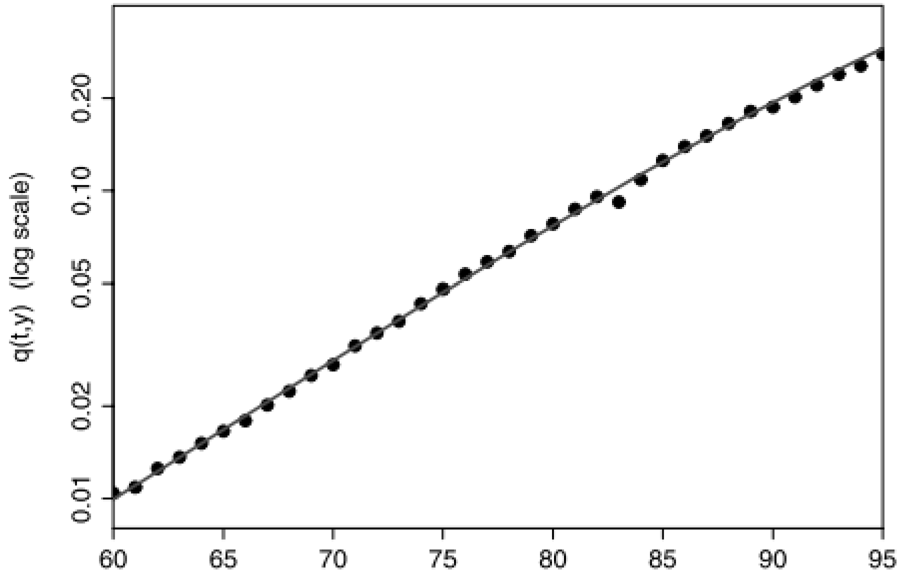


# Stochastic Mortality Modeling

## Cairns-Blake-Dowd Model

Another widely used mortality model is the model of Cairns, Blake und Dowd (2006).

- observation: In higher ages, the log of the death probabilities is almost a perfect straight line.
  - Cairns et al. (2009):




Modeling of the logit-death probabilities for a single year with a straight line.

