Institut für Finanz- und Aktuarwissenschaften ifa

The role of inflation in retirement planning

Why reducing nominal risk can increase real risk

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Motivation and research question

Modelling approach

Results

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Contact details

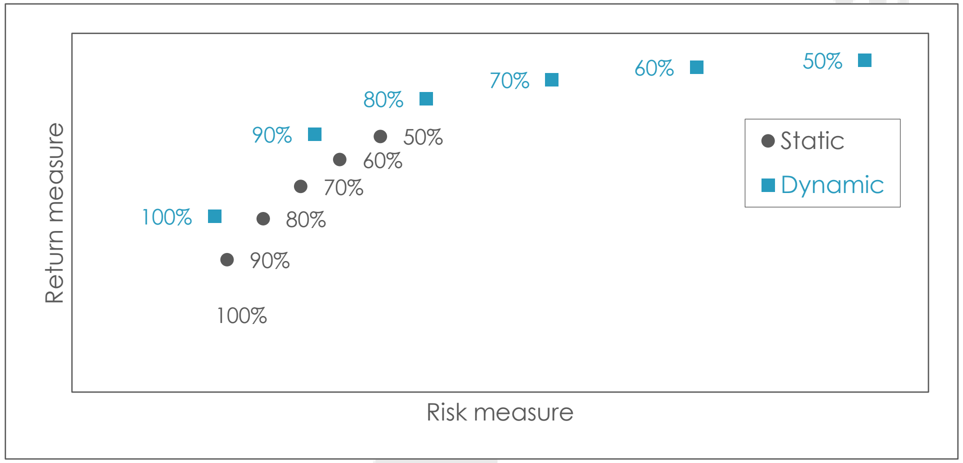
Motivation and research question

Research question

Starting point

- We analyse old age provision (savings) products and especially focus on products with embedded investment guarantees.
- generally accepted "common sense"
  - Guarantees reduce the product‘s return potential (e.g. measured by the expected return).
  - Guarantees reduce the product‘s risk (e.g. measured by some risk measure).

Sneak peek of results



The common sense holds true, when nominal returns are considered.

But what about **inflation-adjusted ("real") returns**?

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Modelling approach

Requirements for the model

- We will consider different types of portfolio insurance strategies which provide a guarantee at maturity.
- These strategies (at least) require stochastic modelling of interest rates and equity returns.

Further, we focus on the risk arising from future inflation and hence require stochastic modelling of inflation.

- Additionally, there is "one more thing" that needs to be considered.

➔ Equity returns and inflation empirically show **a "positive correlation" over the long run**.

Intuitive explanation

- Consider a listed company:
  The higher (lower) inflation realizes, the higher (lower) nominal profits, dividend payments and hence equity returns tend to be.

Some research underpinning this intuitive explanation, e.g.

- Boudoukh and Richardson (1993)
- Lothian and McCarthy (2001)

Consequences of this positive correlation for long term savings processes

- opposing effects on the risk of long-term savings products when the equity ratio is increased
  - **volatility risk increases** (under nominal and real terms)
  - **inflation risk decreases** (only visible under real terms)

Modelling approach

Capital market model ("cascade approach")

- 1st cascade: inflation $i(t)$ and real interest rate $r_r(t)$
  - $i(t)$ follows a Vasiçek (1977)-model.
  - $r_r(t)$ follows a so-called "G2++-model" (cf. Brigo and Mercurio, 2006).
- 2nd cascade: nominal interest rate $r(t)$
  - Set $r(t) \coloneqq i(t) + r_r(t)$ and derive
  $$CPI(t) = e^{\int_0^t i(s)ds}.$$
- 3rd cascade: modelling of equity
  - Assume a generalized Black-Scholes-model with drift $\mu(t) = r(t) + \lambda_S$.

a bit more detailed view …

- Let $S(t)$ denote the spot price of the underlying equity process at time t.

