The role of inflation in retirement planning

Why reducing nominal risk can increase real risk

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- Convention A Europe 2024
- March 2024
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Research question

Starting point

- We analyse old age provision (savings) products and especially focus on products with embedded investment guarantees.
- Generally accepted “common sense”
  - Guarantees reduce the product’s return potential (e.g. measured by the expected return).
  - Guarantees reduce the product’s risk (e.g. measured by some risk measure).

Sneak peek of results

The common sense holds true, when nominal returns are considered.

But what about inflation-adjusted (“real”) returns?
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Requirements for the model

- We will consider different 
  types of portfolio 
  insurance strategies which provide a guarantee 
  at maturity.

- These strategies (at least) require 
  stochastic 
  modelling of interest rates and equity returns.

Further, we focus on the risk arising from future 
inflation and hence require stochastic modelling of 
inflation.

- Additionally, there is “one more thing” that 
  needs to be considered.

➔ Equity returns and inflation empirically show a 
  “positive correlation” over the long run.

Intuitive explanation

- Consider a listed company: 
  The higher (lower) inflation realizes, the higher 
  (lower) nominal profits, dividend payments and 
  hence equity returns tend to be.

Some research underpinning this intuitive 
explanation, e.g.

- Boudoukh and Richardson (1993)
- Lothian and McCarthy (2001)

Consequences of this positive correlation for long term savings processes

- opposing effects on the risk of long-term savings products when the equity ratio is increased
  - volatility risk increases (under nominal and real terms)
  - inflation risk decreases (only visible under real terms)
Modelling approach

Capital market model ("cascade approach")

1st cascade: inflation \(i(t)\) and real interest rate \(r(t)\)
- \(i(t)\) follows a Vasiçek (1977)-model.
- \(r(t)\) follows a so-called "G2++-model" (cf. Brigo and Mercurio, 2006).

2nd cascade: nominal interest rate \(r(t)\)
- Set \(r(t) := i(t) + r(t)\) and derive

\[ CPI(t) = e^{\int_0^t i(s) ds}. \]

3rd cascade: modelling of equity
- Assume a generalized Black-Scholes-model with drift \(\mu(t) = r(t) + \lambda_S\).

a bit more detailed view ...
- Let \(S(t)\) denote the spot price of the underlying equity process at time \(t\).

We then obtain

\[ S(t) = \exp\left(\int_0^t i(s) ds + \int_0^t r(s) ds + (\lambda_S - 0.5\sigma_S^2)t + \sigma_S W_S(t)\right) \]

and hence the model implies a (long-term) correlation between equity returns and inflation,
- especially depending on the volatility \(\sigma_S\) of the underlying equity process.
Modelling approach

Model-implied correlation of equity returns and inflation over an investment horizon of 30 years for different volatilities of the underlying equity investment and different instantaneous correlation coefficients
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Results: Utility maximization

Maximization of expected utility (for a “Merton-like” investment problem)

- Consider a CRRA utility function, i.e. $u_\gamma(x) = \frac{x^{1-\gamma}}{1-\gamma}$ ($\gamma \neq 1$) and $u_1(x) = \log(x)$ in what follows.

Problem

- **Investment strategy**: continuous rebalancing between bank account and an equity investment, i.e.
  
  $$dA_\alpha(t) = A_\alpha(t)(r(t) + \alpha \lambda_S) dt + \alpha \sigma_S dW_S$$

- For fixed $\gamma$, derive the equity quota $\alpha$ which maximizes the expected utility after taking inflation into account, i.e.
  
  $$\arg\max_\alpha \mathbb{E} \left[ u_\gamma \left( \frac{A_\alpha(t)}{CPI(t)} \right) \right]$$

Solution

- We obtain the optimal equity quota as
  
  $$\alpha = \frac{\lambda_S}{\gamma \sigma_S^2}.$$ 

  ➔ Same solution as for the “nominal” problem, why?

- Bank account and equity only differ by their volatility, not by their exposure to inflation.

  ➔ Be careful with the bank account in this model.
Now, we analyse products with embedded investment guarantees.

- We consider CPPI-like products (cf. Black and Perold, 1992) and assume different multipliers $m$ (static $m = 1$, dynamic $m = 5$) for different levels of guarantee, i.e. 50%, ..., 100%.

- Instead of expected utility, we focus on the expected return and the return derived from a conditional tail expectation.

In sharp contrast to the nominal view, **investment guarantees reduce expected return and may increase risk** (up to a certain point) if **inflation is taken into account**.
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Conclusion

- We introduced a stochastic model, especially taking the long-term “correlation” between inflation and equity into account.
- Based on this, we showed that products with “lower” (nominal) investment guarantees can be suitable even for risk-averse investors, when inflation-adjusted returns are considered.

Outlook

- analyses of other portfolio insurance strategies (esp. option-based portfolio insurance, OBPI)
- What about the decumulation phase?

In case you want to dive deeper

- The (working) paper will be available via https://www.ifa-ulm.de/inflation_in_retirement_planning.
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References