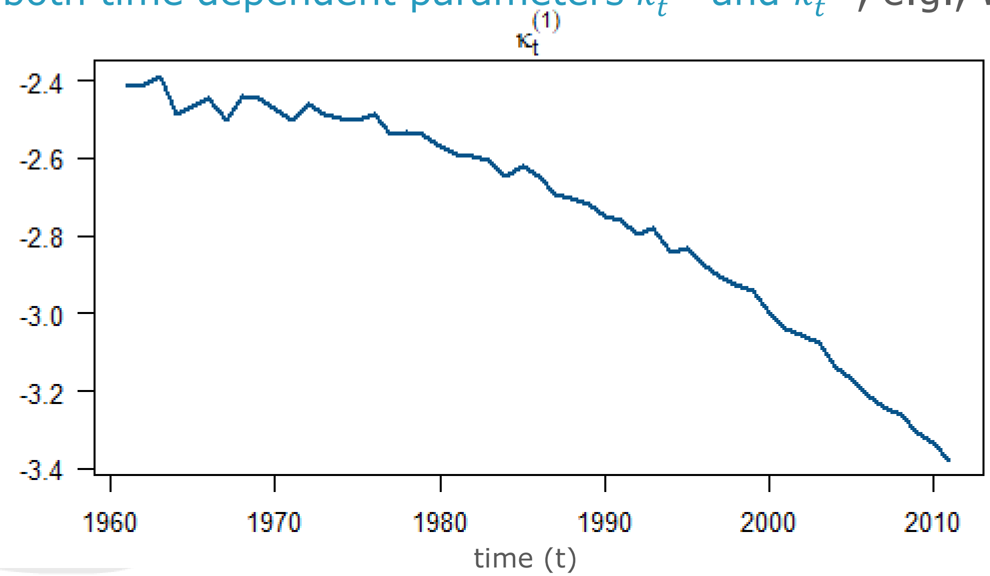
# Stochastic Mortality Modeling

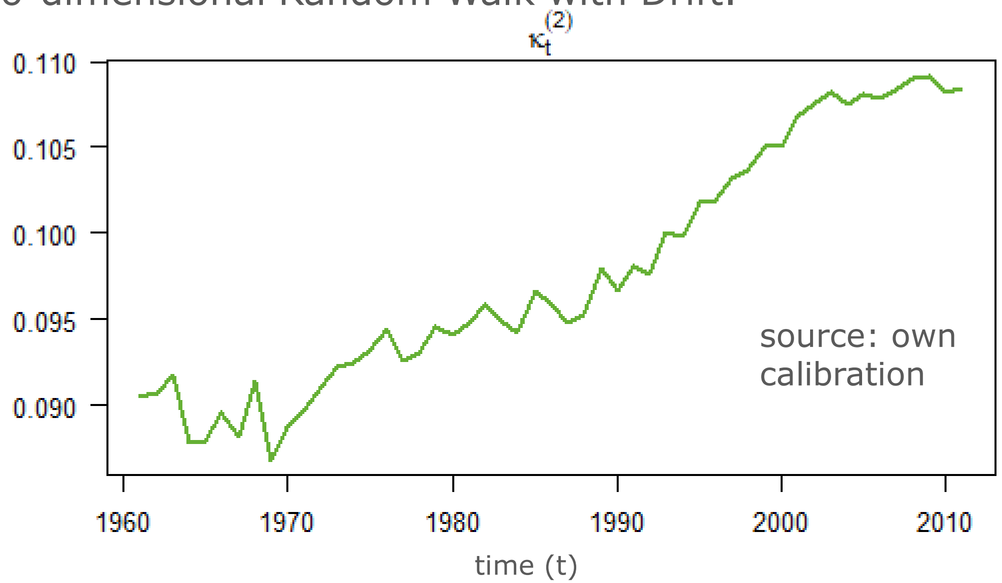
## Cairns-Blake-Dowd Model

Another widely used mortality model is the model of Cairns, Blake und Dowd (2006).

$$\operatorname{logit}(q_{x,t}) = \ln\left(\frac{q_{x,t}}{1-q_{x,t}}\right) = \kappa_t^{(1)} + \kappa_t^{(2)} \cdot (x - \bar{x})$$

- $\boldsymbol{\kappa_t^{(1)}}$ describes the general level of mortality over time.
- $\boldsymbol{\kappa_t^{(2)}}$ describes the evolution of the slope of the mortality curve over time for different ages x.
- $\bar{x}$ is the median age in the considered age span.
- Again, stochastic scenarios of the future evolution of mortality can be generated by a simulation of the further evolution of both time dependent parameters $\kappa_t^{(1)}$ and $\kappa_t^{(2)}$, e.g., with a two-dimensional Random Walk with Drift.



# Stochastic Profit Testing

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Motivation

Stochastic Mortality Models for Life Insurance

**Case Study: Analysis of guarantees at annuitization**

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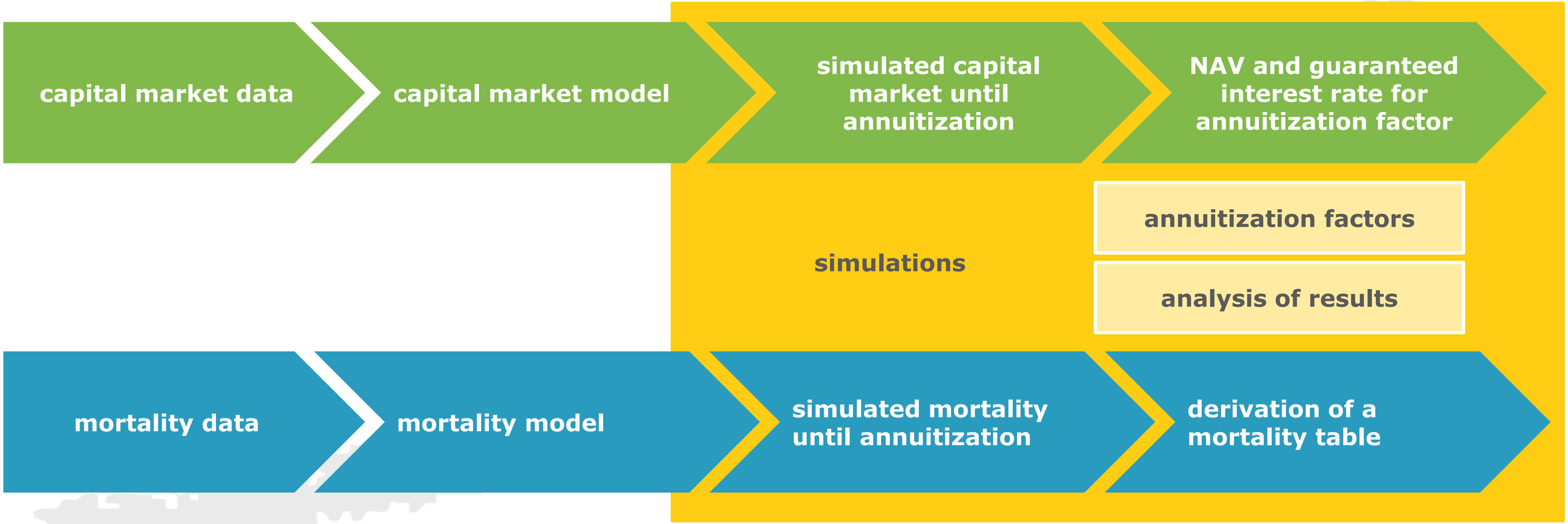
# Case Study: Analysis of guarantees at annuitization