We then obtain

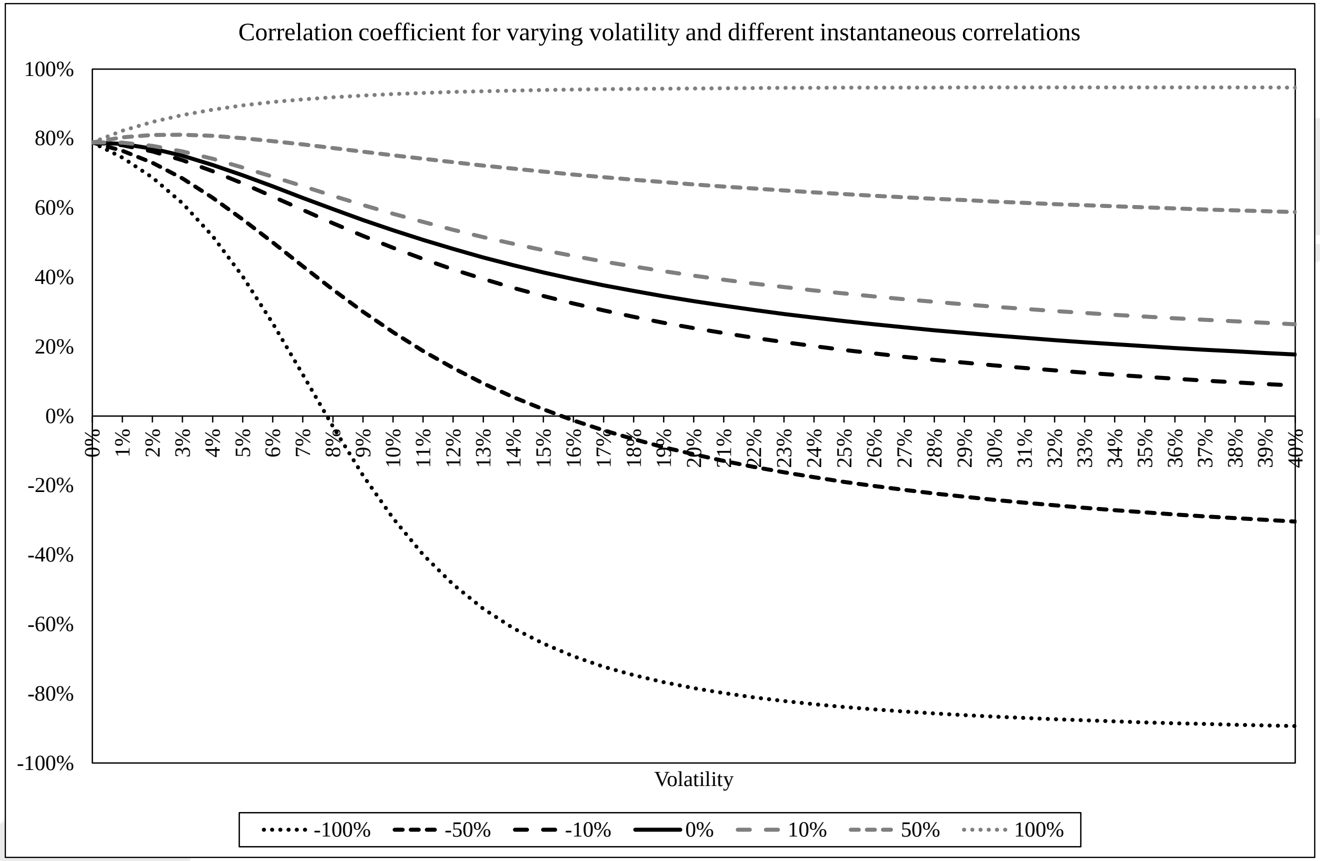
$$S(t) = \exp\left(\int_0^t i(s)ds + \int_0^t r_r(s)ds + \left(\lambda_S - 0.5\sigma_S^2\right)t + \sigma_S W_S(t)\right)$$

and hence the model implies a (long-term) **correlation between equity returns and inflation,**

- especially depending on the volatility σ_S of the underlying equity process.

Modelling approach

Model-implied correlation of equity returns and inflation over an investment horizon of 30 years for different volatilities of the underlying equity investment and different instantaneous correlation coefficients



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Results: Utility maximization

Maximization of expected utility (for a "Merton-like" investment problem)

- Consider a CRRA utility function, i.e. $u_\gamma(x) = \frac{x^{1-\gamma}}{1-\gamma}$ $(\gamma \neq 1)$ and $u_1(x) = \log(x)$ in what follows.

Problem

- Investment strategy: continuous rebalancing between bank account and an equity investment, i.e.

$$dA_\alpha(t) = A_\alpha(t)\big((r(t) + \alpha\lambda_S)dt + \alpha\sigma_S dW_S\big)$$

- For fixed γ, derive the equity quota α which maximizes the expected utility after taking inflation into account, i.e.

$$\underset{\alpha}{\operatorname{argmax}}\, \mathbb{E}\left[u_\gamma\left(\frac{A_\alpha(t)}{CPI(t)}\right)\right]$$

Solution

- We obtain the optimal equity quota as

$$\alpha = \frac{\lambda_S}{\gamma\sigma_S^2}.$$

➔ **Same solution as for the "nominal" problem, why?**

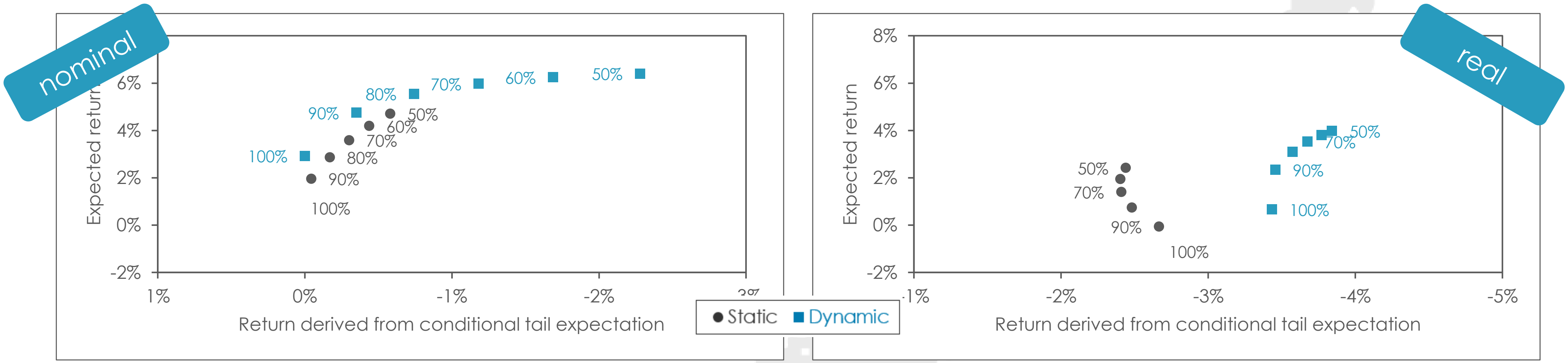
- Bank account and equity only differ by their volatility, not by their exposure to inflation.

➔ Be careful with the bank account in this model.

Results: Products with investment guarantees

Now, we analyse products with embedded investment guarantees.

- We consider CPPI-like products (cf. Black and Perold, 1992) and assume different multipliers m (static ➔ $m = 1$, dynamic ➔ $m = 5$) for different levels of guarantee, i.e. $50\%, \dots, 100\%$.
- Instead of expected utility, we focus on the expected return and the return derived from a conditional tail expectation.



In sharp contrast to the nominal view, **investment guarantees reduce expected return and may increase risk** (up to a certain point) if **inflation is taken into account**.

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Conclusion

- We introduced a stochastic model, especially taking the long-term “correlation” between inflation and equity into account.
- Based on this, we showed that products with “lower” (nominal) investment guarantees can be suitable even for risk-averse investors, when inflation-adjusted returns are considered.

Outlook

- analyses of other portfolio insurance strategies (esp. option-based portfolio insurance, OBPI)
- What about the decumulation phase?

In case you want to dive deeper

- The (working) paper will be available via https://www.ifa-ulm.de/inflation_in_retirement_planning.

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References

Black, F. and Perold, F. (1993). Theory of constant proportion portfolio insurance. *Journal of Economic Dynamics and Control*, 16(3-4): 403-426.

Boudoukh, J. and Richardson, M. (1993). Stock Returns and Inflation: A Long-Horizon Perspective. *The American Economic Review*, 83(5): 1346-1355.

Brigo, D. and Mercurio, F. (2006). Interest Rate Models – Theory and Practice. *Springer Finance.*

Lothian, J.R. and McCarthy, C. H. (2001). Equity Returns and Inflation: The Puzzlingly Long Lags. *Research in Finance and Banking*, 2: 149-166.

Vasiçek, O. (1977). An equilibrium characterization of the term structure. *Journal of Financial Economics*, 5(2): 177-188.