## Overview

- The analysis of different guarantees requires a probability distribution of the (unknown) annuitization factor after the deferment period.
  - Depending on the (simulated) capital market and the (simulated) prevailing mortality, there is a different annuitization factor.



# Case Study: Analysis of guarantees at annuitization

## Wrapup: assumptions and parameters

| | |
|---|---|
| **capital markets** | Black-Scholes for assets, Shadow-Rate interest rate model |
| | independence between capital market and mortality |
| | guaranteed interest rate based on returns of 10-year government bonds |
| **mortality data** | data for the total population; adjusted to fit insureds mortality |
| **mortality model** | LC and CBD model |
| **simulated mortality** | (multidimensional) Random Walk with drift; Simulation of realized mortality numbers with Binomial / Poisson distr. see Börger, Ruß und Schupp (2020) |
| **mortality table** | similar approach as proposed by the DAV |

loss for guarantee **GAO**:

$$L_T^{\mathrm{GA}} = g \cdot A_T \cdot \max\{\frac{1}{RF_T} - \frac{1}{g}, 0\}$$

loss for guarantee **GAOWL**:

$$L_T^{\mathrm{GAWL}} = g \cdot \min\{A_T; L\} \cdot \max\{\frac{1}{RF_T} - \frac{1}{g}, 0\}$$

loss for guarantee **GMIB**:

$$L_T^{GMIB} = \max\left\{g \cdot G \cdot \frac{1}{RF_T} - A_T, 0\right\}$$



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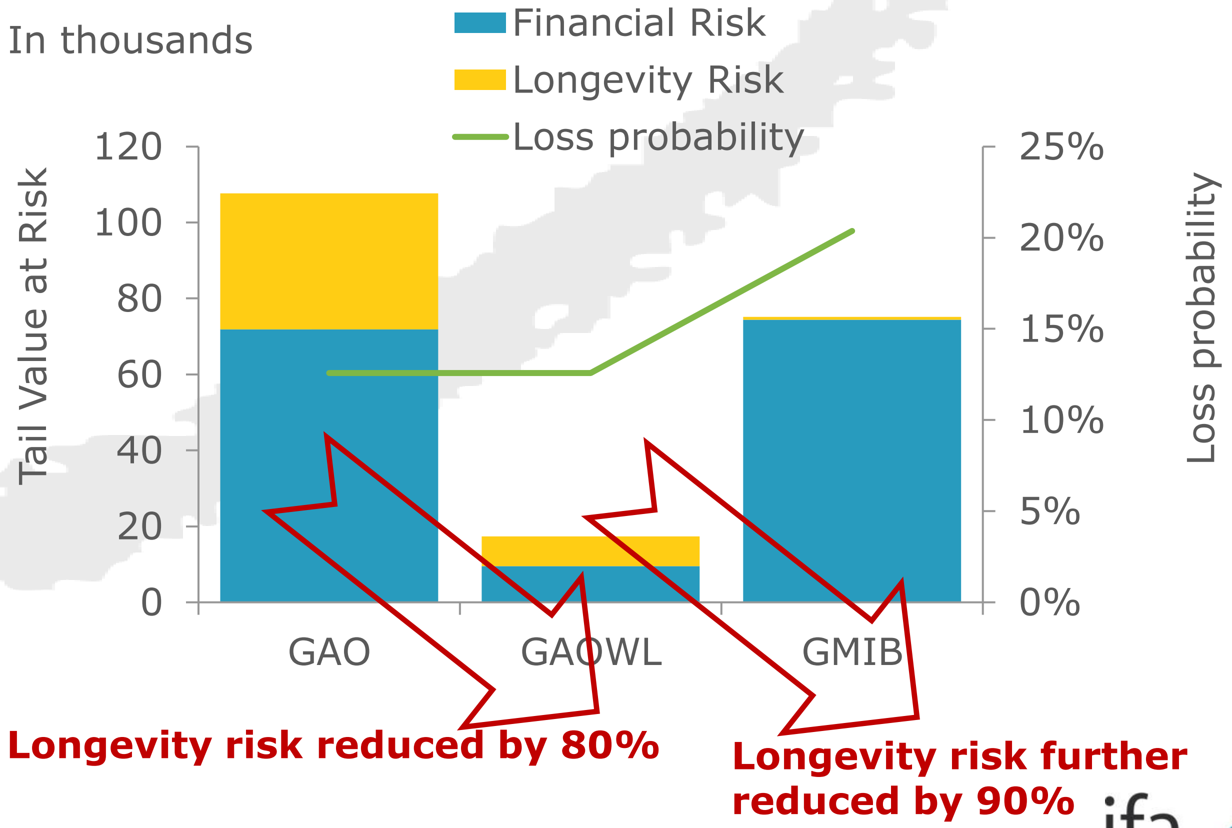
# Case Study: Analysis of guarantees at annuitization

## Results

Comparison for the insurers risk for the three different guarantees

| | GAO | GAOWL | GMIB |
|---|---|---|---|
| Loss probability | 12,6% | 12,6% | 20,4% |
| Expected loss | 2.140 | 540 | 4.990 |
| VaR (99,5%) | 78.900 | 15.600 | 71.840 |
| TVaR (99%) | 107.680 | 17.360 | 75.110 |
| Proportion (TVaR) **longevity risk** | 33,3% | 45,4% | 0,9% |
| Proportion (TVaR) **financial risk** | 66,7% | 54,6% | 99,1% |

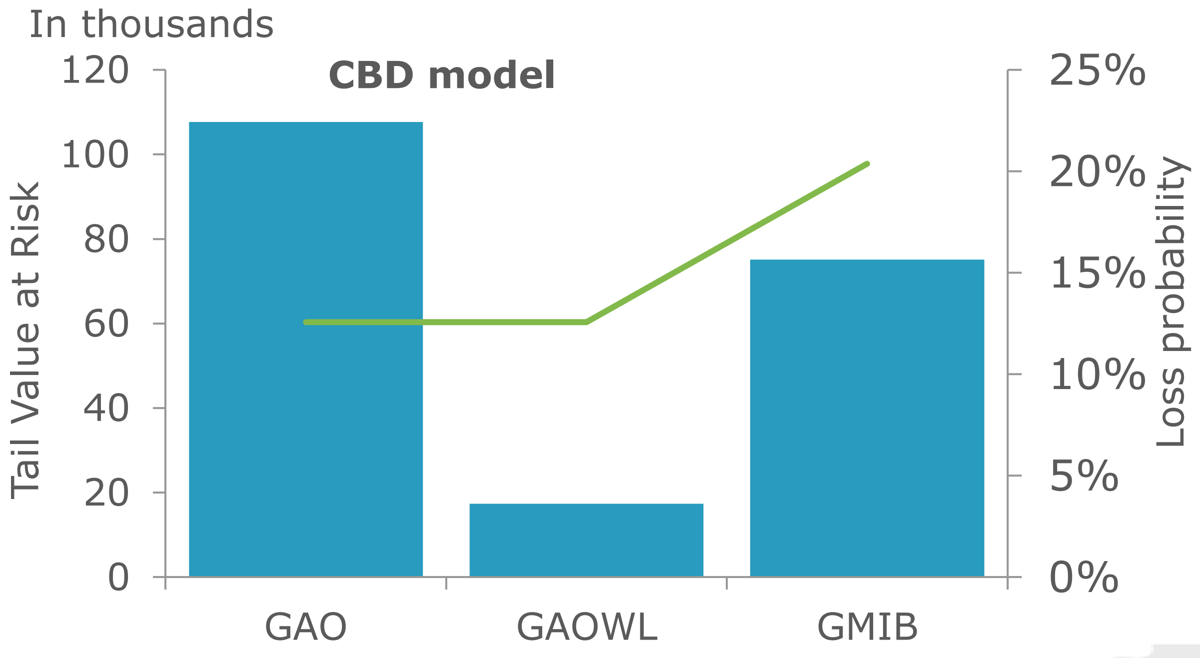
decomposition of the TVaR in financial and longevity risk

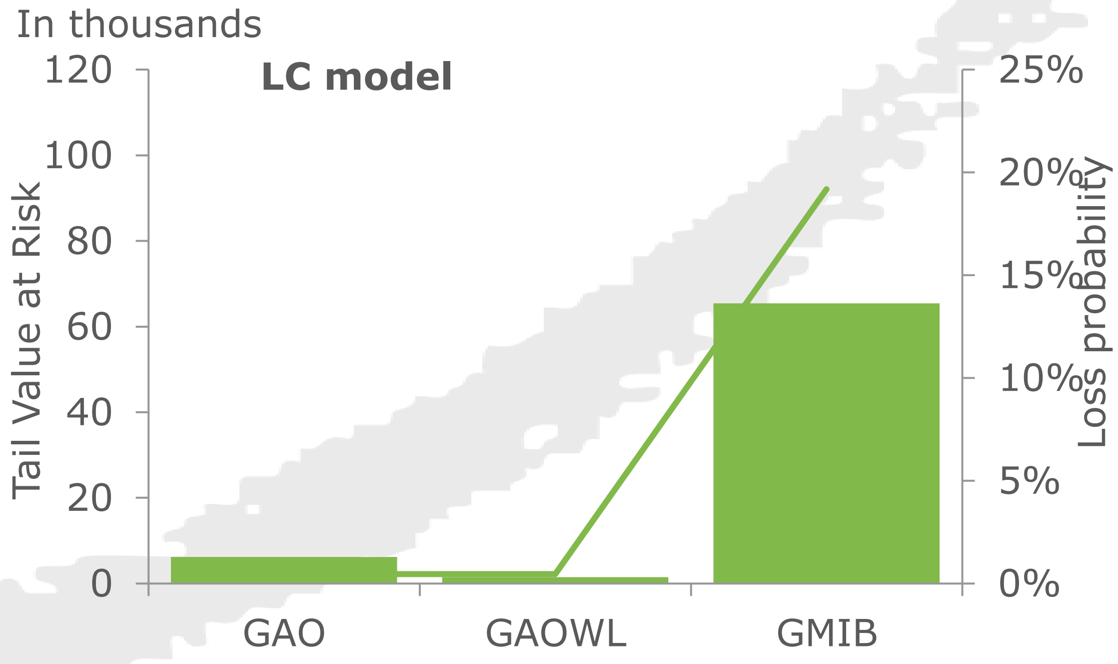


# Case Study: Analysis of guarantees at annuitization

## Results

Comparison for the insurers risk for the three different guarantees analysed with the **CBD model** and the **LC model**




- GMIB only has financial risk that can be handled by product design or hedging
- Choice of the mortality model is crucial for a thorough understanding of the insurers risk.

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# Stochastic Profit Testing

## Summary

**Longevity** and **mortality risk** are key risk drivers of life insurance business

- increasing importance of retirement products and higher retirement rates of deferred annuities
- We observe a variety of new retirement products and guarantees

A **sound risk management** requires to properly **quantify** and **model** the risk of products and guarantees.

- **This is only possible with a stochastic mortality model.**
- We have seen, that the choice of the mortality model is crucial for a thorough understanding of the insurers risk.
  - A deep understanding of the mortality models is necessary to identify a suitable model.

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# Contact

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